Application Specific Integrated Circuits for Digital Signal Processing
Lecture 3

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Today's topic

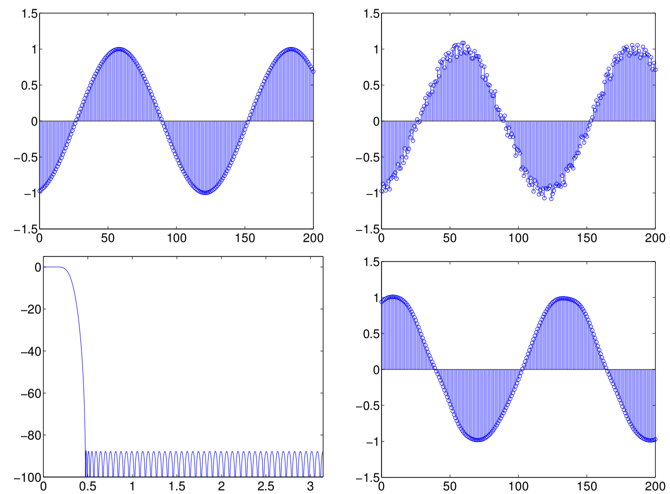
- Digital filters

Applications of Digital Filters

- Frequency-selective digital filters
  - Removal of noise and interfering signals
  - Separating/extracting signals
  - Sample rate changes
- Matched filters
  - Detect signal shape, filter impulse response is time-reversed signal
  - Used in e.g. radar
- Wavelets etc used for signal classification
- Adaptive filters
  - Filter coefficients are updated depending on current conditions
  - Track a disturbing signal
  - Adaptive noise removal
  - Communication channel adaptation

Noise removal example

- Additive white gaussian noise, e.g., from a transmission channel



Digital filters

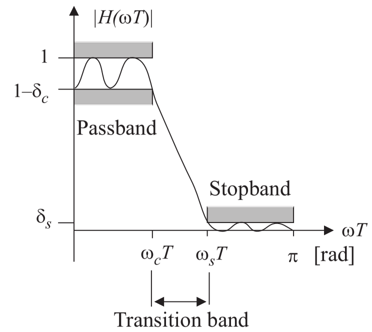
- A linear frequency-selective digital filter computes a weighted linear combination of inputs and/or previous outputs
- The weighting factors are selected to transmit some frequencies and attenuate some frequencies
- Transfer function

$$H(z) = \frac{\sum_{i=0}^{N} a_i z^{-i}}{\sum_{j=0}^{M} b_j z^{-j}} \quad (1)$$

- Filter order – $\max\{N, M\}$
- If more than one b_j is non-zero, the filter is an infinite-length impulse response (IIR) filter
- An IIR filter is a recursive algorithm
- If only one b_j (b_0) is non-zero, the filter is a finite-length impulse response (FIR) filter
- An FIR filter can be realized using either a non-recursive (preferred) or a recursive algorithm
- Often the denominator part is neglected for FIR filters

Digital filters

- Specifications (lowpass filter)



- Passband and stopband ripples – δ_c and δ_s
- Passband and stopband angles/edges – $\omega_c T$ and $\omega_s T$
- Passband attenuation – $A_{\text{max}} = -20\log_{10}(1-\delta_c)$
- Stopband attenuation – $A_{\text{min}} = -20\log_{10}(\delta_s)$

Digital filters

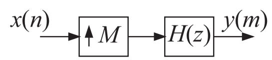
- All filters meeting the specification are equally good from a filtering point of view
- Need to determine an algorithm realizing the transfer function
- Many different algorithms proposed
- Algorithms differ in computational properties
  - Computational complexity
  - Stability
  - Sensitivity
  - Round-off noise
- Possibly additional optimization criteria, e.g., maximize SNR

Sample rate change

- Increase sample rate with an integer factor – Interpolation
- Insert zeros – expansion

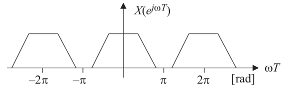
$$y(m) = \begin{cases} x(n) & n = 0, \pm\frac{m}{M}, \pm 2\frac{m}{M}, \dots \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

