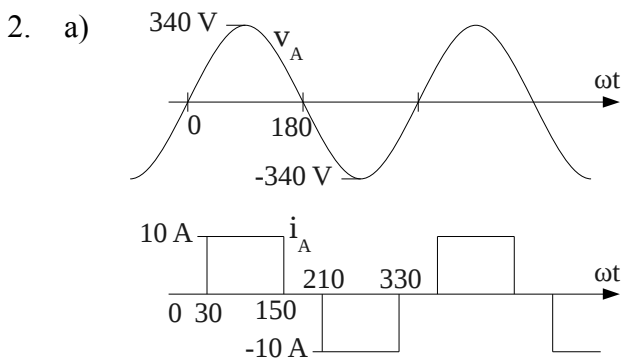


1. a) Shorter on-time for switches make the inductor currents have less time to build up. The maximum value of the generated magnetic flux is therefore less, and the physical size can thus be reduced.
- b) The frequency of the driving voltage controls the speed of the synchronous motor.
- c) Yes, example of this is that the fundamental of a square wave is 1.27 times the peak value.
- d) IGBT = Insulated Gate Bipolar Transistor
- e) The Cúk converter generates a negative output voltage.



- b) The pulse shape of  $i_A(t)$  is odd and half-wave.

1st harmonic (also known as the fundamental,  $h=1$ ):

$$a_1 = 0$$

$$b_1 = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} f(t) \sin(h\omega t) d(\omega t) = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} 10 \cdot \sin(\omega t) d(\omega t) = \frac{4 \cdot 10}{\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin(\omega t) d\omega t =$$

$$= \frac{4 \cdot 10}{\pi} [-\cos(\omega t)]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{4 \cdot 10}{\pi} \cdot \frac{\sqrt{3}}{2} = 11 \text{ A}$$

2nd harmonic ( $h=2$ ):

$$a_2 = 0; b_2 = 0; \text{ 1st harmonic amplitude} = 0 \text{ V}$$

3rd harmonic ( $h=3$ ) (not really part of the question):

$$a_3 = 0$$

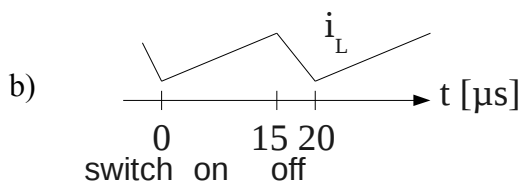
$$b_3 = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} f(t) \sin(3\omega t) d(\omega t) = \frac{4}{\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 10 \sin(3\omega t) d(\omega t) = \frac{4 \cdot 10}{\pi \cdot 3} \int_{\frac{3 \cdot \pi}{6}}^{\frac{3 \cdot \pi}{2}} \sin(\omega t) d(\omega t) =$$

$$= \frac{4 \cdot 10}{3 \pi} [-\cos(\omega t)]_{\frac{\pi}{2}}^{\frac{3 \pi}{2}} = 0$$

The 1<sup>st</sup> harmonic (fundamental) = 11 A, the 2<sup>nd</sup> harmonic = 0A.

3. a) Efficiency  $\eta = P_{out} / P_{in} = (250 * 6) / (400 * 4) = 1500 / 1600 = 93.75 \%$
- b) Apparent output power is  $250 * 6$  A, maximum switch current = 6A (simplified assumption), maximum switch voltage =  $\sqrt{(2)} \cdot 400V$ , 4 switches. Switch utilization ratio = apparent output power / total switching capability =  $\frac{250 \cdot 6}{4 \cdot \sqrt{(2)} \cdot 400 \cdot 6} = 11 \%$
- c) Total power loss equal to  $1600 - 1500 = 100$  W. That flows from the heat sink to the ambient. Heat sink to ambient thermal resistance =  $0.8$  C/W  $\Rightarrow$  increase of heat sink temperature by  $0.8 * 100 = 80$  C. The heat sink will then have a temperature of  $50 + 80 = 130$  C. Each thyristor dissipates  $100/4 = 25$  W. The junction to heatsink thermal resistance =  $2$  C/W gives an increase of  $2 * 25 = 50$  C between heat sink and junction. Junction temperature is then  $130 + 50 = 180$  C.

4. a)  $v_o = \frac{v_d}{1-D} = \frac{12}{1-0.75} = \frac{12}{0.25} = 48V$



- c)  $R = 80 \Omega$ ,  $v_o = 48V \Rightarrow I_o = 48/80 = 0.6$  A. While the switch on is the voltage across L  $v_L = 12V$ . Current increase during that time is  $\frac{v_L \cdot T_{sw} \cdot D}{L} = \frac{12 \cdot 0.75}{50 \cdot 10^{-3} \cdot 50 \cdot 10^{-6}} = 3.6A$

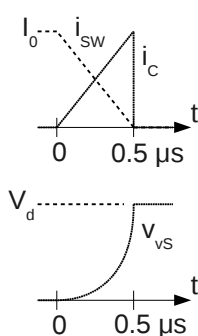
Power in should equal power out.  $P_{out} = 0.6 * 48$ ,  $P_{in} = 12 * I_{in} \Rightarrow I_{in,avg} = \frac{0.6 \cdot 48}{12} = 2.4A$

