# TSTE19

# POWER ELECTRONICS

EXAMINATION (TEN1)

# SOLUTIONS

Time: Wednesday 8 April 2015 at 8.00 - 12.00

Place: U4, U6

Resonsible teacher: Martin Nielsen-Lönn, ISY, 070-361 52 44 (martin.nielsen.lonn@liu.se)

Will visit exam location at 8:45 and 10.

Number of tasks: 6

Number of pages: 9

Allowed aids: Calculator

Total points: 70

Notes: A pass on the exam requires approximately 30 points.

Remember to indicate the steps taken when solving problems.

Exam presentation: Friday 17 April 2015 12:00-13:00 (Filtret, B-building)

# Instructions

Write as detailed as you can and describe what you are doing. It is better to write if you are unsure about some step or equation instead of just using it.

# Questions

## Question 1

(a) Will an ideal capacitor connected to a 220 V 50 Hz line outlet voltage get warm due to the current flowing through it? Motivate your answer.

## Solution:

(b) Why do you want electrical isolation in a power supply?

(2)

(2)

#### Solution:

For example: safety requirements, different ground levels is required etc.

(c) What does the acronyms ZCS and ZVS mean?

(2)

## **Solution:**

Zero Current Switching and Zero Voltage Switching

(d) When and why can a transistor used in a rectifier be better than a diode even if its behavior(on-/off times) is exactly the same.

#### Solution:

It has a lower voltage drop which is preferable in a low voltage operation to increase efficiency.

(e) What component (resistor, capacitor, inductor) can, in the right circumstances, be modeled as an ideal current source?

(2)

#### Solution:

An inductor since it, if large enough, will have an almost constant current running through it.

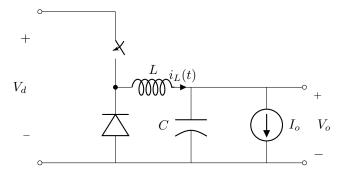


Figure 1: Circuit for question 2

#### Question 2

Consider the circuit in figure 1 with the following values:  $V_d = 6$  V, L = 37.5  $\mu$ H, D = 0.25,  $I_o = 1.5$  A and  $f_s = 100$  kHz. Assume that C is large and that it works in continuous conduction mode.

(a) What kind of circuit is this and what is the ratio between the output and input voltages? Hint: Is  $V_{out} > V_{in}$ , vice versa or is the ratio arbitrary? (2)

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## Solution:

Step-down converter

(b) Mention two other types of DC-DC converters and what their ratio between the output and input voltage are? *Hint: See part 1*.

## **Solution:**

For example: Step-up(boost) which has a higher output voltage than input voltage and buck-boost which can have both a higher or lower output voltage than the input voltage.

(c) What is the output voltage?

(2)

(3)

(4)

#### Solution:

$$V_o = V_d D = 6 \times 0.25 = 1.5 \ V$$

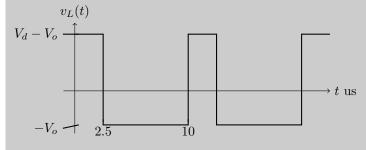
(d) Sketch the waveforms for  $v_L(t)$  and  $i_L(t)$ . Indicate the times, average values, all voltages and the peak-to-peak current.

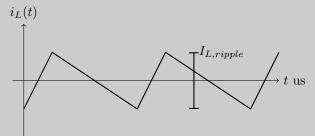
#### Solution:

Beginning by calculating the average  $i_L$  as,

$$I_L = I_o = 1.5 \ A$$

and drawing a sketch of them,





Note that the baseline in the current waveform is equal to  $I_o$ .

The ripple in the current,  $I_{L,ripple}$ , can be calculated by integrating the voltage over the inductor from time 0 to  $DT_s$ ,

$$I_{L,ripple} = \frac{1}{L_s} \int_0^{DT_s} v_L(t) dt = \frac{DT_s(V_o - V_d)}{L_s} = \frac{0.25(6 \ V - 1.5 \ V)}{37.5 \ \mu H \times 100 \ kHz} = 0.6 \ A.$$

(e) Calculate the minimum switching frequency to keep continuous conduction mode.