- Cascaded with lowpass filter

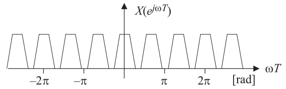


Interpolation spectrum

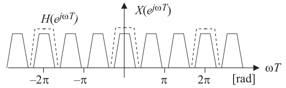
- Initial spectrum



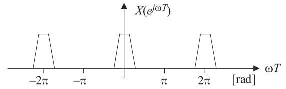
- After zero-insertion



- Filter specification

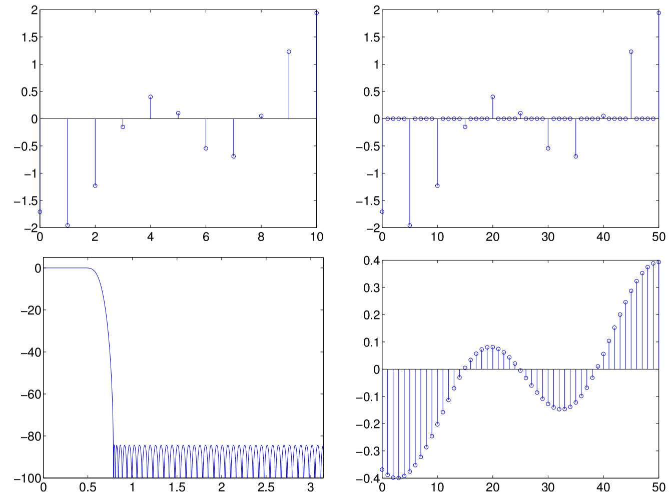


- Final spectrum



Interpolation example

- Interpolate by 5

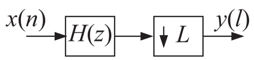


Sample rate changes

- Decrease sample rate with an integer factor – Decimation
- Throw signals away – compression

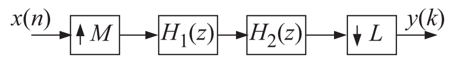
$$y(l) = x(l/M), \; l = 0, \pm M, \pm 2M, \ldots \qquad (3)$$

- Require a bandlimited signal to avoid aliasing
- Preceded by lowpass filter

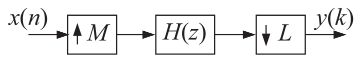


Sample rate changes

- Change sample rate with a rational factor $\frac{M}{L}$
- Solved by first interpolating with M and then decimating by L



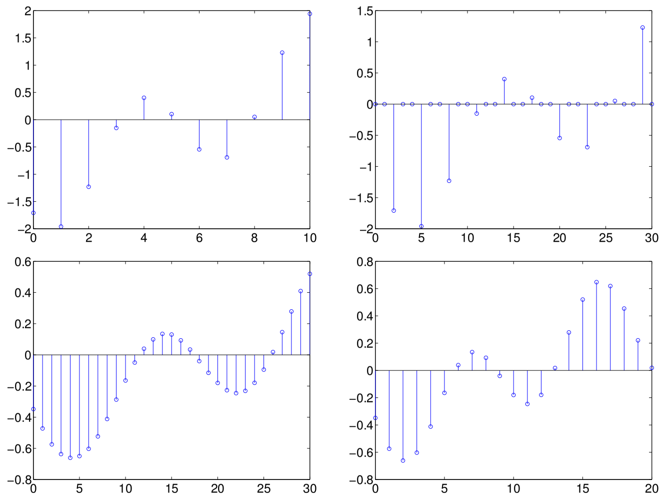
- The filter specifications can be merged to a single filter



- Intermediate filtering at M times the input rate
- For large (non-prime) M and L it is advantageous to use several stages
- Keep intermediate sample rate higher than signal bandwidth

