

**TSTE19**

**Power Electronics**

**Examination (TEN1)**

Time: Wednesday 30<sup>th</sup> March 2016 at 8:00 – 12:00

Place: TER4

Responsible teacher: Tomas Jonsson

Will visit the exam location at 8:45 and 10.

Number of tasks: 6

Number of pages: 12

Allowed aids: Calculator

Total points: 70

Notes: A pass on the exam requires approx. 30 points.

Describe your calculations clearly and detailed, explaining your methods, assumptions and equations used.

When a question is dependent on result from an earlier question which you fail to complete, then its better to assume some value for this result in order to present the calculation methodology.

### Question 1

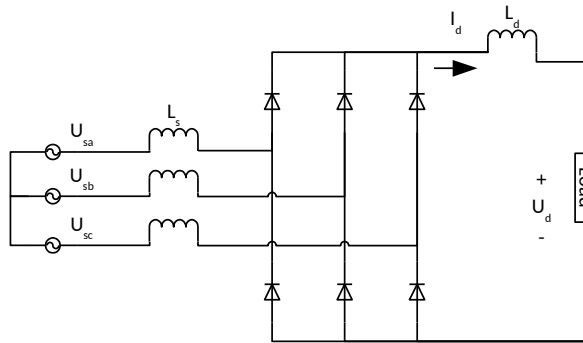


Figure 1

- a) A three-phase diode rectifier according to Figure 1 is connected to the sinusoidal ac-voltages  $U_{sa}$ ,  $U_{sb}$  and  $U_{sc}$ . The ac-voltages have 120 degree phase shift in the order a-b-c. Define the order in which the diodes will conduct by assigning numbers (1-6) to each diode.

(2)

**Answer: The upper diodes: 1, 3, 5. The lower diodes: 4, 6, 2**

- b) A diode rectifier according to Figure 2: has an inductance,  $L$ , and a voltage source  $E_d$  on the dc-side. How will the current,  $i$ , change if the dc-voltage  $E_d$  is increased.

(2)

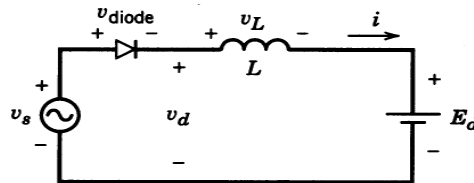


Figure 2:

**Answer: The current will decrease.**

- c) A full-bridge inverter is producing an output voltage,  $v_o$ , with the harmonic spectra according to Figure 3, where  $m_f$  is the frequency modulation index. Draw the voltage waveform and state the name of the PWM switching scheme used.

**Answer: See waveform W2 in question 2. Unipolar switching.**

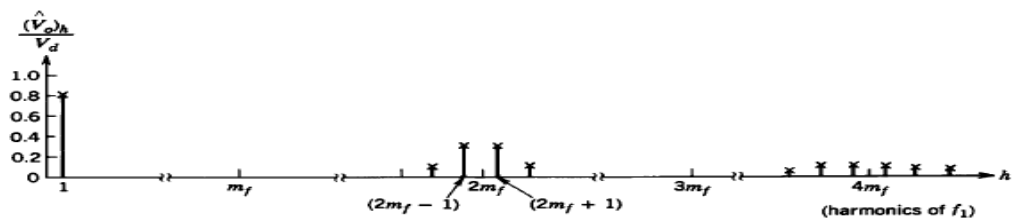


Figure 3

(2)

- d) List three types of semiconductors which has the capability to turn-off current by the action of a dedicated control input.

