NOISE

Look at a signal voltage $v(t)$ that interferes from a noise voltage v_n, which means that $v_{tot}(t) = v(t) + v_n(t)$. The power of the signal denotes P_{signal} and the power of the noise P_{noise}.

Following performance measures is defined:

- Signal Noise Ratio, $SNR = 10 \cdot {}^{10}\log \frac{P_{signal}}{P_{noise}}$
- Dynamic Range, $DR = 20 \cdot {}^{10}\log \frac{|v_{in,max}(t)|}{|v_{in,min}(t)|}$

- If $|v_{in}(t)| > |v_{in,max}(t)|$ you get distorsion.
- If $|v_{in}(t)| < |v_{in,min}(t)|$ the signal gets drowned in the noise.

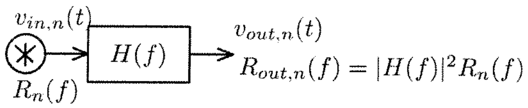
- Noise power, P_{noise}, defines as $P_{noise} = \frac{1}{T}\int_0^T v_n^2(t)dt$
- Also, if $V_n(f)$ is the Fourier transform of $v_n(t)$, $P_{noise} = \int_{-\infty}^{\infty} V_n^2(f)df$
- $V_n^2(f)$ is the *spectral density* $R_n(f)$ and $R_n(f)$ is the Fourier transform of the *autocorrelation function* $r_n(t)$. I.e. $V_n^2(f) = R_n(f) = \mathcal{F}\{r_n(t)\}$

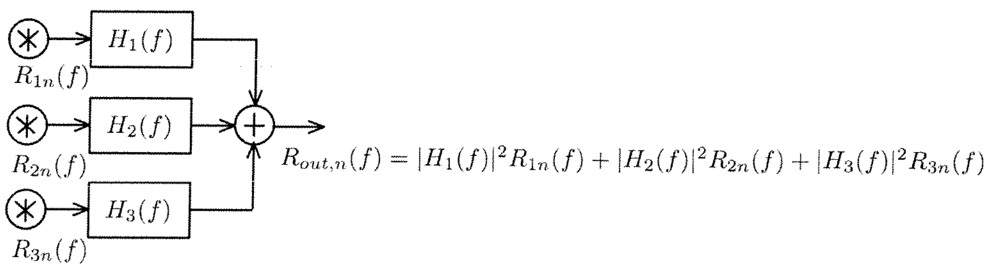
If the input signal to a linear system, with transfer function $H(f)$, has a noise component with spectal density $R_n(f)$ the output signal will get a noise component with spectral density $R_{out,n}(f)$ and

$$R_{out,n}(f) = |H(f)|^2 R_n(f)$$

If we have say three systems, $H_1(f)$, $H_2(f)$ and $H_3(f)$ with noise, $R_{1n}(f)$, $R_{2n}(f)$ and $R_{3n}(f)$ respectivly, on their inputs and the noise sources are *uncorrelated*, and if the output signals from the systems are added the spectal density of the output signal will be:

$$R_{out,n}(f) = |H_1(f)|^2 R_{1n}(f) + |H_2(f)|^2 R_{2n}(f) + |H_3(f)|^2 R_{3n}(f)$$





Noise bandwidth concept
Regard a one-pole system with transfer function

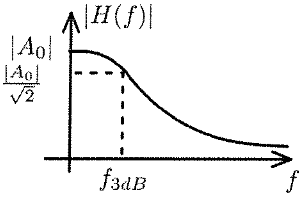
$$H(s) = \frac{A_0}{1 + \frac{s}{p_1}} \Rightarrow |H(f)| = \frac{|A_0|}{\sqrt{1 + \left(\frac{2\pi f}{p_1}\right)^2}}$$

This equation gives the 3dB cut-off frequency: $f_{3dB} = \frac{|p_1|}{2\pi}$.