Rational sample rate change example

- Interpolate by 3/2

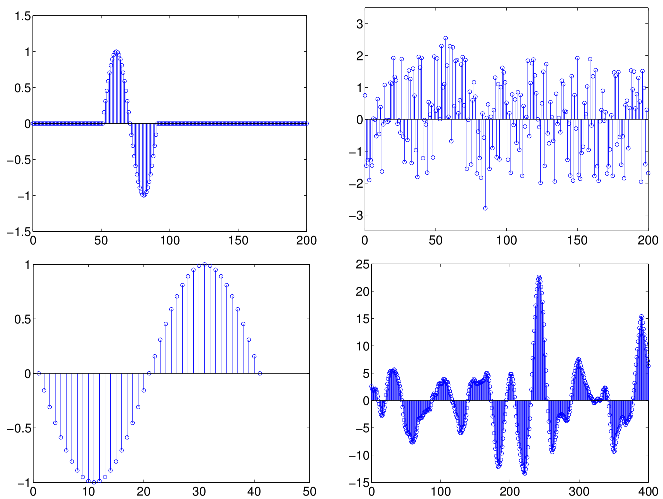


Matched filter

- A matched filter is used to detect a particular signal wave form in a received signal
- The matched filter is typically implemented as convolution with the time reversed wave form
- Used e.g. in RADAR to detect the reflected signal

Matched filter example

- Use a one period sinusoid as wave form

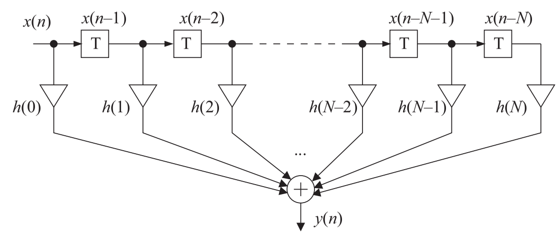


FIR filters

- Transfer function for N:th-order FIR filter

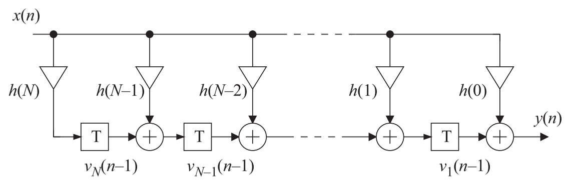
$$H(z) = \sum_{n=0}^{N} h(n)z^{-n} \tag{4}$$

- Direct form FIR filter



FIR filters

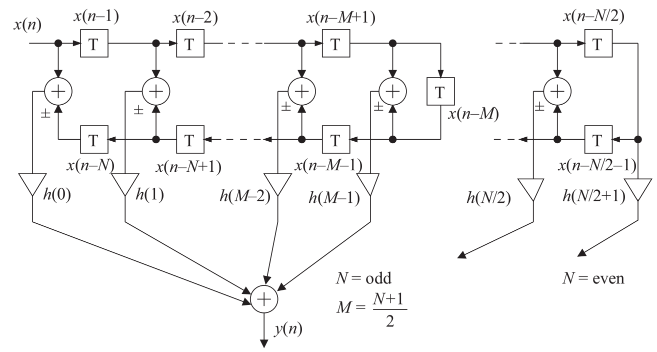
- Transposition – reverse signal flow graph
  - Input ⇔ Output
  - Adder ⇔ Branch
  - Multiplier and delay input ⇔ output
- A single input single output SFG keep the same transfer function when transposed
- Transposed direct form FIR filter



- Possibly different computational properties

FIR filters

- Complexity of N:th-order FIR filter
  - $N+1$ multiplications
  - N additions
  - N delays
- Linear-phase ⇔ coefficient symmetry/anti-symmetry



- $\left\lfloor \frac{N+1}{2} \right\rfloor$ multiplications

Half-band FIR filters

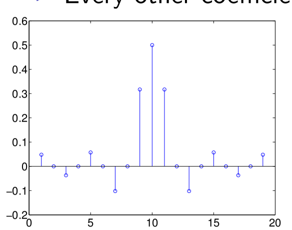
- Useful in interpolation and decimation by 2
- Even order FIR filters with complementary anti-symmetry around $\pi/2$

