FIR FILTERS

$$H(z) = \sum_{n=0}^{N} h(n)z^{-n} = \frac{h(0)z^{N} + h(1)z^{N-1} + \ldots + h(N-1)z + h(N)}{z^{N}}$$

LINEAR-PHASE FIR FILTERS

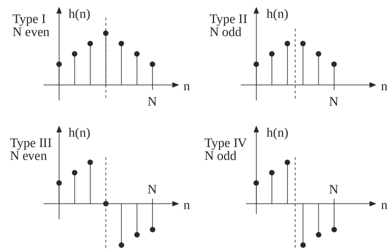
Linear-phase FIR filters exhibits either *symmetry* or *antisymmetry* around $n = N/2$, i.e.,

$$h(n) = h(N-n), \quad n = 0, 1, \ldots, N,$$

and for the *antisymmetic case*,

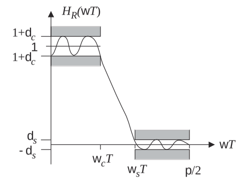
$$h(n) = -h(N-n), \quad n = 0, 1, \ldots, N$$

Type I: $h(n) = h(N-n)$, N even
Type II: $h(n) = h(N-n)$, N odd
Type III: $h(n) = -h(N-n)$, N even
Type IV: $h(n) = -h(N-n)$, N odd



Specification of Linear-Phase FIR Filters

Typically, an FIR specification is expressed in terms of the zero-phase function $H_R(\omega T)$ as shown in the figure for a linear-phase lowpass filter. The acceptable deviations are $\pm\delta_c$ and $\pm\delta_s$ in the passband and stopband, respectively.



Example

Synthesize a linear-phase FIR filter with $\omega_c T = 0.3\pi$, $\omega_s T = 0.6\pi$, $\delta_c = 0.02$, and $\delta_s = 0.0025$ (lowpass). Use McClellan-Parks-Rabiner's algorithm `remez.m`

```
% Example Linear-Phase, lowpass FIR
wT = [0.3 0.6]*pi;               % Band edges
bands = [1 0];                   % Gain in the bands
deltas = [0.02 0.0025];          % Acceptable deviations
fsample = 2*pi;
[N, Be, D, W] = remezord(wT, bands, deltas, fsample);
N                                % Estimated filter order

h = remez(N, Be, D, W);
points = 16;
w = linspace(0, 180, 180*points+1);% wT axis
H = freqz(h,1, w*pi/180);        % Transfer function
Mag = 20*log10(abs(H));          % Magnitude response [dB]
Attenu = -Mag;                   % Attenuation [dB]
Phase = angle(H)*180/pi;         % Phase response [deg]
gd = grpdelay(h, 1, w*pi/180);   % Group delay
```


In this case we get from `remezord`

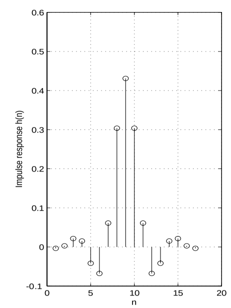
$N = 13$ $Be = [0\ 0.3\ 0.6\ 1]^T$ $D = [1\ 1\ 0\ 0]^T$ $W = [1\ 8]^T$

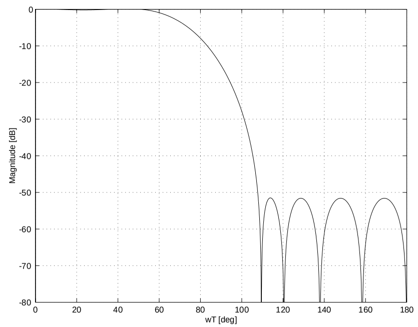
The filter order N that is obtained from `remezord` is only an estimation. It may therefore be necessary to try different filter orders in order to find a filter that satisfies the specification.

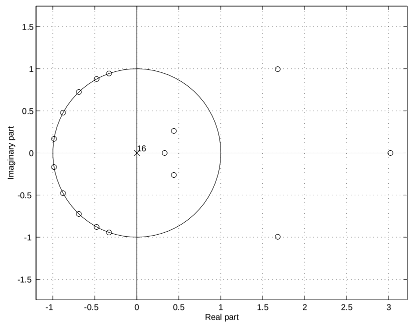
$N = 13 \Rightarrow A_{min} \approx 51.7$ dB and $A_{max} \approx 0.366$ dB
$N = 14 \Rightarrow A_{min} \approx 50.9$ dB and $A_{max} \approx 0.384$ dB
$N = 15 \Rightarrow$ neither acceptable
$N = 16 \Rightarrow A_{min} \approx 54.9$ dB and $A_{max} \approx 0.26$ dB.

The symmetric impulse response is

$h(0) = -0.003428964 = h(16)$
$h(1) = 0.002941935 = h(15)$
$h(2) = 0.021664496 = h(14)$
$h(3) = 0.015363454 = h(13)$
$h(4) = -0.041601459 = h(12)$
$h(5) = -0.067766346 = h(11)$
$h(6) = 0.061181730 = h(10)$
$h(7) = 0.303702325 = h(9)$
$h(8) = 0.430966079$







Half-Band FIR Filters

If the zero-phase function of an **even**-order lowpass, linear-phase FIR filter is antisymmetric with respect to $\pi/2$, i.e.,

$$H_R(e^{j\omega T}) = 1 - H_R(e^{j(\pi - \omega T)})$$

then every other coefficient in the impulse response is zero except for the one in the center, which is 0.5.

Delay-Complementary FIR Filters

A pair of filters, $H(z)$ and $H_c(z)$ are *delay-complementary* if

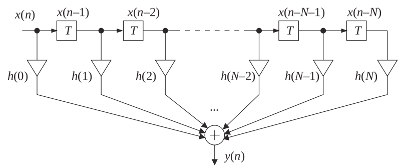
$$H(z) + H_c(z) = z^{-M}$$

where M is a nonnegative integer.

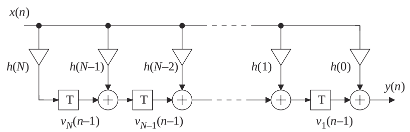
FIR Structures

$$y(n) = \sum_{k=0}^{N} h(k)x(n-k)$$

Direct form FIR structure



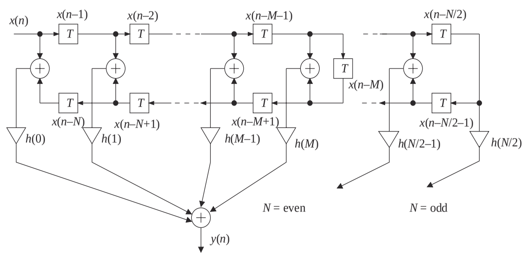
Transposed direct form FIR structure



Direct form linear-phase FIR structure

$2M = N+1$

$$y(n) = \sum_{k=0}^{\frac{N}{2}} h(k)[x(n-k) \pm x(n-N+k)]$$



Delay-Complementary FIR structure

N = even and $2M = N–1$