Hand-in problems 2 for TSTE18 Digital Arithmetic

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The solutions to the hand-in problems should be submitted at most one week after the corresponding seminar to result in prioritized corrections.

These problems should be solved on an **individual** basis. Each student has a consecutive number assigned during the first seminar (or through email contact with the examiner) and should solve the problems using the corresponding data.

Note that the problems should be solved **"by hand"**. Hence, you will need to provide some evidence that you actually solved the problem and not just ran some software for it.

On each sheet of paper write name, personal id number, and student-id, as well as the consecutive number assigned to you.

Preliminaries

Variable	Meaning
r	Radix
M	Number of integer digits
L	Number of fractional digits
x_i	Digit
α	Magnitude of most negative digit value
β	Magnitude of most positive digit value

1 Determine the stated numerical properties for the following unsigned number systems.

- Largest representable number
- Number of representable numbers
- Unit of least position, ulp

System no.	r	M	L	x_i
1	2	4	4	$\{0,1\}$
2	2	5	3	$\{0,1,2\}$
3	4	0	7	$\{0,1,2,3\}$
4	4	2	7	$\{0,1,2\}$
5	4	2	5	$\{0,1,2,3,4,5\}$

2 Determine the stated numerical properties for the following signed number systems.

- Most positive representable number
- Most negative representable number
- Number of representable numbers
- Unit of least position, ulp

System no.	Sign-handling	r	M	L	x_i
1	Sign-magnitude	2	4	4	$\{0, 1\}$
2	Diminished-radix (one's) complement	2	4	4	$\{0, 1\}$
3	Radix (two's) complement	2	4	4	$\{0, 1\}$
4	Sign-magnitude	5	4	4	$\{0, 1, 2, 3, 4\}$
5	Diminished-radix (four's) complement	5	4	4	$\{0, 1, 2, 3, 4\}$
6	Radix (five's) complement	5	4	4	$\{0, 1, 2, 3, 4\}$

3 Determine the stated numerical properties for the following signed-digit systems.

- Most positive representable number
- Most negative representable number
- Number of representable numbers
- Unit of least position, ulp
- Redundancy index

System no.	r	M	L	α	β
1	2	4	4	-1	1
2	4	4	4	-2	2
3	4	4	4	-3	3
4	7	3	3	-4	6

4 Derive the following numerical ranges

Assume that a radix-r number system with M integer and L fractional digits has an initial digit set of $x_i \in \{0, 1, 2, \ldots, r-1\}$. Now, assume that the digit set is modified such that all digits are reduced by d, $d < r - 1$. Determine the most negative representable number in the new number representation.

5 Perform generalized carry-free addition of the following numbers

For denomination purposes: $\bar{c} = -c$.

	$r = 4$, $\alpha = 0$, $\beta = 6$		$r = 7$, $\alpha = -5$, $\beta = 6$	
Student no.	Number 1	Number 2	Number 1	Number 2
1	2415142	5335236	$5\bar{6}32121$	$245\bar{4}6\bar{2}2$
2	2122212	1516202	$3413\bar{5}\bar{4}\bar{2}$	$\bar{4}\bar{3}33\bar{5}1\bar{4}$
3	6523645	3332434	$\bar{2}\bar{3}0\bar{1}442$	$\bar{4}\bar{5}1\bar{2}56\bar{2}$
4	5545634	3034326	$44\bar{1}2552$	$\bar{3}2\bar{4}\bar{1}254$
5	3536652	3155534	$\bar{3}50\bar{3}535$	$\bar{2}22\bar{4}\bar{1}\bar{2}3$
6	2552345	4323454	$\bar{5}200\bar{4}4\bar{1}$	$\bar{2}\bar{1}\bar{1}11\bar{1}\bar{1}$
7	3464251	0112203	$\bar{4}\bar{3}24616$	$11\bar{1}00\bar{4}5$
8	0352455	6125434	$41\bar{3}0062$	$33\bar{1}1\bar{1}\bar{3}1$
9	4354232	5434654	$\bar{4}\bar{1}10\bar{4}\bar{2}4$	$\bar{5}5\bar{4}2\bar{1}\bar{3}1$
10	1440010	3543515	$\bar{4}1\bar{3}1\bar{5}34$	$561\bar{2}\bar{4}11$
11	5043164	3643533	$2\bar{1}\bar{2}15\bar{1}\bar{4}$	$0\bar{2}\bar{1}4\bar{1}\bar{1}\bar{1}$
12	1213223	1341502	$\bar{5}\bar{3}\bar{5}\bar{3}\bar{3}\bar{2}\bar{3}$	$\bar{3}255\bar{3}0\bar{1}$
13	3203106	3650444	$1\bar{3}3\bar{2}\bar{1}54$	$\bar{1}\bar{2}2552\bar{1}$
14	5350341	2122303	$\bar{2}\bar{2}\bar{2}\bar{3}654$	$3\bar{3}\bar{1}\bar{3}\bar{5}\bar{2}3$
15	4332355	3441321	$\bar{3}\bar{3}20\bar{3}\bar{2}\bar{5}$	$1\bar{2}55\bar{1}\bar{5}2$
16	5603344	4423106	$\bar{2}26\bar{3}\bar{3}\bar{1}5$	$\bar{1}\bar{2}\bar{3}\bar{1}\bar{1}40$
17	1403513	3034156	$41\bar{2}3\bar{2}62$	$2\bar{3}\bar{4}\bar{2}453$
18	4135261	2304335	$44\bar{2}0\bar{3}\bar{4}\bar{4}$	$\bar{2}163\bar{2}\bar{2}4$
19	5421546	0425132	$42\bar{3}\bar{2}\bar{4}21$	$\bar{3}\bar{3}05\bar{1}\bar{5}\bar{4}$
20	5325234	5246042	$\bar{2}\bar{2}36\bar{3}\bar{4}\bar{3}$	$6403\bar{0}4\bar{1}$