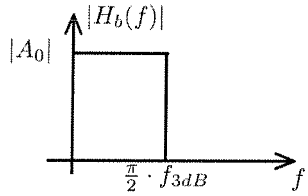
If you feed the system with *white noise*, that is noise with constant (independent of f) spectral density $R_i(f) = R_i$, the noise power on the output will be

$$\begin{aligned} P_{out,noise} &= \int_0^\infty |H(f)|^2 R_i df = R_i \int_0^\infty \frac{|A_0|^2}{1+\left(\frac{2\pi f}{p_1}\right)^2} = R_i \cdot \frac{|A_0|^2|p_1|}{2\pi}\left[\arctan\frac{2\pi f}{|p_1|}\right]_0^\infty \\ &= R_i \cdot \frac{|A_0|^2|p_1|}{2\pi}\cdot\frac{\pi}{2} = R_i|A_0|^2\cdot\frac{\pi}{2}\cdot\frac{|p_1|}{2\pi} = R_i|A_0|^2\cdot\frac{\pi}{2}\cdot f_{3dB} \end{aligned}$$

If you have a *brick-wall filter* with $|H_b(f)| = |A_0|$ and bandwidth $\frac{\pi}{2}\cdot f_{3dB}$ you get the same power $P_{out,noise}$. Therefore $\frac{\pi}{2}\cdot f_{3dB}$ is said to be the **noise-bandwidth** of this one-pole system.



$R_{out}(f) = R_i|H(f)|^2$



$R_{out}(f) = R_i|H_b(f)|^2$

Noise in CMOS-circuits

Noise in CMOS-circuits is *inherent* noise, not *interference* noise.
There are three different types of inherent noise.

1) Thermal noise - due to thermal excitation of charge carriers. Thermal noise is white noise.

2) Flicker noise - due to traps in the semiconductor that hold carriers, which normally gives the DC-current, for some while and than release them. (DC-current doesn't float smooth.) $R_n(f) \sim \frac{1}{f}$ (moore accurate $R_n(f) \sim \frac{1}{f^\alpha}$ where $0.8 < \alpha < 1.3$).

3) Shot noise - DC-current is a result of individual carriers, which yields a current that actually is pulsed and not smooth.

Noise models CMOS: (Regard the saturated region)

Flicker noise (spektral density $V_g^2(f) = \frac{K}{WLC_{ox}f}$) and thermal noise (spectral density $I_d^2(f) = 4kT\frac{2}{3}g_m$) dominates in CMOS-circuits.

As $i_d(t) \approx g_m v_{gs}(t)$ then $I_d(f) \approx g_m V_{gs}(f)$ and the thermal noise with spectral density $I_d^2(f) = 4kT\frac{2}{3}g_m$ can be transformed to an equivalent noise voltage on the input with the spectral density $V_{gs}^2(f) = 4kT\frac{2}{3}\frac{1}{g_m}$. $V_{gs}^2(f)$ and $V_g^2(f)$ are uncorrelated and can thus be added.