(6)

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## Solution:

If the converters operate at the boundary between continuous and discontinuous mode the inductor current,  $i_L$ , will touch 0 at the end of each period. This gives that the average output current will equal half of the peak inductor current.

$$\hat{I}_L = 2I_o = 2 \times 1.5 \ A = 3 \ A.$$

When the switch is closed the voltage across the inductance,  $v_L$ , is,

$$v_L = V_d - V_o = 6 \ V - 1.5 \ V = 4.5 \ V.$$

The current change during this time is,

$$\hat{I}_L = \frac{T_s D v_L}{L}$$

Rearranging and solving for  $f_s = 1/T_s$  gives,

$$f_s = \frac{Dv_L}{\hat{I}_L L} = \frac{0.25 \times 4.5 \ V}{3 \ A \times 37.5 \ uH} = 20000 \ Hz = 20 \ kHz.$$

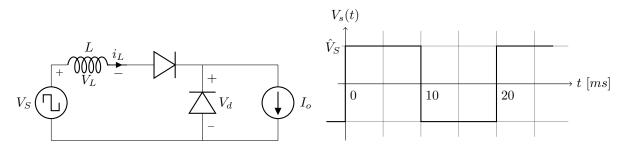
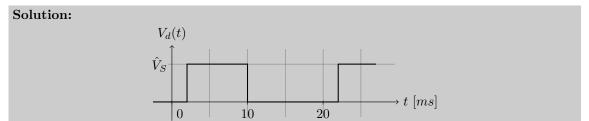


Figure 2: Circuit and waveform for question 3

## Question 3

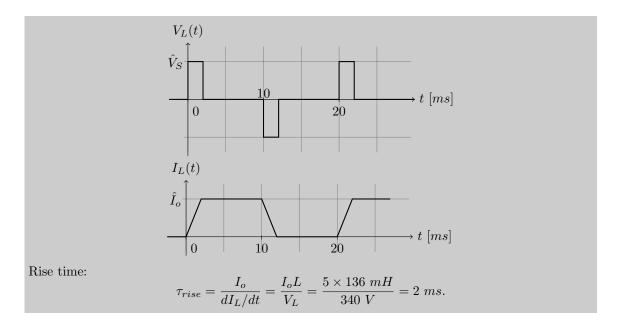
Consider the circuit and voltage graph in figure 2. The input voltage,  $V_S$ , is shown in the graph on the right and has a peak voltage of 340 V. L = 136 mH and  $I_o = 5$  A.

(a) Draw the inductor voltage  $V_L$  and current  $I_L$ .



(3)

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(b) How long time does it take for the currenct commutation to complete?

Solution:

$$\frac{dI_L}{dt} = \frac{V_L}{L} = \frac{340 \ V}{136 \ mH}$$

(5)

(3)

 $I_L$  reaches  $I_o$  after

$$\tau_{rise} = \frac{I_o}{dI_L/dt} = \frac{I_oL}{V_L} = \frac{5\times136~mH}{340~V} = 2~ms. \label{eq:taurise}$$

(c) What is the average output voltage  $V_d$ ?

Solution:

$$V_{avg,o} = \frac{(10~ms - 2~ms) \times 340~V}{20~ms} = 136~V.$$

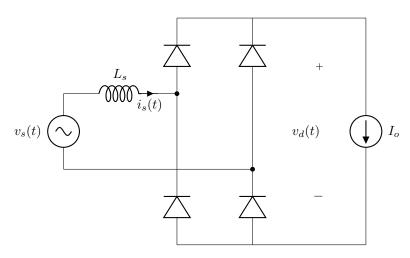


Figure 3: Circuit for question 4

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## Question 4

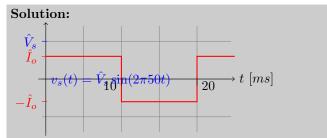
The rectifier in figure 3 have a load current  $I_o$  of 2 A. Assume that all the diodes are ideal.

(a) Draw the waveforms of  $v_s(t)$  and  $i_s(t)$  if  $L_s$  would be zero and  $v_s$  is a sinusoidal voltage with 240  $V_{RMS}$  at 50 Hz, and describe which diodes are conducting at each interval and why.

(2)

(3)

(3)



(b) Calculate the 3 first harmonics of  $i_s$ , that is the fundamental and the two following harmonics, and present them in table with frequency and amplitude. Assume  $L_s$  is still zero.