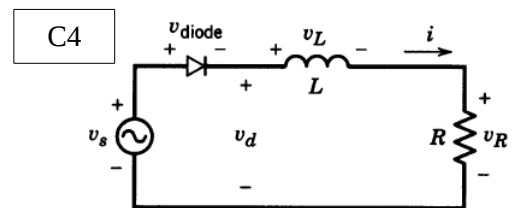
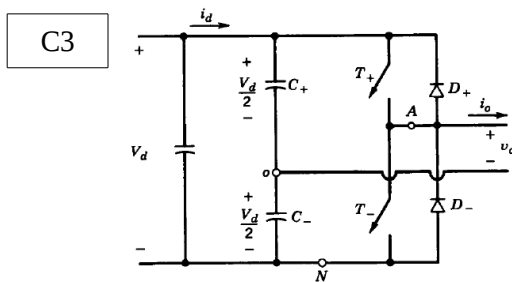
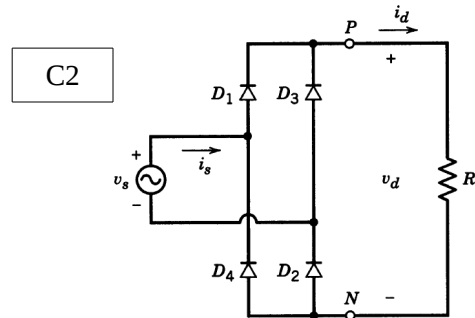
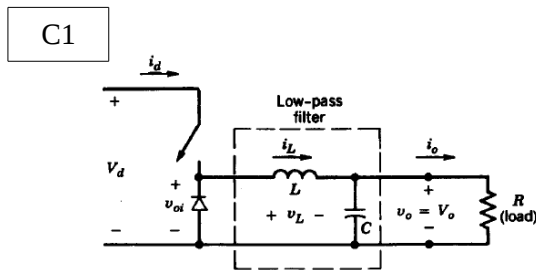
(3)

**Answer: BJT, MOSFET, JFET, IGBT, GTO, IGCT**

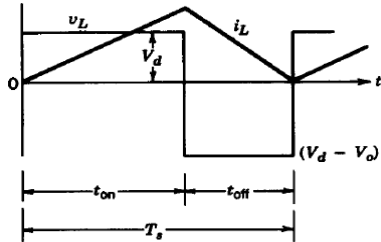
- e) For each of the four converter circuits below find the corresponding waveform figure that presents the main voltage and/or current. One wave form for each circuit only.

(3)

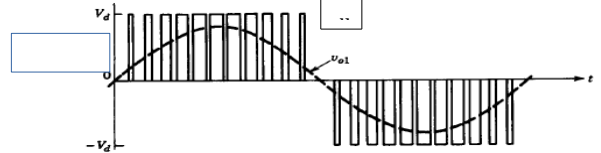
**Answer: C1 - W8, C2 - W5, C3 - W3, C4 - W6**



