5.12 a) The node $v(n)$ must be scaled since it is input (after the delay element) to the multiplier with non-integer coefficient. Insert a scale coefficient, c, in front of the filter. The impulse response from the input to the node is:

$$h_v(n) = \begin{cases} 0 & , n < 0 \\ c\, b^n & , n \geq 0 \end{cases}$$

We get: $S_v = \sum_{n=0}^{\infty} h_v(n)^2 = \sum_{n=0}^{\infty} c^2\, b^{2n} = \dfrac{c^2}{1 - b^2} = 25.2525c^2$

Now, let $S_v = 1 \Rightarrow c = \dfrac{1}{\sqrt{25.2525}} = 0.198997$

The impulse response from the input to the output is

$$h(n) = a_0\, h_v(n) + a_1 h_v(n-1) = \begin{cases} 0 & , n < 0 \\ a_0\, c & , n = 0 \\ a_0\, c\, b^n + a_1\, c\, b^{n-1} & , n \geq 1 \end{cases}$$

$$S_y = \sum_{n=0}^{\infty} h(n)^2 = (a_0 c)^2 + \sum_{n=1}^{\infty} (a_0 c)^2\, [b^n - b^{n-1}]^2 =$$

$$= (a_0 c)^2 [1 + \frac{(b-1)^2}{b^2} \sum_{n=1}^{\infty} b^{2n}] = (a_0 c)^2 [1 + \frac{(b-1)^2}{b^2} \frac{b^2}{1 - b^2})] =$$

$$= (a_0 c)^2 [1 + \frac{1-b}{1+b}] = a_0^2\, 0.02$$

Let $S_y = 1 \Rightarrow a_0 = \dfrac{1}{\sqrt{0.02}} = 7.07107$

b) All of the coefficients are non-integers. Hence, there are two noise sources to each "adder". The contribution from c and b is:

$$\sigma_{y1}^2 = 2\, \sigma_0^2 \sum_{n=0}^{\infty} h_{noise}(n)^2 = 2\, \sigma_0^2 \sum_{n=0}^{\infty} (\frac{h(n)}{c})^2 =$$

$$\sigma_{y1}^2 = \frac{2\, \sigma_0^2}{c^2} \sum_{n=0}^{\infty} h(n)^2 = \frac{2\, \sigma_0^2}{c^2}$$

Since the filter is scaled.

$$\sigma_{y1}^2 = \frac{2\, \sigma_0^2}{c^2} = 100.5026 \frac{Q^2}{12}$$

The contribution from σ_0 and σ_1 is: $\sigma_{y2}^2 = 2 \dfrac{Q^2}{12}$

$$\sigma_y^2 = \sigma_{y1}^2 + \sigma_{y2}^2 = 102.5026 \frac{Q^2}{12} \quad \text{where } Q = 2^{-7}$$