## Instructions

Describe your calculations clearly and detailed, explaining your methods, assumptions and equations used.
Total no of points $=70$. A pass on the exam requires approx. 30 points.

## Question 1



Figure 1
a) A three phase diode rectifier acoording to Figure 1 has inductance, $\mathrm{L}_{\mathrm{s}}$, on the ac-side-and $\mathrm{L}_{\mathrm{d}}$ on the dc-side. Which inductance shall be large in order to obtain continuous current through the dc-load.
$L_{d}$ shall be large to create continuous dc-side current with filtering of the ripple present in the voltage from the converter.
b) List the three most important parameters that defines the commutation of current between two diodes in a rectifier as of Figure 1. Give a short motivation why.
Us, Ls and Id are the decisive parameters for the commutation angle. Us is the voltage magnitude driving the commutation, giving shorter commutation for higher magnitude. Ls is the inductance to commutate and Id defines the magnitude of the current to commutate. Both Ls and Id gives longer commutation duration with increasing values.
c) In a DC/ DC converter, inductance is commonly used for energy transfer between low and high-voltage sides. If the average voltage across the inductance is greater than zero during a time interval, what can you say
about the shape of the inductor current during this interval?


Figure 2
The inductor current will be increasing if the average voltage across the inductor is $>0$.
d) What device parameters are required to determine the conduction losses of a MOSFET if the drain current is known?
The on-state resistance of the MOSFET.
e) List three types of semiconductors with turn-off capability.

MOSFET, IGBT, GTO, BJ T, IGCT has the capability of forced turnoff. Thyristors and diodes has the capability to go to off-state if the current is reduced to zero.

## Question 2



Figure 3
A three phase thyristor rectifier as shown by Figure 3 is connected to a three phase voltage source with the phase-phase voltage Us=410 Vrms. The commutation inductance can be neglected.
a) Draw the waveform of the converter dc-side voltage $\mathrm{U}_{\mathrm{v}}$ (before the inductor $\mathrm{L}_{\mathrm{d}}$ ) for a firing angle $\alpha=30$ deg.

b) Draw the waveform of the source current, $\mathrm{i}_{\text {sa }}$, in one phase.

c) Determine the displacement power factor.

## $D P F=\cos$ (angle between ac-voltage and current) $=\cos (\alpha)=\cos (30)$

## $=0.87$

d) Calculate the dc-load voltage, $\mathrm{U}_{\mathrm{d}}$, and dc-power considering a resistive load of 25 ohm.
$\mathrm{Ud}=1.35 * 410 * \cos (30)=479 \mathrm{~V}$

$$
\mathrm{Id}=479 / 25=19.2 \mathrm{~A}
$$

$$
\begin{equation*}
P d=479^{\wedge} 2 / 25=9.2 \mathrm{~kW} \tag{3}
\end{equation*}
$$

e) Calculate the fundamental frequency source rms current (isa). Assume zero losses of the thyristor converter.

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sqrt(3)*Us*Is1*DPF=Pd, => Is1=9200/410/0.87/sqrt(3)=15A,
Is1=0.78Id
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## Question 3