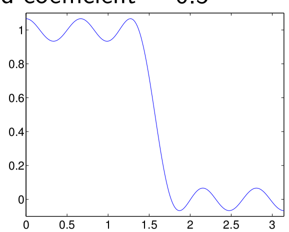
$$H_R(\omega T) = 1 - H_R(\pi - \omega T) \quad (5)$$

where H_R is the zero-phase magnitude function

$$H\left(e^{j\omega T}\right) = e^{j\phi(\omega T)} H_R(\omega T) \Rightarrow \left| H\left(e^{j\omega T}\right) \right| = |H_R(\omega T)| \quad (6)$$

- Every other coefficient = 0, mid-coefficient = 0.5



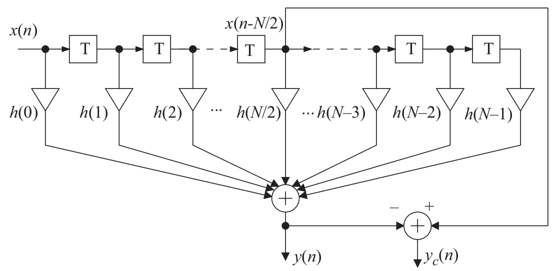


Complementary FIR filters

- Even order FIR filter

$$\left| H\left(e^{j\omega T}\right) + H_c\left(e^{j\omega T}\right) \right| = 1 \quad (7)$$

$$H(z) + H_c(z) = z^{-\frac{N}{2}} \Rightarrow H_c(z) = z^{-\frac{N}{2}} - H(z) \quad (8)$$



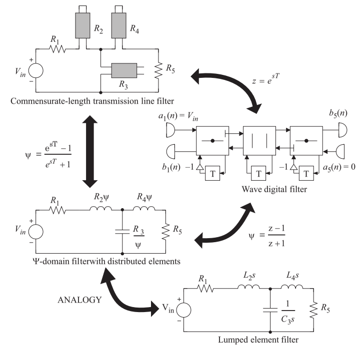
- One extra subtraction required to obtain both standard and complementary output

FIR vs. IIR filters

FIR	Feature	IIR
Easy Symmetry	Linear-phase	Not possible "Near" linear-phase
Stable	Stability	Possibly unstable
Small	Round-off noise	?
Small	Sensitivity	?
High	Complexity	Low

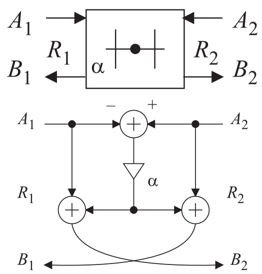
Wave digital filters (WDF)

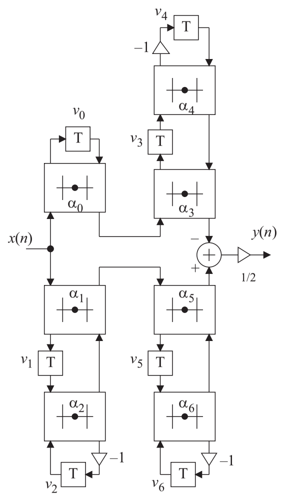
- Class of IIR filters derived from analog reference filters
- Inherit sensitivity from reference filter
- Guarantee stability



Lattice wave digital filters (LWDF)

- Composed of two parallel allpass filters
- Allpass filters composed of first- and second-order sections
- Symmetric two-port adaptor

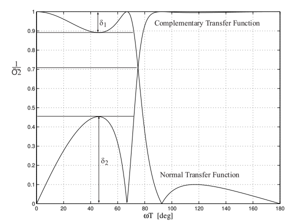




Properties of LWDFs

- Lowpass and highpass filters must be of odd order
- Number of multiplications = number of delays = filter order (canonic)
- Very low passband sensitivity and very high stopband sensitivity
- Simple modular building blocks
- Power complementary, add and subtract allpass branches
- Feldtkeller's equation

$$\left|H\left(e^{j\omega T}\right)\right|^2 + \left|H_c\left(e^{j\omega T}\right)\right|^2 = 1 \qquad (9)$$