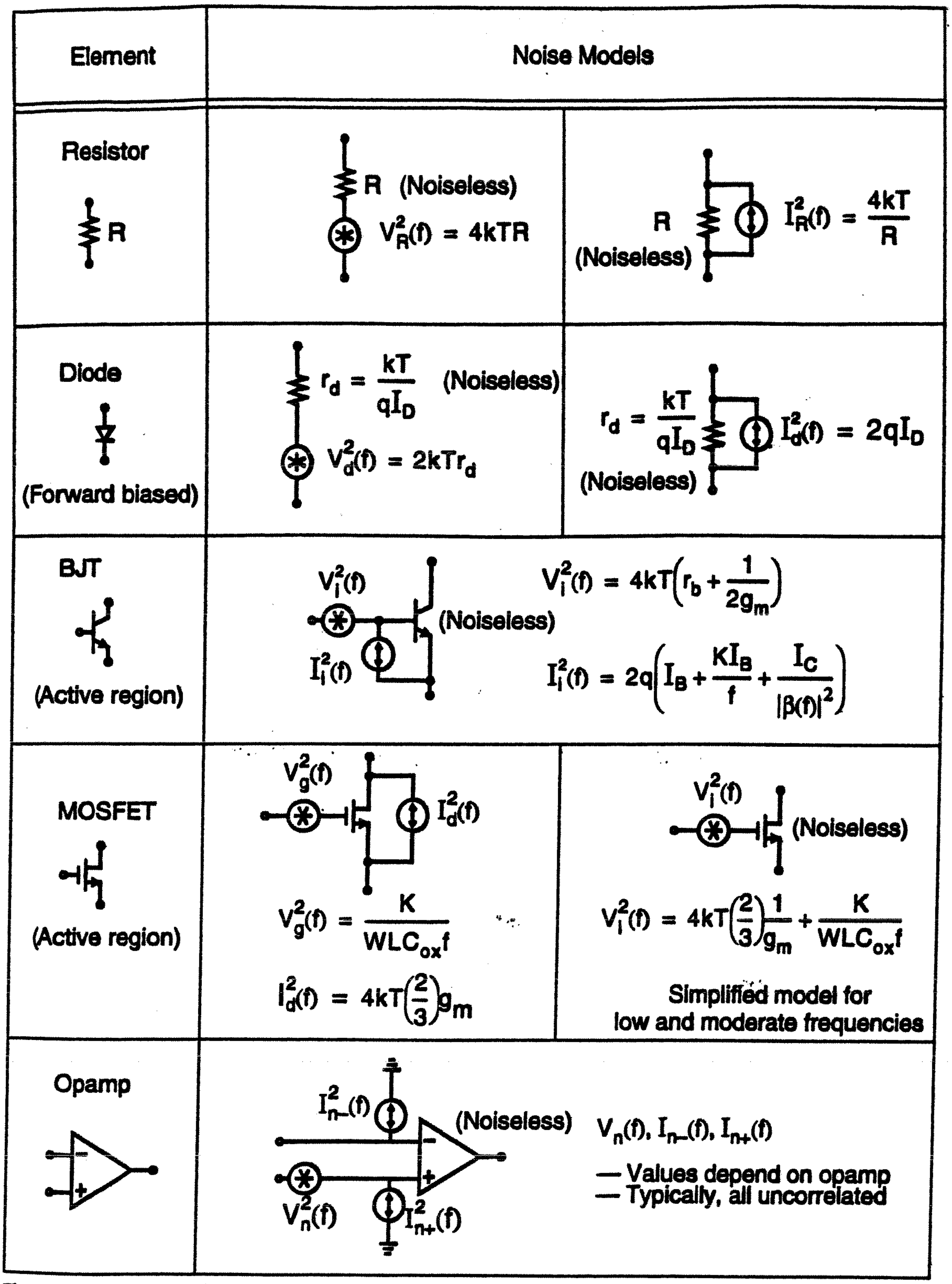


Fig. 4.11 Circuit elements and their noise models. Note that capacitors and inductors do not generate noise.

DISTORSION

- **Harmonic distorsion**

  A result of non-linearities of the circuit (system). This means that e.g. a sinusoidal input signal $x(t)$ doesn't generate a sinusoidal output signal. Instead it gives a periodical non-sinusoidal output signal $y(t)$. $y(t)$ can be described by a fourier serie.

  $$x(t) = \widehat{X}\sin\omega_1 t \Rightarrow y(t) = Y_0 + \sum_{k=1}^{\infty} \widehat{Y}_k \sin(k\omega_1 t + \varphi_k)$$

  Different distorsion metrics:

  THD (Total Harmonic Distorsion)

  $$THD = \frac{(\sum_{k=2}^{\infty} Y_{ke}^2)^{1/2}}{(\sum_{k=1}^{\infty} Y_{ke}^2)^{1/2}} = \frac{(Y_e^2 - Y_0^2 - Y_{1e}^2)^{1/2}}{(Y_e^2 - Y_0^2)^{1/2}}$$

  $Y_{ke} = \dfrac{\widehat{Y}_k}{\sqrt{2}}$ and $Y_e^2 = \frac{1}{T}\int_T |y(t)|dt$

  $SFDR$ (Spurious-Free Dynamic Range) $= P_1 - max\{P_k\}_{k\geq 2}$ [dB]

- **Bandwidth distorsion, D_b**

  Reason: The bandwidth of the reconstructed signal is lower than the bandwidth of the original signal.

  $$D_b = \frac{W_{eb}}{W_x}$$

  Energi of the original signal: $W_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$

  Energi of the error-signal caused by bandwidth distorsion:

  $$W_{eb} = \frac{1}{2\pi}\int_{|\omega|>\pi f_s} |X(\omega)|^2 d\omega$$

- **Aliasing distorsion, D_v**

  Caused by undersampling ($f_s < 2f_{max}$), when different periods of spectrum from the sampled signal are overlapping.

  $$D_v = \frac{W_{ev}}{W_x}$$

  Energi of error-signal caused by aliasing:

  $$W_{ev} = \frac{1}{2\pi}\int_{-\pi f_s}^{\pi f_s} |\sum_{k\neq 0} X(\omega - k2\pi f_s)|^2 d\omega$$