## Figure 4

In the buck converter in Figure 4, the current iL is continuous with an average of 5 A , and with a negligible ripple magnitude. The MOSFET T1 is operated with a switching frequency $\mathrm{f}_{\mathrm{sw}}=120 \mathrm{kHz}$ and a duty cycle in order to keep the capacitor voltage $U_{c}=10 \mathrm{~V}$ for an input voltage $U_{d}=24 \mathrm{~V}$.
a) Determine the duty cycle of the MOSFET T1. $D=10 / 24=0.42$
b) Calculate the conduction losses in the MOSFET T1 if the on-state resistance $\mathrm{R}_{\mathrm{ds}(\mathrm{on})}=0.05 \mathrm{ohm}$. Pc=D* $\mathbf{R}_{\mathrm{ds}(o n)} *_{\mathrm{i}_{\mathrm{L}}}{ }^{\wedge} 2=0.417 * 0.05 * 25=0.52 \mathrm{~W}$
c) Calculate the turn-on losses in the MOSFET T1 if the rise time of the drain current is 40 ns . Current rises linearly and voltage is constant.
Pon=fsw ${ }^{*}$ Eon $=f_{\text {sw }} * U_{d} * 0.5 t_{r} * \mathbf{i}_{\mathbf{L}}=0.29 \mathbf{W}$
d) Calculate the turn-off losses in the MOSFET T1 if the fall time of the
drain current is 60 ns . Current rises linearly and voltage is constant.
Poff $=\mathbf{f s w}^{*}$ Eoff $=\mathbf{f}_{\text {sw }} * \mathbf{U}_{\mathrm{d}} * \mathbf{0 . 5 t} \mathbf{t}^{*} \mathbf{i}_{\mathrm{L}}=\mathbf{0 . 4 3} \mathbf{~ W}$
e) Determine the maximum allowed thermal resistance of the heatsink
( $\mathrm{R}_{\mathrm{thHA}}$ ) for the MOSFET T1 in order to keep the heatsink temperature, $\mathrm{T}_{\mathrm{H}} \leq 60^{\circ} \mathrm{C}$ and thejunction temperature, $\mathrm{T}_{\mathrm{J}} \leq 100^{\circ} \mathrm{C}$. The MOSFET has a thermal resistance $\mathrm{R}_{\text {thj }}=45.0^{\circ} \mathrm{C} / \mathrm{W}$. The ambient temperature, $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$. Note: $\mathrm{T}_{\mathrm{H}}$ or $\mathrm{T}_{\mathrm{J}}$ will equal the given limits, the other shall be lower.
Ptot $=1.24 W$. RthHAmax $=(60-25) / 1.24=28{ }^{\circ} \mathrm{C} / \mathrm{W}$. RthJ $_{\text {taxax }}=(100-$ 25) $/ 1.24=60{ }^{\circ} \mathrm{C} / \mathrm{W} . R_{\text {thHAmax }}=60-45=15{ }^{\circ} \mathrm{C} / \mathrm{W}$.

Consequently, the heatsink must have $\mathrm{R}_{\mathrm{thHA}}<15{ }^{\circ} \mathrm{C} / \mathrm{W}$. With $R_{\text {thHA }}=15{ }^{\circ} \mathrm{C} / \mathrm{W}, \mathrm{T}_{\mathrm{H}}=25+1.24 * 15=44^{\circ} \mathrm{C}, \mathrm{T}_{\mathrm{J}}=44+1.24 * 45=99.4^{\circ} \mathrm{C}$.

## Question 4



Figure 5
A half-bridge voltage source converter is connected between a dc-source and an acload as shown by Figure 5. The control of the switched output voltage is done through pulse width modulation (PWM) with a switching frequency $\mathrm{f}_{\mathrm{sw}}=950 \mathrm{~Hz}$, in order to obtain a 50 Hz voltage component with a defined magnitude.
a) What is the minimum required dc-side voltage, $U_{d}$, required if the magnitude of the 50 Hz voltage component shall be 24 V rms when the amplitude modulation ratio, ma=0.9.
$U_{\mathrm{vpk}}=\mathrm{m}_{\mathrm{a}} * \mathrm{U}_{\mathrm{d}} / 2,=>\mathrm{Ud}=2 * \mathrm{U}_{\mathrm{vpk}} / \mathrm{m}_{\mathrm{a}}=2 *$ sqrt(2)*24/0.9=75.4 V
b) Calculate the current ripple in the output current, $\mathrm{I}_{\mathrm{v}}$, during the time interval shown in Figure 6. The time is defined based on the switching frequency cycle, $\mathrm{T}_{\mathrm{sw}}=1 / \mathrm{f}_{\text {sww }}$. During this time interval $\mathrm{U}_{\mathrm{ac}}=0.4^{*} \mathrm{U}_{\mathrm{d}}$, for the value of $U_{d}$ calculated in a). The inductance $L=3 \mathrm{mH}$. The initial current $\mathrm{I}_{\mathrm{v}}(\mathrm{t}=0)=0$.


Figure 6
The voltage drop across the inductance $L$ defines the current ripple. $\Delta i_{v}=\frac{u_{L}}{L} \Delta t$. The current ripple can be calculated from one time instant to the next, building the waveform. Starting from to we calculate the current at $\mathbf{t}_{1}$ as: $i_{v}\left(t_{1}\right)=i_{v}\left(t_{0}\right)+\Delta i_{v}\left(t_{1}-t_{0}\right)=i_{v}\left(t_{0}\right)+\frac{\left(u_{v}-u_{a c}\right)\left(t_{1}-t_{0}\right)}{L}$ The time instants are defined by Figure 6, where time is given relative the switching cycle $T_{\text {sw }}$. The time in seconds is calculated by the relative value divided by $f_{\text {sw. }}$. Discrete time and current values are given below the figure.



