Lösningsförslag till tentamen TMEL53 Digitalteknik M 2016-03-18

1a)

$$231/2 = 115 \quad \text{REST } 1 \quad \text{LSB}$$
$$115/2 = 57 \quad \text{REST } 1$$
$$57/2 = 28 \quad \text{REST } 1$$
$$28/2 = 14 \quad \text{REST } 0$$
$$14/2 = 7 \quad \text{REST } 0$$
$$7/2 = 3 \quad \text{REST } 1 \quad \uparrow$$
$$3/2 = 1 \quad \text{REST } 1$$
$$1/2 = 0 \quad \text{REST } 1 \quad \text{MSB}$$

$$231_{10} = 11100111_2$$

1b) $\underbrace{11}\,\underbrace{100}\,\underbrace{111} = 347_8$

1c) $\underbrace{1110}\,\underbrace{0111} = E7_{16}$

1d) $231_{10} = 001000110001$ NBCD

1e)

$$0{,}6 \cdot 2 = 0{,}2 + 1 \quad \text{MSB}$$
$$0{,}2 \cdot 2 = 0{,}4 + 0 \quad \downarrow$$
$$0{,}4 \cdot 2 = 0{,}8 + 0$$
$$0{,}8 \cdot 2 = 0{,}6 + 1$$
$$0{,}6 \cdot 2 = 0{,}2 + 1$$

ETC.

$$0{,}6_{10} \approx 0.10011001\ldots \text{ ETC}$$

1f) MULTIPLIKATION MED TVÅ UTFÖRS ENKLAST GENOM ATT SKIFTA DET BINÄRA TALET ETT STEG TILL VÄNSTER

00010100 GÅNGER TVÅ = 00101000

1g)

```
          11111100

                11
          00000011
   +             1
   ---------------
          00000100   ← +4
```

ALLTSÅ 11111100 = −4 ENLIGT TVÅKOMPLEMENTMETODEN.

2a)

$$\bar{x}_2 + x_3\bar{x}_1 = (x_3+\bar{x}_3)\bar{x}_2(x_1+\bar{x}_1) + x_3(x_2+\bar{x}_2)\bar{x}_1 =$$

$$= (x_3\bar{x}_2 + \bar{x}_3\bar{x}_2)(x_1+\bar{x}_1) + (x_3x_2 + x_3\bar{x}_2)\bar{x}_1 =$$

$$= x_3\bar{x}_2x_1 + x_3\bar{x}_2\bar{x}_1 + \bar{x}_3\bar{x}_2x_1 + \bar{x}_3\bar{x}_2\bar{x}_1 +$$

$$+ x_3x_2\bar{x}_1 + \cancel{x_3\bar{x}_2\bar{x}_1} = P_5 + P_4 + P_1 + P_0 + P_6$$

FINNS REDAN.

STRYK TERMEN.

2b) P_7, P_3 OCH P_2 SAKNAS I SP-FORMEN OVAN. ALLTSÅ BLIR PS-FORMEN

$$S_7 \cdot S_3 \cdot S_2 = (\bar{x}_3+\bar{x}_2+\bar{x}_1)(x_3+\bar{x}_2+\bar{x}_1)(x_3+\bar{x}_2+x_1)$$

2c) $F = (A \oplus C) + \overline{B} + BC\overline{D} + \overline{ACD}$

A	B	C	D	$A \oplus C$	$\overline{B}$	$BC\overline{D}$	$\overline{ACD}$	F
0	0	0	0	0	1	0	1	1
0	0	0	1	0	1	0	1	1
0	0	1	0	1	1	0	1	1
0	0	1	1	1	1	0	1	1
0	1	0	0	0	0	0	1	1
0	1	0	1	0	0	0	1	1
0	1	1	0	1	0	1	1	1
0	1	1	1	1	0	0	1	1
1	0	0	0	1	1	0	1	1
1	0	0	1	1	1	0	1	1
1	0	1	0	0	1	0	1	1
1	0	1	1	0	1	0	0	1
1	1	0	0	1	0	0	1	1
1	1	0	1	1	0	0	1	1
1	1	1	0	0	0	1	1	1
1	1	1	1	0	0	0	0	0

