3.27 We form a new, complex-valued sequence from the two real-valued sequences.

$z(n) = x(n) + j\,y(n)$

The DFT is $Z(k) = \sum_{n=0}^{N-1} z(n)\, W^{nk} = \sum_{n=0}^{N-1} z(n)\, e^{-j2\pi nk/N}$

Now, we compute the complex conjugate of the rotated $Z(k)$ values

$$Z^*(N-k) = \sum_{n=0}^{N-1} [x(n) - j\,y(n)]\, e^{j2\pi n(N-k)/N}$$

but

$$e^{-j2\pi n(N-k)/N} = e^{-j2\pi n}\, e^{-j2\pi nk/N} = e^{-j2\pi nk/N}$$

Hence, we get $Z^*(N-k) = \sum_{n=0}^{N-1} [x(n) - j\,y(n)]\, e^{-j2\pi nk/N}$

Adding these two expressions we get $Z(k) + Z^*(N-k) =$

$$= \sum_{n=0}^{N-1} [x(n) + j\,y(n)]\, e^{-j2\pi nk/N} + \sum_{n=0}^{N-1} [x(n) - j\,y(n)]\, e^{-j2\pi nk/N}$$

=

$$= \sum_{n=0}^{N-1} 2x(n)\, e^{-j2\pi nk/N}$$

Hence, $Z(k) + Z^*(N-k) = 2X(k)$ and $X(k) = 0.5[Z(k) + Z^*(N-k)]$

Similarly, subtracting the two expressions we get

$$Y(k) = -0.5\, j[Z(k) - Z^*(N-k)]$$

Hence, the DFTs of two real-valued sequences can be computed simultaneously without any significant additional cost. The inverse DFT can be computed in the same way.