3.3(a) $x(n-n_0) \leftrightarrow z^{-n_0}X(z)$

$$Z\{x(n-n_0)\} = \sum_{n=-\infty}^{\infty} x(n-n_0)z^{-n} = \sum_{n=-\infty}^{\infty} x(n-n_0)z^{-(n-n_0)-n_0}$$

$$= \sum_{n=-\infty}^{\infty} x(n-n_0)z^{-(n-n_0)}z^{-n_0} = z^{-n_0}\sum_{n=-\infty}^{\infty} x(n-n_0)z^{-(n-n_0)} = z^{-n_0}X(z)$$

(b) $a^n x(n) \leftrightarrow X\left(\frac{z}{a}\right)$

$$Z\{a^n x(n)\} = \sum_{n=-\infty}^{\infty} a^n x(n)z^{-n} = \sum_{n=-\infty}^{\infty} x(n)\left(\frac{z}{a}\right)^{-n} = X\left(\frac{z}{a}\right)$$

(c) $x(-n) \leftrightarrow X\left(\frac{1}{z}\right)$

$$Z\{x(-n)\} = \sum_{n=-\infty}^{\infty} x(-n)z^{-n} = \sum_{m=-\infty}^{\infty} x(m)z^{m} = \sum_{m=-\infty}^{\infty} x(m)\left(\frac{1}{z}\right)^{-m} = X\left(\frac{1}{z}\right)$$

(d) $x^*(n) \leftrightarrow X^*(z^*)$

$$Z\{x^*(n)\} = \sum_{n=-\infty}^{\infty} x^*(n)z^{-n} = \sum_{n=-\infty}^{\infty} x^*(n)((z^{-n})^*) = \sum_{n=-\infty}^{\infty} x^*(n)(z^{-n*})^*$$

$$= \sum_{n=-\infty}^{\infty} [x(n)(z^*)^{-n}]^* = X^*(z^*)$$

Note: consider a complex number $z = ae^{i\theta}$, where $a = |z|$ and θ is the argument,
$(z^n)^* = (a^n e^{in\theta})^* = (a^n e^{-in\theta})^* = (ae^{-i\theta})^n = (z^*)^n$

(e) $Re\{x(n)\} \leftrightarrow 0.5[X(z) + X^*(z^*)]$

$Re\{x(n)\} = 0.5[X(n) + x^*(n)]$

$$Z\{Re\{x(n)\}\} = \sum_{n=-\infty}^{\infty} 0.5[x(n) + x^*(n)]z^{-n} = 0.5\sum_{n=-\infty}^{\infty} [x(n)z^{-n} + x^*(n)z^{-n}]$$

$$= 0.5\left(\sum_{n=-\infty}^{\infty} x(n)z^{-n} + \sum_{n=-\infty}^{\infty} x^*(n)z^{-n}\right) = 0.5[X(z) + X^*(z^*)]$$

(f) $Im\{x(n)\} \leftrightarrow -0.5j[X(z) - X^*(z^*)]$

$Im\{x(n)\} = -0.5j[x(n) - x^*(n)]$, in the same manner as (e), we have

$Z\{Im\{x(n)\}\} = -0.5j[X(z) - X^*(z^*)]$.

Alternatively, we can derive it from (e), since $x(n) = Re\{x(n)\} + j \cdot Im\{x(n)\}$, and $Z\{x(n)\} = X(z)$, $Z\{Rex(n)\}\} = 0.5[X(z)+X^*(z^*)]$, with the linear property of z-transform,

$$Z\{Im\{x(n)\}\} = \frac{1}{j}[Z\{x(n) - Re\{x(n)\}\}] = -0.5j[X(\mathrm{z}) - X^*(z^*)].$$