2d) DET ENKLASTE ÄR ATT TITTA I FUNKTIONSTABELLEN. DÅ SER MAN ATT $F = 0$ OM $A = 1$ OCH $B = 1$ OCH $C = 1$ OCH $D = 1$,

DVS $\overline{F} = ABCD \Rightarrow F = \overline{ABCD}$

OM MAN ANVÄNDER KARNAUGH-DIAGRAM OCH RINGAR IN 0:OR SÅ GER DET SAMMA SAK.
OM MAN RINGAR IN 1:OR SÅ FÅR KARNAUGHDIAGRAMMET FÖLJANDE UTSEENDE :

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	0	1
10	1	1	1	1

$$F = \bar{A} + \bar{B} + \bar{C} + \bar{D}$$

DE MORGANS TEOREM $\Rightarrow$

$$F = \overline{ABCD}$$

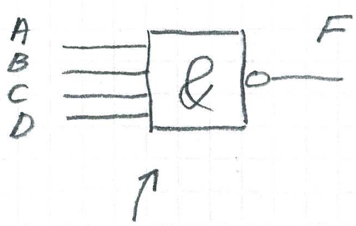
OM MAN DÄREMOT JOBBAR MED BOOLESK ALGEBRA SÅ MEDFÖR DET EN STÖRRE ARBETSINSATS:

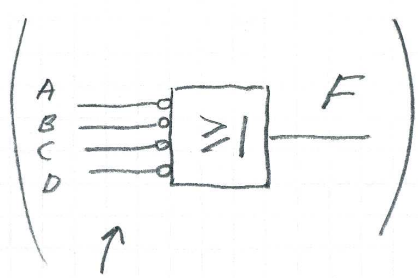
$$F = (A \oplus C) + \bar{B} + BC\bar{D} + \overline{ACD} =$$

$$= A\bar{C} + \bar{A}C + \bar{B} + BC\bar{D} + \bar{A} + \bar{C} + \bar{D}$$

$$= \bar{A}(C+1) + \bar{B} + \bar{C}(A+1) + \bar{D}(BC+1) =$$

$$= \bar{A} + \bar{B} + \bar{C} + \bar{D} = \overline{ABCD}$$





BILLIGARE

DYRARE PGA INVERTERARNA

2d)

NOR-GRINDAR ⇒ RINGA 0:OR

A = 0

BC \ DE	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	1	1	0
10	0	0	0	0

A = 1

BC \ DE	00	01	11	10
00	0	1	1	0
01	1	1	1	1
11	1	1	1	1
10	0	1	1	0

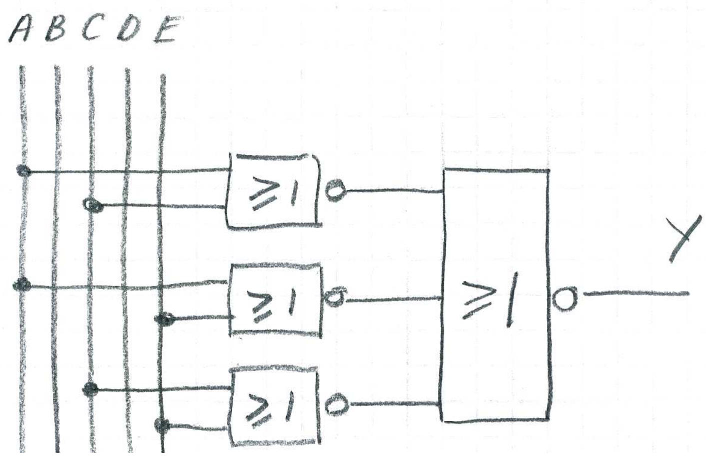
$$\overline{Y} = \bar{A}\bar{C} + \bar{A}\bar{E} + \bar{C}\bar{E} \Rightarrow$$

$$Y = \overline{\bar{A}\bar{C} + \bar{A}\bar{E} + \bar{C}\bar{E}} =$$

$$= (A+C)(A+E)(C+E) =$$

$$= \overline{\overline{(A+C)(A+E)(C+E)}} =$$

$$= \overline{\overline{(A+C)} + \overline{(A+E)} + \overline{(C+E)}}$$



3a)