#### Solution:

The current is odd and a half-wave which makes the calculations easier.  $a_h$  is zero for all h and  $b_h$  is only non-zero for odd h,

$$b_h = \frac{4}{\pi} \int_0^{pi/2} \hat{I}_o \sin(h\omega t) d(\omega t) = 2 A \frac{4}{\pi h} \text{ for odd h.}$$

Harmonic	Frequency	Amplitude
0	DC	0
1	50	2.5464
2	100	0
3	150	0.8488

(c) Draw the waveforms of  $v_s(t)$  and  $i_s(t)$  if  $L_s$  is 150 mH and  $v_s$  is a square wave with 300 V amplitude at 50 Hz, and describe which diodes are conducting at each interval and why.

## **Solution:**

Since the current increases linearly through an inductor which have a constant voltage across it the current waveforms is a bit different. By setting the maximum current through the inductor to  $I_o$  we can calculate the risetime as,

$$2I_o = \frac{1}{L_s} \int_0^{t_{rise}} \hat{V}_s dt = \frac{\hat{V}_s t_{rise}}{L_s}$$
 
$$t_{rise} = \frac{2L_s I_o}{\hat{V}_s} = \frac{2 \times 200 \text{ mH} \times 2 \text{ A}}{300 \text{ V}} = 2.67 \text{ ms.}$$
 
$$\hat{I}_o \longrightarrow t \text{ [ms]}$$

#### Question 5

Recently smart watches has become more and more popular. They commonly use Li-Ion batteries with

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3.7 V between the poles and a SoC (System-on-chip) which runs at a lower supply voltage. Because of this a DC-DC converter has to be used to step-down the voltage. Assuming the supply voltage to the SoC is 1.2 V and the watch consumes 400 mW in total (including the power converter), it is your job to design the heatsink for the power converter. You can assume that the efficiency always is 90% and the ambient temperature can be up to  $50^{\circ}\text{C}$ .

(a) What is the maximum power dissipated by the DC-DC converter?

(2)

(5)

(6)

#### **Solution:**

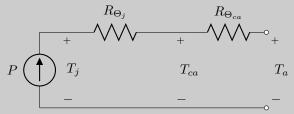
Since the efficiency is 90% the dissipated power is,

$$P_{\text{diss}} = 0.4 \ W \times 0.1 = 0.04 \ W = 40 \ mW.$$

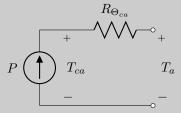
(b) Draw the thermal equivalent circuit and calculate the required  $\Theta_{ca}$  for the heat sink assuming that  $\Theta_i$  is 1°C/W and that the maximum case temperature is 70°C.

#### Solution:

The thermal equivalent circuit is,



Where P is the power dissipated in the circuit and  $T_a$  the ambient temperature. Seeing that we do not care about the junction temperature the circuit boils down to,



Which gives the following equation,

$$R_{\Theta_{ca}} = \frac{T_c - T_a}{P} = \frac{70^{\circ}C - 50^{\circ}C}{0.04 \ W} = \frac{20^{\circ}C}{0.04 \ W} = 500^{\circ}C/W$$

## Question 6

The thyristor based inverter showed in figure 4 have the input voltage  $v_s$  shown to the right in the same figure. All thyristor have a 30° firing angle. The current source load  $I_o$  is 3 A and the inductor L is 110 mH.

(a) Draw the output voltage  $v_o$  and the source current  $i_s$ , indicating which thyristor is on (conducting) and off (not conducting).

## Solution:

The firing angle is 30° which is equal to 1.67 ms (20  $ms \times 30/360$ ). Assume thyristors  $T_2$  and  $T_3$  are on, and  $v_s$  is changing from negative to positive. Then the current  $i_s = -I_o$ . When  $v_s$  goes positive, the current still continuous to run through  $T_2$  and  $T_3$  (thyristors do not turn off unless the current is zero). When  $T_1$  and  $T_4$  are fired, then all thyristors will conduct. The voltage  $v_o$ 

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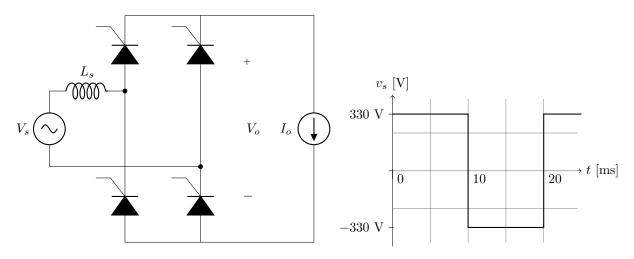
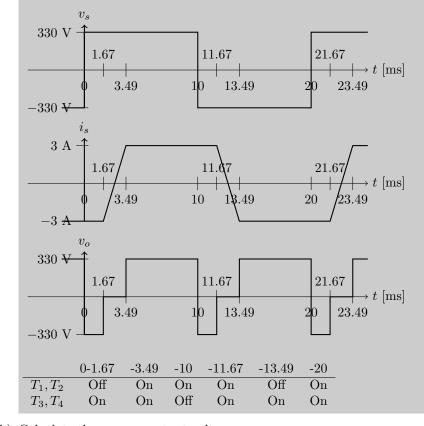