|  | t 0 | t 1 | t 2 | t 3 | t 4 | t 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| $\mathrm{t}[\mathrm{s}]$ | 0.0 | $0.37 \mathrm{e}-3$ | $0.68 \mathrm{e}-3$ | $1.42 \mathrm{e}-3$ | $1.74 \mathrm{e}-3$ | $2.10 \mathrm{e}-3$ |
| Iv [A] | 0.0 | 0.926 | -6.2 | -4.4 | -11.5 | -10.6 |

## Question 5

The half-bridge converter in Figure 5 has a parasitic inductance, $\mathrm{L}_{\mathrm{c}}$, between the dcsource and the half-bridge. Figure 7 the switching waveform of the current through the MOSFET switch T1. The current $\mathrm{I}_{\mathrm{v}}=12$ A flows through L out of the converter.


Figure 7
a) Draw the waveform of the voltage across MOSFET T1, related to the current given in Figure 7 and considering the inductance $\mathrm{L}_{\mathrm{c}}=40 \mathrm{nH}$. The dc-voltage $\mathrm{U}_{\mathrm{d}}=110 \mathrm{~V}$.
The voltage drop of the inductor at turn-on: Ulon $=\mathbf{L c}{ }^{*} \mathbf{d I t} 1 / \mathrm{dt}=$ $40 \mathrm{e}-9 * 12 /(20 \mathrm{e}-9)=24 \mathrm{~V}$.
The MOSFET voltage at turn-on: $\mathrm{U}_{\mathrm{t1}}=110-\mathrm{U}_{\mathrm{Lon}}=86 \mathrm{~V}$.
The voltage drop of the inductor at turn-off: $\mathrm{U}_{\text {Loff }}=$
$\mathrm{Lc}^{*} \mathrm{dIt} 1 / \mathrm{dt}=-40 \mathrm{e}-9 * 12 /(10 \mathrm{e}-9)=-48 \mathrm{~V}$.
The MOSFET voltage at turn-off:
$\mathrm{U}_{\mathrm{t} 1}=110-\mathrm{U}_{\mathrm{Loff}}=110+48=158 \mathrm{~V}$.

(4)
b) What is the peak voltage across the MOSFET?
$\mathrm{U}_{\text {tipk }}=$ Ud $-\mathrm{Lc}^{*}$ dIt $1 / \mathrm{dt}=110-40 \mathrm{e}-9 *(0-12) /(10 \mathrm{e}-9)=158 \mathrm{~V}$

## Question 6

A parallel capacitive snubber shall be designed for limitation of the peak voltage across the MOSFET switches of a half-bridge converter. The snubber, as shown by Figure 8, consists of a diode $D_{s}$, which will charge the snubber capacitor $C_{s}$ during over-voltage but prevent discharge when the MOSFET turns on. The dc-side voltage $\mathrm{U}_{\mathrm{d}}=110 \mathrm{~V}$. The design shall be based on the switching conditions related to a load current $\mathrm{I}_{\mathrm{v}}=12 \mathrm{~A}$.


Figure 8
a) Draw the waveforms of the current and voltage related to $\mathrm{T} 1, \mathrm{D} 2$ and Cs .

Assume the T1 current turn-off to be instantaneous as shown above. The snubber capacitor is initially charged to $\mathrm{U}_{\mathrm{c}}=\mathrm{U}_{\mathrm{d}}$ at the instant of T1 turn-off.

b) Calculate the required snubber capacitance in order to limit the over voltage to $20 \%$ when T 1 is turned off.
Stored energy in the inductance $L_{c}$ : $W_{L}=0.5^{*} L_{c} * I_{v}{ }^{\wedge} 2$. All stored inductor energy will be transferred to the snubber capacitor, giving a voltage increase defined by the capacitor energy relation:

Selecting $\mathrm{C}_{\mathrm{s}} *$ to limit $\mathrm{U}_{\mathrm{c}}=1.2 \mathrm{U}_{\mathrm{d}}$ yields: $\mathrm{C}_{\mathrm{s}}=\mathrm{L}_{\mathrm{c}} *($
$\left.I_{v} / \Delta U_{c}\right)^{\wedge} 2=40 *\left(12 /\left(0.2^{*} 110\right)\right)^{\wedge} 2=11.9 \mathrm{nF}$.
Another way of addressing it to consider the LC circuit below, where at the instant of T1 turn-off, the inductor carries the current 12 A . The capacitor Cs is initially charged to Ud. The current through the LC circuit will be a quarter-wave of a sinusoidal oscillation with a frequency defined by the series resonance $\omega=\frac{1}{\sqrt{L C}}$. The current starts at its peak value and drops towards zero, following a cosine function. At $\mathrm{iL}=0$, the diode blocks the current after a quarter of a cycle.


