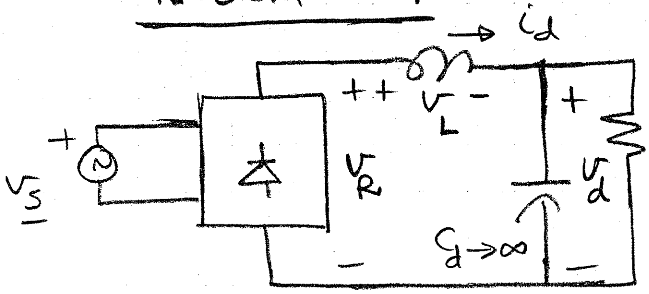
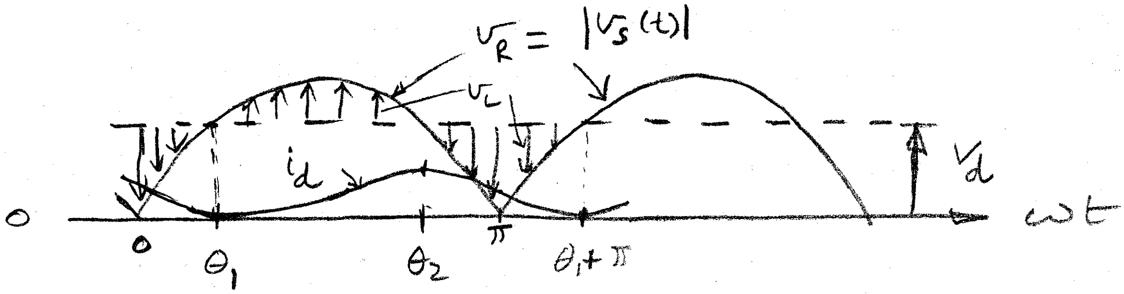
Problem 5-7



$v_d(t) \simeq V_d = 0.9 V_s$



For $0 < \theta < \pi$

$$i_d(\theta) = i_d(0) + \frac{1}{\omega L_d}\int_0^\theta v_L \cdot d(\omega t)$$

$$= i_d(0) + \frac{1}{\omega L_d}\left[\int_0^\theta (\sqrt{2} V_s \sin \omega t - V_d) \cdot d(\omega t)\right]$$

$$= i_d(0) + \frac{1}{\omega L_d}\left[\sqrt{2} V_s \cos(\omega t)\Big|_\theta^0 - V_d \theta\right]$$

$$= i_d(0) + \frac{1}{\omega L_d}\left[\sqrt{2} V_s (1 - \cos \theta) - V_d \theta\right]$$

To obtain $i_d(0)$; we know that $i_d(\theta_1) = 0$ in the limiting case. Therefore, at $\theta = \theta_1$

$$i(\theta_1) = 0 = i_d(0) + \frac{\sqrt{2} V_s}{\omega L_d} - \frac{\sqrt{2} V_s}{\omega L_d}\cos\theta_1 - \frac{V_d \theta_1}{\omega L_d}$$

[Note: $\sqrt{2} V_s \cos\theta_1 = V_d$]

$$= i_d(0) + \frac{\sqrt{2} V_s}{\omega L_d} - \frac{V_d}{\omega L_d}(1 + \theta_1)$$

$$\therefore \quad i_d(0) = \frac{V_d}{\omega L_d}(1+\theta_1) - \frac{\sqrt{2}V_s}{\omega L_d}$$

and,

$$i_d(\theta) = \frac{V_d}{\omega L_d}(1+\theta_1) - \cancel{\frac{\sqrt{2}V_s}{\omega L_d}} + \cancel{\frac{\sqrt{2}V_s}{\omega L_d}} - \frac{\sqrt{2}V_s}{\omega L_d}\cos\theta - \frac{V_d}{\omega L_d}\theta$$

$$= \frac{V_d}{\omega L_d}(1+\theta_1 - \theta) - \frac{\sqrt{2}V_s}{\omega L_d}\cos\theta$$

Average current I_d:

$$I_d = \frac{1}{\pi}\int_0^{\pi} i_d(\theta)\cdot d\theta$$

$$= \frac{1}{\pi}\left[\frac{V_d}{\omega L_d}\left\{(1+\theta_1)\pi - \frac{\pi^2}{2}\right\} - \cancel{\frac{\sqrt{2}V_s}{\omega L_d}\sin\theta\Big|_0^{\pi}} \rightarrow 0\right]$$

$$= \frac{V_d}{\omega L_d}\left(1+\theta_1 - \frac{\pi}{2}\right)$$

$$V_d = 0.9\,V_s \quad \text{and} \quad \theta_1 = \cos^{-1}\left(\frac{V_d}{\sqrt{2}V_s}\right) = 0.881 \text{ rad}$$

$$\therefore \quad I_d = \frac{0.9V_s}{\omega L_d}\left(1 + 0.881 - \frac{\pi}{2}\right) = 0.279\frac{V_s}{\omega L_d}$$

$$\therefore \quad L_{d,min} = 0.279\frac{V_s}{\omega I_d}$$