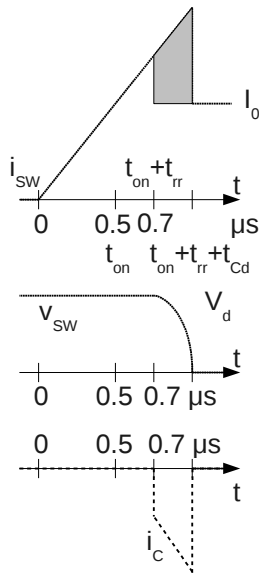
Input current is only flowing during  $T_{on}$ , and is triangle shaped  $\Rightarrow$

$$I_{in,avg} = \frac{(I_{in,low} + I_{in,high}) \cdot 0.75}{2} \Rightarrow \text{Combine equations} \Rightarrow I_{in,high} = \frac{\frac{I_{in,avg} \cdot 2}{0.75} + 3.6}{2} = 5A$$

This also gives  $I_{low} = 1.4$  A.

5. a)  $V_{sw}$  increase quadratically from 0 to  $V_d$  after  $0.5 \mu s$ .





- b)  $i_{SW}$  increase linearly for 0.5  $\mu s$  reaching  $I_0$ . The  $i_{SW}$  increase continuous for another 0.2  $\mu s$  due to D reverse recovery. Finally is the capacitor discharged while the switch current is still increasing. Once C is discharged the  $i_{SW}$  current is back to  $I_0$ .  $v_{SW}$  does not start to decrease until the capacitor starts to discharge after  $t_{on} + t_{rr}$ .

- c) Calculate charge stored in C. Linearly increasing current at switch turn-off charges C.

Should charge to 380V after 0.5  $\mu s$ , where peak current is 10 A.

$$i_C(t) = \begin{cases} \frac{I_0 \cdot t}{t_{on}} & 0 < t < t_{on} \quad (t_{on} = 0.5 \cdot 10^{-6}) \\ 0 & \text{otherwise} \end{cases}$$

$$Q_C = \int_0^{t_{on}} i_C(t) dt = \left[ \frac{I_0 \cdot t^2}{2 \cdot t_{on}} \right]_0^{t_{on}} = \frac{I_0 \cdot t_{on}^2}{2 t_{on}} = \frac{I_0 \cdot t_{on}}{2}$$

$$v_C = \frac{Q}{C} \rightarrow C = \frac{Q}{v_C} = \frac{I_0 \cdot 0.5 \cdot 10^{-6}}{2 \cdot V_d} = \frac{10 \cdot 0.5 \cdot 10^{-6}}{2 \cdot 380} = 6.6 \text{ nF}$$

At switch turn-on is  $i_{SW}$  increasing to  $I_0$  after  $t_{on}$ , then further increase during  $t_{rr}$ . Then is C discharged while  $i_{SW}$  still increases during  $t_{cd}$ . The current through the capacitance will be zero until  $t_{on} + t_{rr}$  when it makes a step increase, then increase linearly, and drop back to zero at  $t_{on} + t_{rr} + t_{cd}$ .

$$i_C(t) = \begin{cases} 0 & t < t_{on} + t_{rr} \\ I_0 \left(1 + \frac{t_{rr} + t}{t_{on}}\right) & t_{on} + t_{rr} < t < t_{on} + t_{rr} + t_{Cd} \\ 0 & t > t_{on} + t_{rr} + t_{Cd} \end{cases}$$

$$Q_C = \int_{t_{on} + t_{rr}}^{t_{on} + t_{rr} + t_{Cd}} i_C(t) dt = I_0 \left( \frac{t_{on} + t_{rr} + t_{Cd}}{2t_{on}} \right) (t_{on} + t_{rr} + t_{Cd}) - I_0 \left( \frac{t_{on} + t_{rr}}{2t_{on}} \right) (t_{on} + t_{rr}) - I_0 t_{Cd}$$

Charge and discharge equal

$$\frac{I_0 \cdot t_{on}}{2} = I_0 \left\{ \left( \frac{t_{on} + t_{rr} + t_{Cd}}{2t_{on}} \right) (t_{on} + t_{rr} + t_{Cd}) - \left( \frac{t_{on} + t_{rr}}{2t_{on}} \right) (t_{on} + t_{rr}) - t_{Cd} \right\}$$

$$1 = \left(1 + \frac{t_{rr}}{t_{on}} + \frac{t_{Cd}}{t_{on}}\right)^2 - \left(1 + \frac{t_{rr}}{t_{on}}\right)^2 - 2 \frac{t_{Cd}}{t_{on}}$$

$$t_{Cd} = \sqrt{t_{on}^2 + t_{rr}^2} - t_{rr}$$

$$i_{SW,peak} = I_0 \frac{t_{on} + t_{rr} + t_{Cd}}{t_{on}} = I_0 \frac{t_{on} + t_{rr} + \sqrt{t_{on}^2 + t_{rr}^2} - t_{rr}}{t_{on}} = \frac{10 \cdot (0.5 + \sqrt{0.5^2 + 0.2^2}) \cdot 10^{-6}}{0.5 \cdot 10^{-6}} = 20.77 A$$