The current and voltage will be defined by the following equations:

$$
i_{c}=i_{L 0} \cos \omega t
$$

$$
u_{c}=U_{d}+\frac{1}{C} \int_{0}^{T / 4} i_{c} d t=U_{d}+\frac{i_{L 0}}{\omega C} \int_{0}^{\pi / 2} \cos \omega t d \omega t=U_{d}+i_{L 0} \sqrt{\frac{L}{C}}
$$

Consequently, limiting uc=1.2Ud gives the fllowing expression for the snubber capacitance: $C=L \frac{i_{L 0}^{2}}{\left(0.2 U_{d}\right)^{2}}$

## Formula collection TSTE19 Power Electronics

## Fourier series coefficients using symmetry,

| Even | $f(-t)=f(t)$ | $b_{h}=0 \quad a_{h}=\frac{2}{\pi} \int_{0}^{\pi} f(t) \cos (h \omega t) d(\omega t)$ |
| :---: | :---: | :---: |
| Odd | $f(-t)=-f(t)$ | $a_{h}=0 \quad b_{h}=\frac{2}{\pi} \int_{0}^{\pi} f(t) \sin (h \omega t) d(\omega t)$ |
| Half-wave | $f(t)=-f\left(t+\frac{1}{2} T\right)$ | $a_{h}=b_{h}=0$ for even h |
|  |  | $a_{h}=\frac{2}{\pi} \int_{0}^{\pi} f(t) \cos (h \omega t) d(\omega t)$ for odd $h$ |
|  |  | $b_{h}=\frac{2}{\pi} \int_{0}^{\pi} f(t) \sin (h \omega t) d(\omega t)$ for odd $h$ |
| Even quart-wave | Even and half-wave | $b_{h}=0$ for all $h$ |
|  |  | $a_{h}=\frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} f(t) \cos (h \omega t) d(\omega t)$ for odd $\mathrm{h}, a_{h}=0$ for even h |
| Odd quarter-wave | Odd and half-wave | $a_{h}=0$ for all $h$ |
|  |  | $b_{h}=\frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} f(t) \sin (h \omega t) d(\omega t)$ for odd $\mathrm{h}, b_{h}=0$ for even h |

## Undamped resonant circuits

Even
$f(-t)=f(t)$
$b_{h}=0$
$a_{h}=\frac{2}{\pi} \int_{0}^{\pi} f(t) \cos (h \omega t) d(\omega t)$
Odd
$f(-t)=-f(t)$
$a_{h}=0$
$b_{h}=\frac{2}{\pi} \int_{0}^{\pi} f(t) \sin (h \omega t) d(\omega t)$
Half-wave

$$
f(t)=-f\left(t+\frac{1}{2} T\right)
$$

$a_{h}=b_{h}=0$ for even h
$a_{h}=\frac{2}{\pi} \int_{0}^{\pi} f(t) \cos (h \omega t) d(\omega t)$ for odd $h$
$b_{h}=\frac{2}{\pi} \int_{0}^{\pi} f(t) \sin (h \omega t) d(\omega t)$ for odd $h$
Even quart-wave Even and half-wave $b_{h}=0$ for all $h$
$a_{h}=\frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} f(t) \cos (h \omega t) d(\omega t)$ for odd $\mathrm{h}, a_{h}=0$ for even h
Odd quarter-wave Odd and half-wave $\quad a_{h}=0$ for all $h$
$b_{h}=\frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} f(t) \sin (h \omega t) d(\omega t)$ for odd $\mathrm{h}, b_{h}=0$ for even h

## Integration rules

$$
\begin{aligned}
& \int_{a}^{b} f(x) d x=\int_{A}^{B} f(g(t)) g^{\prime}(t) d t \text { if } a=g(A), b=g(B) \text { and } g \text { is monotone in }[\mathrm{A}, \mathrm{~B}] \\
& \int_{a}^{b} \sin (x) d x=[-\cos (x)]_{a}^{b} \\
& \int_{a}^{b} \cos (x) d x=[\sin (x)]_{a}^{b}
\end{aligned}
$$