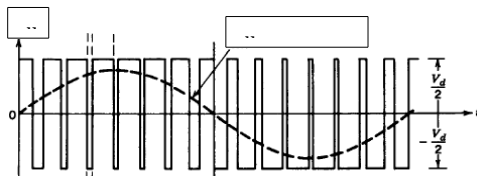
W1



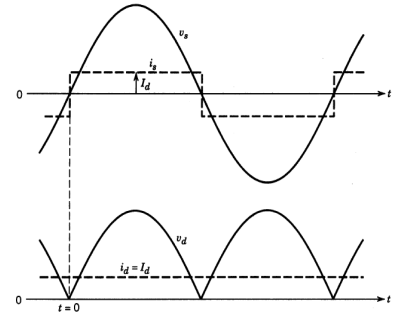
W2



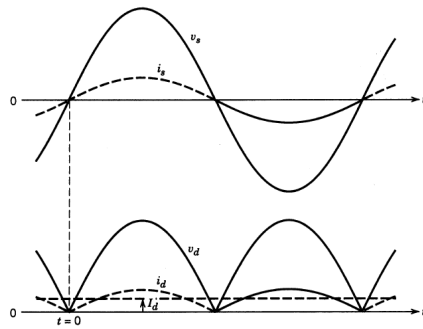
W3



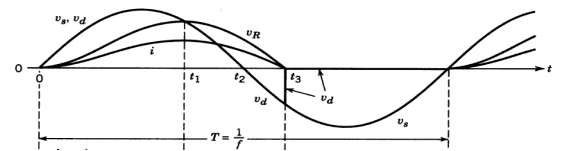
W4



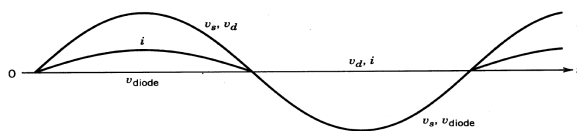
W5



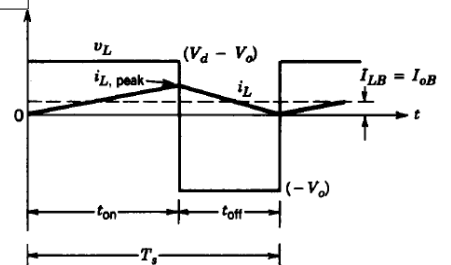
W6



W7



W8



## Question 2

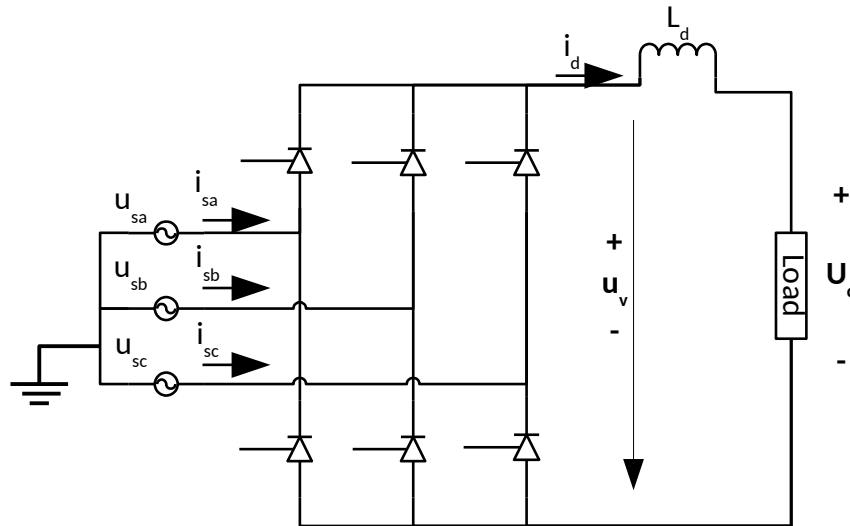


Figure 4

A three phase thyristor rectifier as shown by Figure 4 is connected to a voltage source with a phase-ground voltage  $U_s=240$  Vrms. The commutation inductance can be neglected and the dc-side current considered smooth through the inductance  $L_d$ .

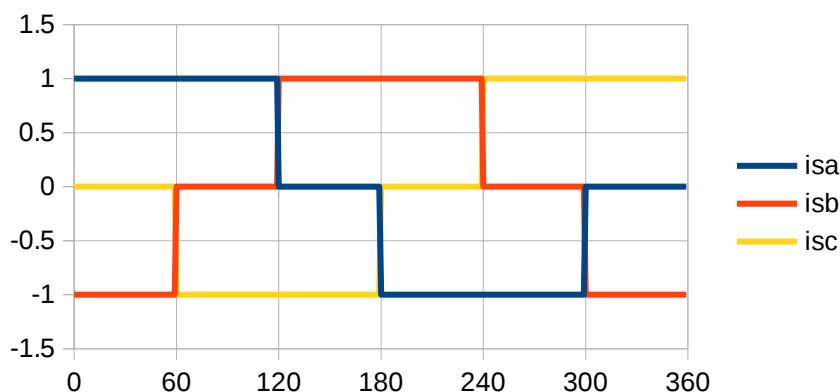
- Calculate the average dc-load voltage,  $U_d$ , for a firing angle  $\alpha=20$  deg.  
**Answer:  $U_d = 1.35 \cdot \sqrt{3} \cdot U_s \cdot \cos(\alpha) = 527$  V.**
- Determine the average dc-current considering a resistive load of 4 kW.  
**Answer:  $I_d = 4000/U_d = 7.6$  A**
- Determine the displacement power factor.  
**Answer:  $DPF = \cos(\alpha) = \cos(20) = 0.94$ .**
- Calculate the ac-side fundamental frequency rms current ( $i_{sa}$ ). Assume zero losses of the thyristor converter.  
**Answer:  $I_{sa} = 4000/DPF/3/240 = 5.9$  A.**
- Draw the waveform of the three phase source currents,  $i_{sa}$ ,  $i_{sb}$  and  $i_{sc}$ .

(2)

(2)

(2)

(2)



(2)

f) Determine the reactive power.