Figure 4: Circuit and waveform for question 6

goes to 0, and the voltage across L is now equal to  $v_s$  and  $i_s$  changes. When  $i_s$  has reached  $I_o$  then  $T_2$  and  $T_3$  turns off. The time to change current direction through L (total change is  $2I_o$ ) is,

$$t_{sw} = \frac{2I_oL}{v_s} = \frac{2 \times 3 \ A \times 110 \ mH}{330 \ V} = 2 \ ms.$$

The waveforms thus look like,



(b) Calculate the average output voltage.

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(6)

Solution:

$$V_{o(avg)} = \frac{1}{T} \int_0^{10} v_o(t)dt = \frac{1}{10} (330(10 - 3.49) - 330 \times 1.67) = \frac{330}{10} (10 - 3.49 - 1.67) \approx 160 \ V$$

(c) Calculate the displacement power factor (DPF) for the input power.

(5)

## **Solution:**

Displacement angle is found by looking at the middle point of high output current,

$$\frac{11.67\ ms - 3.49\ ms}{2} + 3.49\ ms = 7.58\ ms.$$

The voltage peak middle point is 10 ms/2 = 5 ms. This gives us the current displacement angle,

$$\phi = \phi_v - \phi_i = \frac{7.67 \ ms - 5 \ ms}{10 \ ms} 180^\circ = 46.44^\circ.$$

Putting this angle into the formula for DPF gives,

$$DPF = cos(\phi) = cos(46.44^{\circ}) = 0.69$$

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# Formula collection TSTE19 Power Electronics

Fourier series coefficients using symmetri, Table 3.1

Even 
$$f(-t) = f(t)$$
  $b_h = 0$   $a_h = \frac{2}{\pi} \int_0^{\pi} f(t) cos(h\omega t) d(\omega t)$ 

Odd 
$$f(-t) = -f(t)$$
  $a_h = 0$   $b_h = \frac{2}{\pi} \int_0^{\pi} f(t) \sin(h\omega t) d(\omega t)$ 

Half-wave 
$$f(t) = -f\left(t + \frac{1}{2}T\right)$$
  $a_h = b_h = 0$  for even h

$$a_h = \frac{2}{\pi} \int_0^{\pi} f(t) cos(h\omega t) d(\omega t)$$
 for odd h

$$b_h = \frac{2}{\pi} \int_0^{\pi} f(t) sin(h\omega t) d(\omega t)$$
 for odd  $h$ 

Even quart-wave Even and half-wave  $b_h = 0$  for all h

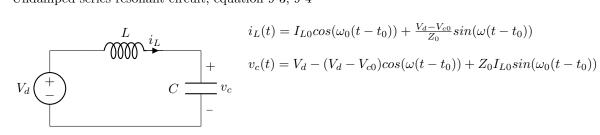
$$a_h = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} f(t) cos(h\omega t) d(\omega t)$$
 for odd h,  $a_h = 0$  for even h

Odd quarter-wave Odd and half-wave  $a_h = 0$  for all h

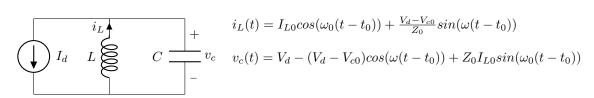
$$b_h=\frac{4}{\pi}\int_0^{\frac{\pi}{2}}f(t)sin(h\omega t)d(\omega t)$$
 for odd h,  $b_h=0$  for even h

## Undamped resonant circuits

Undamped series resonant circuit, equation 9-3, 9-4



Undamped parallel resonant circuit, equation 9-20, 9-21



# Integration rules

$$\int_a^b f(x) dx = \int_A^B f(g(t)) g'(t) dt$$
 if  $a=g(A),\, b=g(B)$  and  $g$  is monotone in [A,B]

$$\int_{a}^{b} \sin(x)dx = \left[-\cos(x)\right]_{a}^{b}$$

$$\int_{a}^{b} \cos(x) dx = \left[ \sin(x) \right]_{a}^{b}$$

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