6 Add the following two binary signed-digit numbers using the limited carry approach

For denomiation purposes: $\bar{1} = -1$.

Student no.	Number 1	Number 2	Number 1	Number 2
1	$000\bar{1}000\bar{1}$	10101011	$0\bar{1}110001$	$1\bar{1}110\bar{1}0\bar{1}$
2	$\bar{1}0011\bar{1}00\bar{1}$	$\bar{1}\bar{1}00\bar{1}0\bar{1}\bar{1}$	10100111	$\bar{1}1010\bar{1}\bar{1}$
3	$0101\bar{0}0\bar{1}0$	$\bar{1}0001110$	$10\bar{1}\bar{1}101\bar{1}$	$00\bar{1}00\bar{1}0\bar{1}$
4	$\bar{1}\bar{1}\bar{1}\bar{1}\bar{1}000$	$0000\bar{1}011$	$0\bar{1}000\bar{1}1\bar{1}$	$\bar{1}\bar{1}\bar{1}00\bar{1}10$
5	$0\bar{1}111110$	$\bar{1}0\bar{1}\bar{1}1000$	$00\bar{1}10100$	$\bar{1}\bar{1}001\bar{1}10$
6	$0100011\bar{1}$	$\bar{1}0\bar{1}00\bar{1}\bar{1}1$	$001\bar{1}000\bar{1}$	$00\bar{1}\bar{1}1\bar{1}\bar{1}\bar{1}$
7	$0\bar{1}1\bar{1}\bar{1}00\bar{1}$	$100\bar{1}0\bar{1}01$	$0\bar{1}\bar{1}110\bar{1}1$	$000\bar{1}\bar{1}000$
8	$00100000\bar{1}$	$1000\bar{1}\bar{1}00$	$110\bar{1}\bar{1}\bar{1}11$	$0\bar{1}0\bar{1}\bar{1}000$
9	$000\bar{1}011\bar{1}$	$00001\bar{0}\bar{1}\bar{1}$	$1\bar{1}00\bar{1}00\bar{1}$	$1\bar{1}0\bar{1}0\bar{1}0\bar{1}$
10	$\bar{1}1001001$	$00\bar{1}1\bar{1}010$	$100\bar{1}0000$	$\bar{1}010\bar{1}\bar{1}\bar{1}\bar{1}$
11	$0000\bar{1}0\bar{1}\bar{1}$	$\bar{1}\bar{1}\bar{1}\bar{1}00\bar{1}0$	$100\bar{1}\bar{1}\bar{1}10$	$00\bar{1}01\bar{1}00$
12	$\bar{1}00100\bar{1}0$	$\bar{1}1100001$	00000000	$00\bar{1}\bar{1}0001$
13	$10\bar{1}1100\bar{1}$	$00\bar{1}\bar{1}1\bar{1}\bar{1}0$	$100\bar{1}100\bar{1}$	$10\bar{1}00000$
14	$\bar{1}0\bar{1}101\bar{1}1$	$10001\bar{1}\bar{1}1$	00000101	$11001\bar{0}\bar{1}1$
15	$001\bar{0}000\bar{1}$	$\bar{1}0\bar{1}00100$	$\bar{1}101\bar{1}0\bar{1}0$	$\bar{1}\bar{1}000000\bar{1}$
16	11010000	$0\bar{1}0\bar{1}11\bar{1}0$	$\bar{1}11\bar{1}1\bar{1}01$	$0110\bar{1}\bar{1}\bar{1}\bar{1}$
17	$\bar{1}0100110$	$1100\bar{1}000$	$00100000\bar{1}$	01010100
18	$\bar{1}\bar{1}0010\bar{1}1$	$11\bar{1}000\bar{1}0$	$000\bar{1}\bar{1}111$	$\bar{1}\bar{1}\bar{1}0\bar{1}1\bar{1}\bar{1}$
19	$010\bar{1}0\bar{1}01$	$0\bar{1}0\bar{1}0011$	$01\bar{1}01100$	$01\bar{1}00100$
20	10010000	10001010	$1\bar{1}000\bar{1}00$	$00\bar{1}00\bar{1}01$

7 Generalized carry-save

A full adder adds three bits, x_i, y_i, z_i and results in a sum bit, s_i and a carry bit c_i. The relation between these bits is $2c_i + s_i = x_i + y_i + z_i$. As all inputs and outputs have positive weights, this is sometimes referred to as a ppp adder. Determine the relation for the three other types of adders, pnp, npn, and nnn, and clearly denote the sign of the output bits. For all four adders types draw a logic level diagram using standard gates.