**Answer:  $Q = 3 \cdot U_s \cdot I_s \cdot \sin(\alpha) = 1.46 \text{ kVAr}$**

(2)

### Question 3

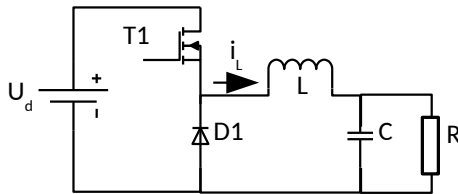


Figure 5

In the buck converter in Figure 5 which is feeding a resistive load,  $R=2 \text{ ohm}$ , the current,  $i_L$ , should be continuous. The MOSFET T1 is operated with a switching frequency  $f_{sw} = 120 \text{ kHz}$  and a duty cycle of 0.75. The input voltage  $U_d = 12 \text{ V}$ .

a) Determine the average output voltage,  $U_o$ , across the load resistance.

**Answer:  $U_o = D \cdot U_d = 9 \text{ V}$**

(3)

b) Determine the average current through the inductance L.

**Answer:  $i_L = U_o / R = 4.5 \text{ A}$**

(2)

c) Determine the minimum inductance related to continuous current and calculate the peak current through the inductance L. Assume the output capacitance C is large enough to get a smooth voltage.

**Answer:  $L_{min} = (U_d - U_o) \cdot D \cdot t_{sw} / (2 \cdot i_L) = 2.1 \text{ } \mu\text{H}$ ,  $i_{L,max} = 2 \cdot i_L = 9 \text{ A}$ .**

(4)

d) Draw the waveforms of the inductor voltage, inductor current and MOSFET current during one switching cycle considering the minimum inductance for continuous current.

**Answer: See figure W8 of question 2.**

(2)

### Question 4

A buck converter as in Figure 5, is operated at the limit between continuous and discontinuous current with a peak inductor current of 20 A. The converter is operated with a switching frequency  $f_{sw} = 50 \text{ kHz}$  and a duty cycle of 0.75 which gives an output voltage of 12 V.

a) Calculate the RMS value of the MOSFET T1 current.

(4)

$$\text{Answer: } I_{t1RMS} = \sqrt{\frac{1}{t_{sw}} \int_0^{Dt_{sw}} \left(\frac{i_{Lmax}}{Dt_{sw}} t\right)^2 dt} = \sqrt{\frac{1}{t_{sw}} \left(\frac{i_{Lmax}}{Dt_{sw}}\right)^2 \left[\frac{t^3}{3}\right]_0^{Dt_{sw}}} = i_{Lmax} \sqrt{\frac{D}{3}}$$

$$I_{t1RMS} = 4.5 \text{ A}$$

- b) Calculate the conduction losses in the MOSFET T1 if the on-state resistance  $R_{ds(on)} = 0.05 \text{ ohm}$ .

(3)

$$\text{Answer: } P_{T1} = R_{ds(on)} \cdot I_{t1RMS}^2 = 5.0 \text{ W}$$

- c) Determine the efficiency of the converter assuming switching losses and diode losses to be negligible.

(2)

**Answer: The output power  $P_o$  is defined by the average inductor current and the output voltage.  $P_o = 12 \cdot 10 = 120 \text{ W}$ . The efficiency  $\eta = P_o / (P_o + P_{T1}) = 120 / (120 + 5) = 0.96$**

- d) Determine the required thermal resistance of the heatsink ( $R_{thHA}$ ) for the MOSFET T1 in order to keep the heatsink temperature,  $T_H \leq 70 \text{ }^\circ\text{C}$  and the junction temperature,  $T_J \leq 100 \text{ }^\circ\text{C}$ . The MOSFET has a thermal resistance  $R_{thJH} = 12.0 \text{ }^\circ\text{C/W}$ . The ambient temperature,  $T_A = 25 \text{ }^\circ\text{C}$ .