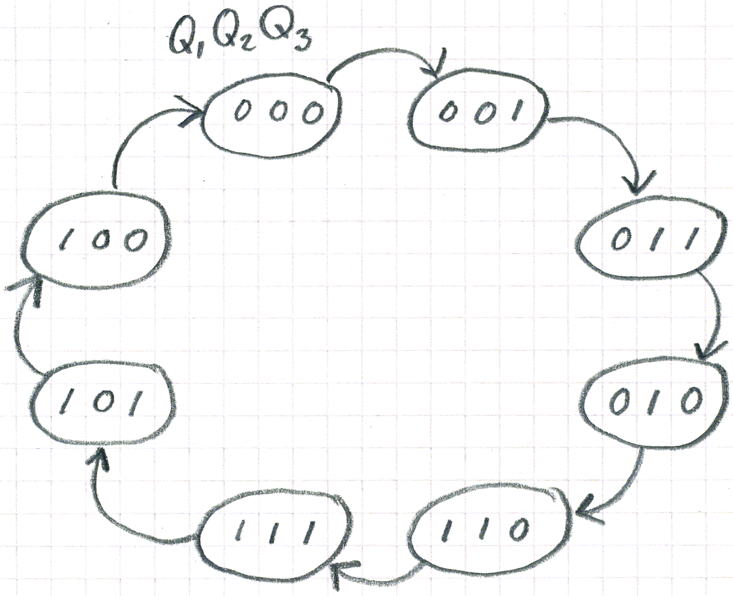
$$T = \overline{Q_3 + \left(\overline{Q_2 \oplus Q_1}\right)}$$

$$S = \overline{Q}_1 Q_3$$

$$R = Q_1 Q_3$$

$$D = \overline{Q_2 \oplus Q_1}$$

Q_1	Q_2	Q_3	T	S	R	D	Q_1^+	Q_2^+	Q_3^+
0	0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	1	0	1	1
0	1	0	1	0	0	0	1	1	0
0	1	1	0	1	0	0	0	1	0
1	0	0	1	0	0	0	0	0	0
1	0	1	0	0	1	0	1	0	0
1	1	0	0	0	0	1	1	1	1
1	1	1	0	0	1	1	1	0	1



3b)

X	Q_1	Q_2	J_1	K_1	J_2	K_2	Q_1^+	Q_2^+
0	0	0	0	–	0	–	0	0
0	0	1	0	–	–	1	0	0
0	1	0	–	1	1	–	0	1
0	1	1	–	0	–	1	1	0
1	0	0	0	–	1	–	0	1
1	0	1	1	–	–	1	1	0
1	1	0	–	0	1	–	1	1
1	1	1	–	0	–	0	1	1

X \ $Q_1 Q_2$	00	01	11	10
0	0	0	–	–
1	0	1	–	–

$J_1 = X Q_2$

X \ $Q_1 Q_2$	00	01	11	10
0	–	–	0	1
1	–	–	0	0

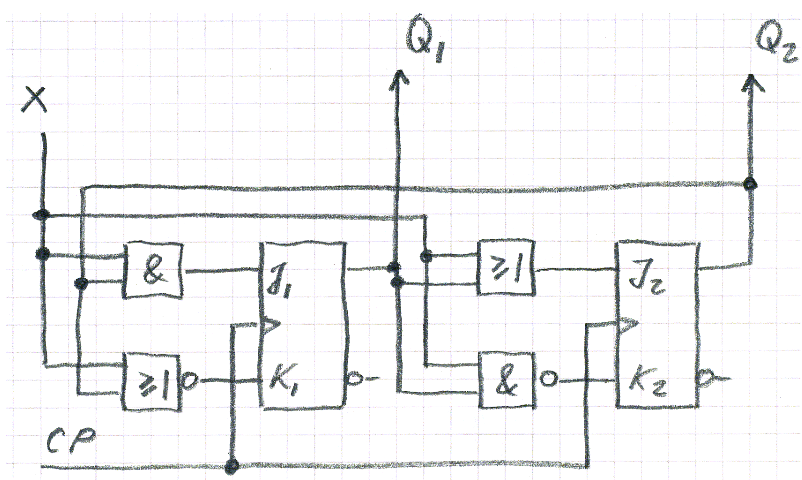
$K_1 = \bar{X}\,\bar{Q}_2 = \overline{X + Q_2}$