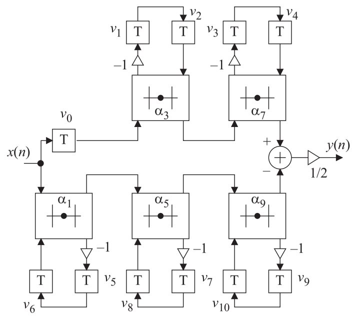


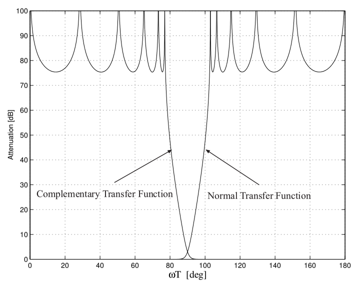
Bireciprocal LWDF

- Anti-symmetric power complementary around $\frac{\pi}{2}$ rad

$$\left|H\left(e^{j\omega T}\right)\right|^2 + \left|H\left(e^{j(\pi-\omega T)}\right)\right|^2 = 1 \qquad (10)$$

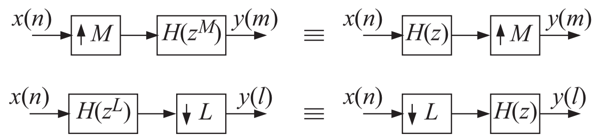
- Every other adaptor coefficient = 0





Polyphase decomposition

- In interpolation many computations are done on zeros
- In decimation many computed samples are thrown away
- More efficient if this is avoided
- Noble identities



- $MK-1$:th-order FIR filter

$$H(z) = \sum_{m=0}^{M-1} H_m(z^M)z^{-m} \qquad (11)$$

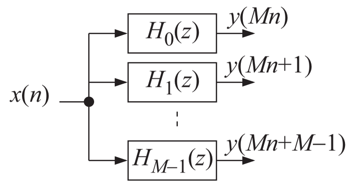
where

$$H_m(z^M) = \sum_{k=0}^{K-1} h(kM+m)z^{kM} \qquad (12)$$

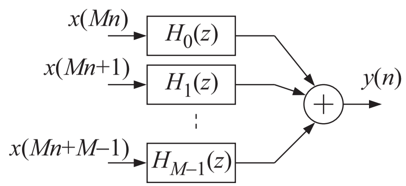
is the m:th polyphase branch

Polyphase decomposition

- Interpolation



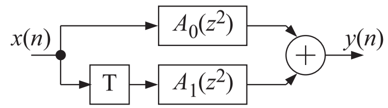
- Decimation



- Both operates at the lower sample rate
- Significant reduction in operations ($\approx M$)

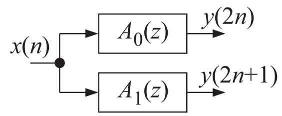
Polyphase decomposition

- Works for BLWDFs when $M=2$

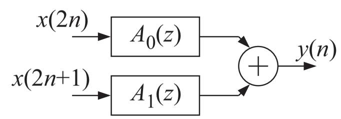


$$H(z) = \frac{A_0\left(z^2\right) + z^{-1}A_1\left(z^2\right)}{2} \qquad (13)$$

- Interpolation

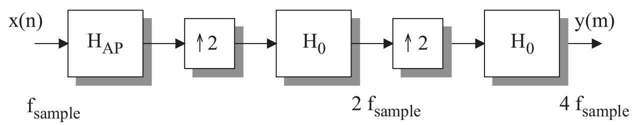


- Decimation



Case study 3: Interpolation filter

- Interpolation by four, from 1.6 to 6.4 MHz
- Constant group delay (linear phase)
- Use eleventh-order BLWDFs and a seventh-order allpass filter for phase compensation



Realization	First	Second	Third	Total
	Adaptor operations per input sample			
Direct	7	2×5	4×5	37
Polyphase	7	5	2×5	22

- With polyphase realization: $22 \times 1.6 \times 10^6 = 35.2$ Madaptors/s

Case study 3: Interpolation filter

- Single-rate realization