(4)

**Answer: Based on  $T_J \leq 100 \text{ }^\circ\text{C}$ :  $R_{thHA} = (T_{Jmax} - T_A) / P_{T1} - R_{thJH} = 3 \text{ K/W}$**

**Based on  $T_H \leq 70 \text{ }^\circ\text{C}$ :  $R_{thHA} = (T_{Hmax} - T_A) / P_{T1} = 9 \text{ K/W}$**

**Consequently, to satisfy both  $T_{Jmax}$  and  $T_{Hmax}$  the minimum  $R_{thHA} = 3 \text{ K/W}$  should be selected.**

## Question 5

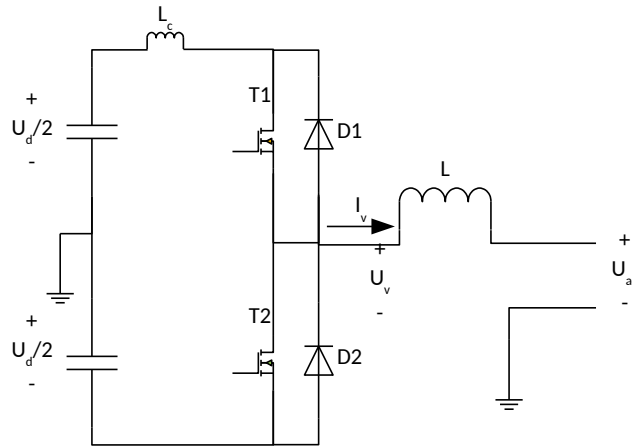


Figure 6

A half-bridge voltage source converter is connected between a dc-source and an ac-load as shown by Figure 6. The dc-side has a parasitic inductance  $L_c = 70 \text{ nH}$ . The control of the switched output voltage is done through pulse width modulation (PWM) with a switching frequency  $f_{sw} = 950 \text{ Hz}$ , in order to obtain a 50 Hz voltage component with a defined magnitude.

- a) What is the maximum amplitude modulation ratio,  $m_a$ , for a dc-side voltage,  $U_d = 48 \text{ V}$  when the rms magnitude of the 50 Hz component,  $U_{v1}$ , shall be controllable between  $10 \text{ V}_{rms}$  and  $15 \text{ V}_{rms}$ .

(4)

**Answer:  $m_{max} = \sqrt{2} * 15 / (48/2) = 0.88$ .**

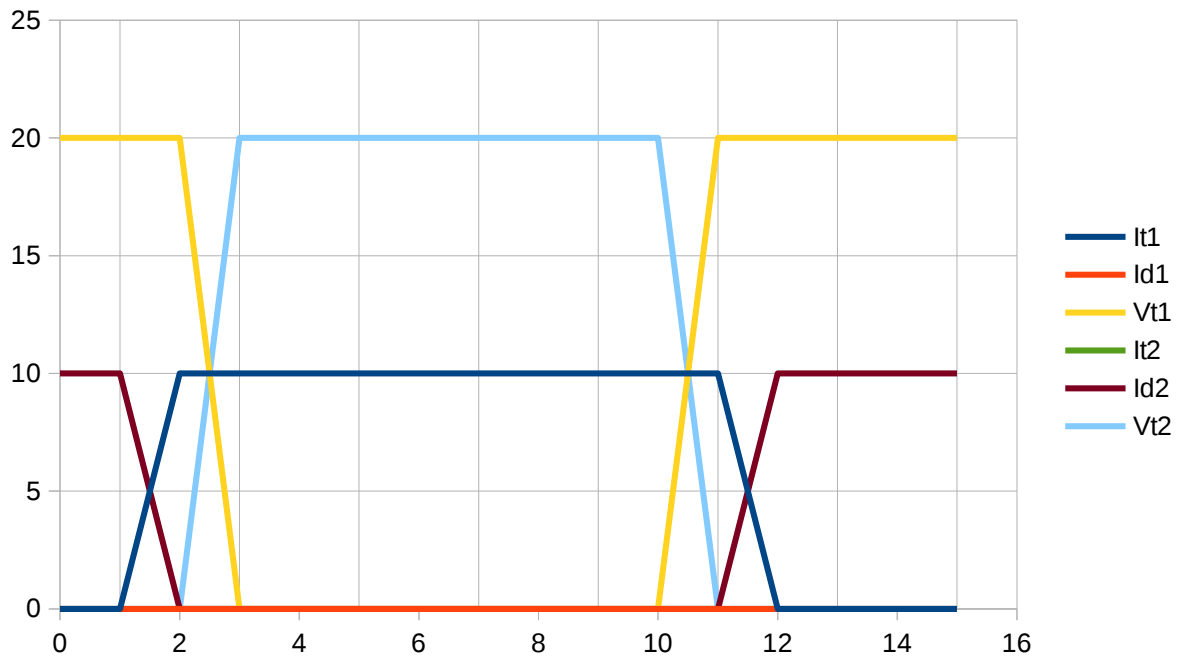
- b) Consider an interval during switching where initially Diode D2 is conducting and consequently the current  $I_v$  is positive. Draw the waveform of current and voltage of T1, D1, T2 and D2 for the switching sequence listed below. Assume the current  $I_v$  is constant during this interval. Consider the rise and fall times for both current and voltage to be  $1 \mu\text{s}$ .