X \ $Q_1 Q_2$	00	01	11	10
0	0	–	–	1
1	1	–	–	1

$J_2 = X + Q_1$

X \ $Q_1 Q_2$	00	01	11	10
0	–	1	1	–
1	–	1	0	–

$K_2 = \bar{X} + \bar{Q}_1 = \overline{X Q_1}$



4a)

c_{2i}	c_{1i}	x_i	c_{2i+1}	c_{1i+1}	s_i
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	0	1
0	1	1	0	1	0
1	0	0	0	1	0
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	1	0	0

c_{2i} \ $c_{1i}x_i$	00	01	11	10
0	0	0	0	0
1	0	0	1	0

$$c_{2i+1} = c_{2i}\, c_{1i}\, x_i$$

c_{2i} \ $c_{1i}x_i$	00	01	11	10
0	0	0	1	0
1	1	1	0	1

$$c_{1i+1} = \overline{c_{2i}\, c_{1i}\, x_i + \overline{c}_{2i}\, \overline{x}_i + \overline{c}_{2i}\, \overline{c}_{1i}}$$

$$= \overline{c_{2i}\, c_{1i}\, x_i + \overline{c_{2i} + x_i} + \overline{c_{2i} + c_{1i}}}$$

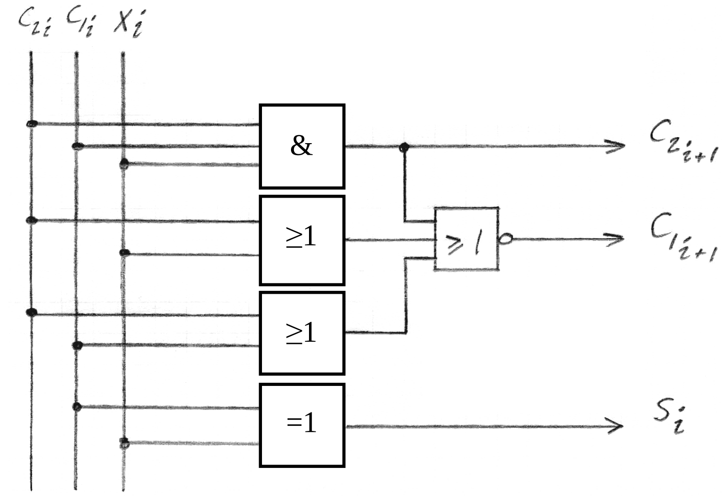
c_{2i} \ $c_{1i}x_i$	00	01	11	10
0	0	1	0	1
1	0	1	0	1

$$s_i = \overline{c}_{1i}\, x_i + c_{1i}\, \overline{x}_i = c_{1i} \oplus x_i$$

När det gäller c_{1i+1} kan man få en ännu smartare lösning genom att ringa in 1:orna i Karnaugh-diagrammet. Man får då:

$$c_{1i+1} = \overline{c_{2i}}\, c_{1i}\, x_i + c_{2i}\, \overline{c_{1i}} + c_{2i}\, \overline{x_i} = \overline{c_{2i}}\, c_{1i}\, x_i + c_{2i}\left(\overline{c_{1i}} + \overline{x_i}\right) = \overline{c_{2i}} \cdot c_{1i}\, x_i + c_{2i} \cdot \overline{c_{1i} x_i} = c_{2i} \oplus c_{1i}\, x_i$$

c_{1i+1} kan alltså skapas med endast en OCH-grind och en EXOR-grind. I lösningen enligt ovan kan grinddelning utnyttjas. Men trots det blir det två grindar mer.



