3.29 $X(k) = \sqrt{\frac{2}{N-1}} \sum_{n=0}^{N-1} c_k \, x(n) \, cos(\frac{\pi n k}{N-1}) \qquad k = 0, 1, .. N-1$

where $c_k = \begin{cases} \frac{1}{2} & \text{for } k = 0 \text{ or } k = N-1 \\ 1 & \text{for } k = 1, 2, \ldots, N-2 \end{cases}$

We can write the DCT in matrix form: $\mathbb{X}(k) = \sqrt{\frac{2}{N-1}} \; \mathbb{C} \; \mathbb{x}(n)$

The rows of $\mathbb{C}$ is referred to as the basis vectors. Desirable properties for the DCT are:

1. The same expression for the DCT transform and the inverse transform. Hence, only one algorithm need to be implemented.
2. The application requires the DC component do not leak into other frequency components. This is due to the high DC content in images. We can view the DCT as a set of FIR filters. Hence, the filers must have a zero at $z = 1$, except for the first lowpass filter.
3. The basis vectors should be symmetric or antisymmetric. This simplifies and reduces the hardware implementation.
4. The transform should be orthogonal in order to efficiently decorrelate the images.

All of this properties cannot be satisfied simultaneously. We therefore relaxes the orthogonality requirement slightly. The matrix $\mathbb{C}$ is

$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\cos(0)$	$\cos(\frac{\pi}{7})$	$\cos(\frac{2\pi}{7})$	$\cos(\frac{3\pi}{7})$	$\cos(\frac{4\pi}{7})$	$\cos(\frac{5\pi}{7})$	$\cos(\frac{6\pi}{7})$	-1
$\cos(0)$	$\cos(\frac{2\pi}{7})$	$\cos(\frac{4\pi}{7})$	$\cos(\frac{6\pi}{7})$	$\cos(\frac{8\pi}{7})$	$\cos(\frac{10\pi}{7})$	$\cos(\frac{12\pi}{7})$	1
$\cos(0)$	$\cos(\frac{3\pi}{7})$	$\cos(\frac{6\pi}{7})$	$\cos(\frac{9\pi}{7})$	$\cos(\frac{12\pi}{7})$	$\cos(\frac{15\pi}{7})$	$\cos(\frac{18\pi}{7})$	-1
$\cos(0)$	$\cos(\frac{4\pi}{7})$	$\cos(\frac{8\pi}{7})$	$\cos(\frac{12\pi}{7})$	$\cos(\frac{16\pi}{7})$	$\cos(\frac{20\pi}{7})$	$\cos(\frac{24\pi}{7})$	1
$\cos(0)$	$\cos(\frac{5\pi}{7})$	$\cos(\frac{10\pi}{7})$	$\cos(\frac{15\pi}{7})$	$\cos(\frac{20\pi}{7})$	$\cos(\frac{25\pi}{7})$	$\cos(\frac{30\pi}{7})$	-1
$\cos(0)$	$\cos(\frac{6\pi}{7})$	$\cos(\frac{12\pi}{7})$	$\cos(\frac{18\pi}{7})$	$\cos(\frac{24\pi}{7})$	$\cos(\frac{30\pi}{7})$	$\cos(\frac{36\pi}{7})$	-1
$\frac{1}{2}$	$\frac{-1}{2}$	$\frac{1}{2}$	$\frac{-1}{2}$	$\frac{1}{2}$	$\frac{-1}{2}$	$\frac{1}{2}$	$\frac{-1}{2}$

We get after simplification

$$\begin{matrix}
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
1 & \cos(\frac{\pi}{7}) & \cos(\frac{2\pi}{7}) & \cos(\frac{3\pi}{7}) & -\cos(\frac{3\pi}{7}) & -\cos(\frac{2\pi}{7}) & -\cos(\frac{\pi}{7}) & -1 \\
1 & \cos(\frac{2\pi}{7}) & \cos(\frac{4\pi}{7}) & \cos(\frac{6\pi}{7}) & \cos(\frac{6\pi}{7}) & \cos(\frac{4\pi}{7}) & \cos(\frac{2\pi}{7}) & 1 \\
1 & \cos(\frac{3\pi}{7}) & \cos(\frac{6\pi}{7}) & \cos(\frac{9\pi}{7}) & -\cos(\frac{9\pi}{7}) & -\cos(\frac{6\pi}{7}) & -\cos(\frac{3\pi}{7}) & -1 \\
1 & \cos(\frac{4\pi}{7}) & \cos(\frac{8\pi}{7}) & \cos(\frac{12\pi}{7}) & \cos(\frac{12\pi}{7}) & \cos(\frac{8\pi}{7}) & \cos(\frac{4\pi}{7}) & 1 \\
1 & -\cos(\frac{5\pi}{7}) & \cos(\frac{10\pi}{7}) & \cos(\frac{15\pi}{7}) & -\cos(\frac{15\pi}{7}) & -\cos(\frac{10\pi}{7}) & -\cos(\frac{5\pi}{7}) & -1 \\
1 & -\cos(\frac{6\pi}{7}) & \cos(\frac{12\pi}{7}) & \cos(\frac{18\pi}{7}) & -\cos(\frac{18\pi}{7}) & -\cos(\frac{12\pi}{7}) & -\cos(\frac{6\pi}{7}) & -1 \\
\frac{1}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{2}
\end{matrix}$$