(3)

- $t = 1 \mu\text{s}$ , T2 turn-off
- $t = 5 \mu\text{s}$ , T1 turn-on
- $t = 10 \mu\text{s}$ , T1 turn-off



○  $t = 15 \mu\text{s}$ , T2 turn-on



(3)

- c) Determine the rate-of-rise of current through a MOSFET at turn-on, considering very fast gate control causing the voltage across the MOSFET to fall to zero instantaneously. E.g. consider the case when D2 is conducting, T2 is off and T1 is turning on. The dc-voltage  $U_d$  shall be considered to be constant, related to a large capacitance.  
**Answer:  $dI/dt = U_d/L_c = 48/70 \text{ [V/ns]} = 0.69 \text{ V/ns}$ .**

## Question 6

A gate drive shall be designed for the half-bridge converter in Figure 6. An equivalent circuit to analyze the switching of the MOSFET T1 is shown in Figure 7. Here the MOSFET is represented by the capacitances  $C_{gd}=150$  pF,  $C_{gs}=1.5$  nF and a current source  $I_{ds}$ . Figure 8 shows the switching waveforms of the voltages and current through the MOSFET. The gate is controlled by the voltage  $V_{GG}$  which switches instantaneously from 0 V to 15 V at turn-on. The current  $I_v$  can be considered to be constant, and flowing out of the converter.

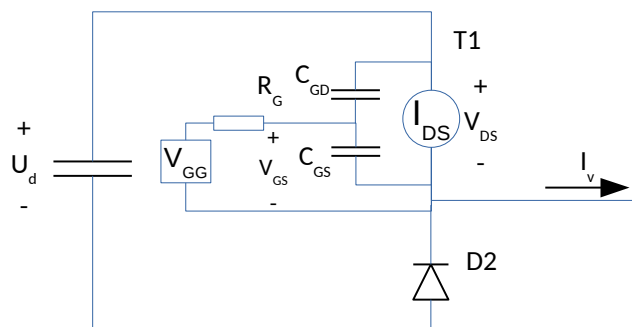


Figure 7

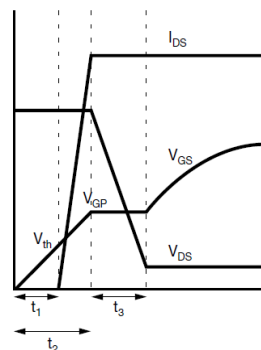


Figure 8

- a) Determine the turn-on delay,  $t_1$  of the MOSFET, from the time when  $V_{GG}$  switches from 0 V to 15 V to the time when the current  $I_{ds}$  starts rising.

$R_g = 680$  ohm,  $V_{th} = 3$  V.

**Answer: The gate voltage rises related to charging of the gate capacitance  $C_{gs}$  through the gate resistor from a constant voltage source**

$$V_{GG} = 15 \text{ V. } V_{GS} = V_{GG} \left( 1 - \exp\left(\frac{-t}{R_g \cdot C_{gs}}\right) \right)$$

(4)

$$t_1 = -R_g \cdot C_{gs} \ln\left(\frac{V_{GG} - V_{th}}{V_{GG}}\right) = 0.23 \mu s$$

- b) Determine the total delay,  $t_2$  of the MOSFET, until full current  $I_v = 12 \text{ A}$  is reached. Consider a rate of rise of drain current  $dI_{ds}/dt = 10 \text{ A}/\mu s$ .