4b)

C_{2i} \ $C_{1i}X_i$	00	01	11	10
0	0	0	0	0
1	0	0	1	0

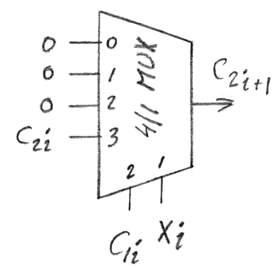
$\Rightarrow$

$C_{1i}X_i = 00 \Rightarrow C_{2i+1} = 0$

$C_{1i}X_i = 01 \Rightarrow C_{2i+1} = 0$

$C_{1i}X_i = 10 \Rightarrow C_{2i+1} = 0$

$C_{1i}X_i = 11 \Rightarrow C_{2i+1} = C_{2i}$



C_{2i} \ $C_{1i}X_i$	00	01	11	10
0	0	0	1	0
1	1	1	0	1

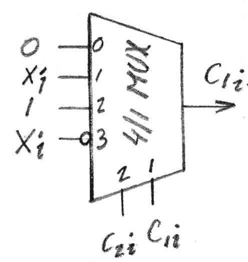
$\Rightarrow$

$C_{2i}C_{1i} = 00 \Rightarrow C_{1i+1} = 0$

$C_{2i}C_{1i} = 01 \Rightarrow C_{1i+1} = X_i$

$C_{2i}C_{1i} = 10 \Rightarrow C_{1i+1} = 1$

$C_{2i}C_{1i} = 11 \Rightarrow C_{1i+1} = \overline{X_i}$



C_{2i} \ $C_{1i}X_i$	00	01	11	10
0	0	1	0	1
1	0	1	0	1

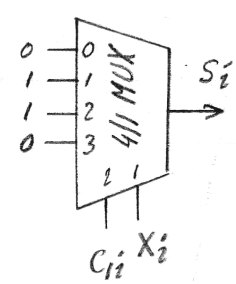
$\Rightarrow$

$C_{1i}X_i = 00 \Rightarrow S_i = 0$

$C_{1i}X_i = 01 \Rightarrow S_i = 1$

$C_{1i}X_i = 10 \Rightarrow S_i = 1$

$C_{1i}X_i = 11 \Rightarrow S_i = 0$



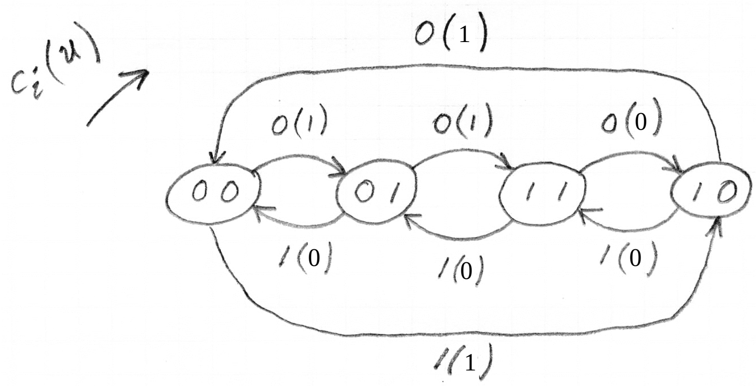
5a)

$$D_1 = q_1 \oplus s_i$$

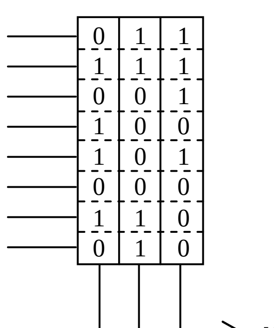
$$D_2 = \overline{c_i \oplus q_1}$$

$$u = \overline{c_{i+1}}$$

FÖRE CP							EFTER CP		
c_i	q_1	q_2	s_i	c_{i+1}	D_1	D_2	q_1^+	q_2^+	u
0	0	0	0	0	0	1	0	1	1
0	0	1	1	0	1	1	1	1	1
0	1	0	1	0	0	0	0	0	1
0	1	1	0	1	1	0	1	0	0
1	0	0	1	0	1	0	1	0	1
1	0	1	0	1	0	0	0	0	0
1	1	0	0	1	1	1	1	1	0
1	1	1	1	1	0	1	0	1	0



5b)



6. TILLSTÅNDSGRAFEN GER :

Q	X=0	X=1
A	H(1)	E(1)
B	C(0)	D(0)
C	D(0)	G(0)
D	H(1)	F(1)
E	G(0)	G(0)
F	D(0)	D(0)
G	H(1)	C(1)
H	F(0)	H(0)
	$Q^+(u)$	

SORTERA TILLSTÅNDEN I GRUPPER MED AV-SEENDE PÅ UTSIGNALERNA.

Σ_{11} (B, C, E, F, H)

Σ_{12} (A, D, G)

Q	X=0	X=1
A	Σ_{11}	Σ_{11}
B	Σ_{11}	Σ_{12}
C	Σ_{12}	Σ_{12}
D	Σ_{11}	Σ_{11}
E	Σ_{12}	Σ_{12}
F	Σ_{12}	Σ_{12}
G	Σ_{11}	Σ_{11}
H	Σ_{11}	Σ_{11}

HÄR FRAMGÅR DET ATT TILLSTÅNDENA B OCH H INTE TILLHÖR SAMMA GRUPP OCH ATT DE MÅSTE SKILJAS FRÅN C, E OCH F.

NY GRUPPINDELNING :

Σ_{21} (B)

Σ_{22} (C, E, F)

Σ_{23} (H)

Σ_{24} (A, D, G)

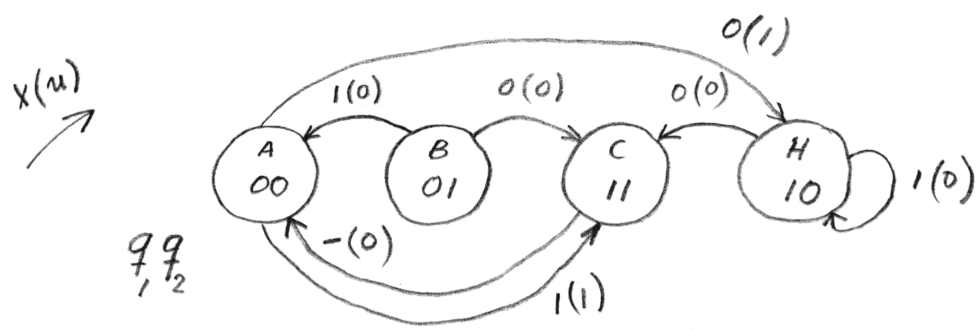
	Q	X=0	X=1
	A	Σ_{23}	Σ_{22}
	B	Σ_{22}	Σ_{24}
	C	Σ_{24}	Σ_{24}
SAMMA SOM A →	D	Σ_{23}	Σ_{22}
SAMMA SOM C →	E	Σ_{24}	Σ_{24}
SAMMA SOM C →	F	Σ_{24}	Σ_{24}
SAMMA SOM A →	G	Σ_{23}	Σ_{22}
	H	Σ_{22}	Σ_{23}

HÄR FRAMGÅR DET ATT GRUPPINDELNINGEN BLIR DENSAMMA SOM FÖREGÅENDE. DÄRMED ÄR VI KLARA OCH KAN KONSTATERA ATT TILLSTÅNDEN D, E, F OCH G ÄR ÖVERFLÖDIGA.

NY TABELL :

Q	X=0	X=1
A	H(1)	~~E~~ C (1)
B	C(0)	~~D~~ A (0)
C	~~D~~ A (0)	~~G~~ A (0)
H	~~F~~ C (0)	H(0)

NY TILLSTÅNDSGRAF OCH FÖRSLAG TILL KODNING :



x	q_1	q_2	D_1	D_2	q_1^+	q_2^+	u
0	0	0	1	0	1	0	1
0	0	1	1	1	1	1	0
0	1	0	1	1	1	1	0
0	1	1	0	0	0	0	0
1	0	0	1	1	1	1	1
1	0	1	0	0	0	0	0
1	1	0	1	0	1	0	0
1	1	1	0	0	0	0	0

$x \backslash q_1q_2$	00	01	11	10
0	1	1	0	1
1	1	0	0	1

$$D_1 = \bar{q}_2 + \bar{x}\bar{q}_1$$

$x \backslash q_1q_2$	00	01	11	10
0	0	1	0	1
1	1	0	0	0

$$D_2 = \bar{x}\bar{q}_1 q_2 + \bar{x} q_1 \bar{q}_2 + x \bar{q}_1 \bar{q}_2$$

$x \backslash q_1q_2$	00	01	11	10
0	1	0	0	0
1	1	0	0	0

$$u = \bar{q}_1 \bar{q}_2$$