(3)

**Answer: The current raises linearly up to the load current  $I_v=12 \text{ A}$ . We assume that the diode has an ideal turn-off at this point.**

$$t_2 = t_1 + \frac{I_v}{dI_{ds}/dt} = 1.4 \mu s$$

- c) Determine the rate of decay of the voltage across the MOSFET,  $dV_{ds}/dt$ , during the time when the gate voltage  $V_{GS}$  is constant at the plateau level  $V_{GP} = 7 \text{ V}$ .

(5)

**Answer: The gate current is  $i_g = \frac{V_{GG} - V_{GP}}{R_g}$ . The voltage derivative**

**$dV_{ds}/dt$  will be equal to the voltage derivative of the  $C_{gd}$  capacitance. The relation between current and voltage derivative for the  $C_{gd}$  capacitance**

**is  $i_{gd} = C_{gd} \cdot \frac{dV_{ds}}{dt}$ . Since the gate voltage is constant all gate current is**

**passing through the  $C_{gd}$  capacitance.**

$$i_{gd} = i_g \Rightarrow \frac{dV_{ds}}{dt} = \frac{V_{GG} - V_{GP}}{R_g \cdot C_{gd}} = 78 \text{ V}/\mu s$$

## Formula collection TSTE19 Power Electronics

### Fourier series coefficients using symmetry,

Even	$f(-t) = f(t)$	$b_h = 0$	$a_h = \frac{2}{\pi} \int_0^{\pi} f(t) \cos(h\omega t) d(\omega t)$
Odd	$f(-t) = -f(t)$	$a_h = 0$	$b_h = \frac{2}{\pi} \int_0^{\pi} f(t) \sin(h\omega t) d(\omega t)$
Half-wave	$f(t) = -f(t + \frac{1}{2}T)$	$a_h = b_h = 0$ for even $h$	
		$a_h = \frac{2}{\pi} \int_0^{\pi} f(t) \cos(h\omega t) d(\omega t)$ for odd $h$	
		$b_h = \frac{2}{\pi} \int_0^{\pi} f(t) \sin(h\omega t) d(\omega t)$ for odd $h$	
Even quart-wave	Even and half-wave	$b_h = 0$ for all $h$	
		$a_h = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} f(t) \cos(h\omega t) d(\omega t)$ for odd $h$ , $a_h = 0$ for even $h$	
Odd quarter-wave	Odd and half-wave	$a_h = 0$ for all $h$	
		$b_h = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} f(t) \sin(h\omega t) d(\omega t)$ for odd $h$ , $b_h = 0$ for even $h$	

### Undamped resonant circuits

Even	$f(-t) = f(t)$	$b_h = 0$	$a_h = \frac{2}{\pi} \int_0^{\pi} f(t) \cos(h\omega t) d(\omega t)$
Odd	$f(-t) = -f(t)$	$a_h = 0$	$b_h = \frac{2}{\pi} \int_0^{\pi} f(t) \sin(h\omega t) d(\omega t)$
Half-wave	$f(t) = -f(t + \frac{1}{2}T)$	$a_h = b_h = 0$ for even $h$	
		$a_h = \frac{2}{\pi} \int_0^{\pi} f(t) \cos(h\omega t) d(\omega t)$ for odd $h$	
		$b_h = \frac{2}{\pi} \int_0^{\pi} f(t) \sin(h\omega t) d(\omega t)$ for odd $h$	
Even quart-wave	Even and half-wave	$b_h = 0$ for all $h$	
		$a_h = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} f(t) \cos(h\omega t) d(\omega t)$ for odd $h$ , $a_h = 0$ for even $h$	
Odd quarter-wave	Odd and half-wave	$a_h = 0$ for all $h$	
		$b_h = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} f(t) \sin(h\omega t) d(\omega t)$ for odd $h$ , $b_h = 0$ for even $h$	

## Integration rules

$$\int_a^b f(x)dx = \int_A^B f(g(t))g'(t)dt \text{ if } a = g(A), b = g(B) \text{ and } g \text{ is monotone in } [A,B]$$

$$\int_a^b \sin(x)dx = [-\cos(x)]_a^b$$

$$\int_a^b \cos(x)dx = [\sin(x)]_a^b$$