Chapter 4 – Problems

4.1) $dBm_{(50\Omega)} = 10 \log \frac{V_{rms}^2}{50\,m}$

$dBm_{(75\Omega)} = 10 \log \frac{V_{rms}^2}{75\,m}$

$\therefore\ dBm_{(50\Omega)} - dBm_{(75\Omega)} = 10 \log \frac{75}{50} \simeq 1.76$

$\therefore\ dBm_{(50\Omega)} = dBm_{(75\Omega)} + 1.76$

4.2) $10 \log \frac{P_1}{1mW} = -20 \Rightarrow P_1 = 10\,\mu W$

$10 \log \frac{P_2}{1mW} = -23 \Rightarrow P_2 = 5\,\mu W$

a) $P_{total} = P_1 + P_2 = 15\,\mu W \Rightarrow P_{total} = -18.24\,dBm$

b) $P_{total} = P_1 + P_2 + 2c\sqrt{P_1 P_2} = 19.24\,\mu W$

$\Rightarrow P_{total} = -17.16\,dBm$

c) $P_{total} = -15.35\,dBm$

d) $P_{total} = -30.67\,dBm$

4.3) Expected dBm for 10 Hz bandwidth $= -40 - 10 \log \frac{30}{10}$

$= -44.77\,dBm$

$-40 = 10 \log \frac{P}{1mW} \Rightarrow P = 0.1\,\mu W$

(cont.)

4.3) (cont.) Spectral density $= \frac{0.1\,\mu W}{30\,Hz} = \frac{3.3}{1000} \frac{(mV)^2}{Hz}$

$\Rightarrow$ root spectral density $= 0.058 \frac{mV}{\sqrt{Hz}}$

4.4) $10 \log \frac{P}{1mW} = -60 \Rightarrow P = 1nW$

$$V_n^2(f)\Big|_{f=0.1\,Hz} = \frac{1nW}{1mHz} = 1 \frac{(mV)^2}{Hz}$$

Assuming $V_n^2(f) = \frac{K_v^2}{f}$ and $V_n^2(f)\Big|_{f=0.1Hz} = 1 \frac{(mV)^2}{Hz}$,

$$V_n^2(f) = \frac{0.1\,(mV)^2}{f}$$

Total noise power in (1mHz, 1Hz) is :

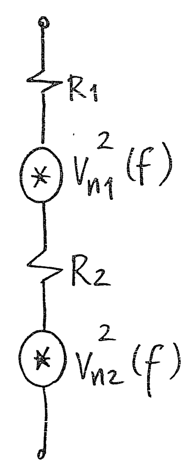
$$\int_{1m}^{1} V_n^2(f)\,df = 0.1\,(mV)^2\,[\ln 1 - \ln 1m] = 0.69\,\mu W$$

$$10 \log \frac{0.69\,\mu}{1m} = -31.6\ dBm$$

4.5) $V_{n1}^2(f) = 4KTR_1$ & $V_{n2}^2(f) = 4KTR_2$

The two noise sources are uncorrelated.

$$\therefore V_n^2(f) = V_{n1}^2(f) + V_{n2}^2(f) = 4KTR_1 + 4KTR_2$$
$$= 4KT(R_1 + R_2)$$



which is equal to the noise spectral density of the series combination of R_1 & R_2.

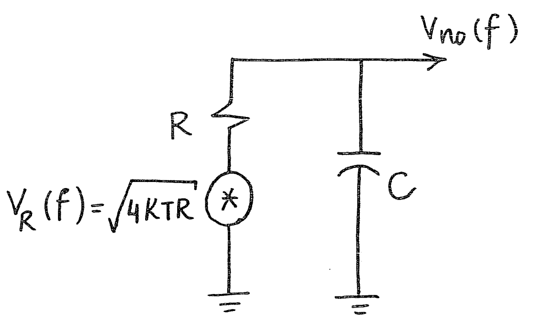
4.6) $V_{no}^2(f) = \dfrac{4KTR}{1+\left(\dfrac{\omega}{\omega_{3dB}}\right)^2}$ where $\omega_{3dB} = \dfrac{1}{RC}$

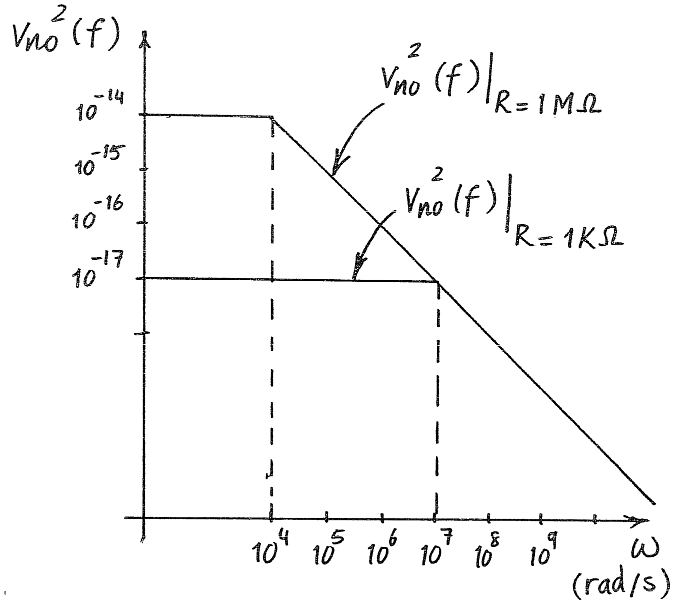
For $C = 100\,pF$ & $R = 1\,K\Omega$, $\omega_{3dB} = 10^7$ rad/sec.

and $V_{no}^2(0) = (4.08\ nV)^2 = 1.66 \times 10^{-17}\ V^2$

For $C = 100\,pF$ & $R = 1\,M\Omega$, $\omega_{3dB} = 10^4$ rad/sec

and $V_{no}^2(0) = (129\ nV)^2 = 1.66 \times 10^{-14}\ V^2$





The area underneath both curves are the same and is equal to $\dfrac{KT}{C} = (6.45\,\mu V)^2$

4.7) $I_{nf}^2 = 8.28 \times 10^{-25} \left(\dfrac{A^2}{Hz}\right)$, $I_{n1}^2 = 1.656 \times 10^{-24} \left(\dfrac{A^2}{Hz}\right)$

$I_{n-}^2(f) = 100 \times 10^{-24} \left(\dfrac{A^2}{Hz}\right)$

$V_{no1}^2(0) = (I_{nf}^2(0) + I_{n1}^2(0) + I_{n-}^2(0))\, R_f^2$

$\simeq I_{n-}^2(0)\, R_f^2 = \left(0.2\ \dfrac{\mu V}{\sqrt{Hz}}\right)^2$

(cont.)

4.7) (cont.) $V_{no1}^2(rms) = (0.2\ \frac{\mu V}{\sqrt{Hz}}) \times \frac{1}{4(20K)(1n)} = \underline{(22.4\ \mu V)^2}$

$$V_{no2}^2(0) = V_n^2(f)(1+\frac{R_f}{R_1}) = (60\ \frac{nV}{\sqrt{Hz}})^2$$

The noise is lowpass filtered at $f_0 = \frac{1}{2\pi R_f C_f} \simeq 8\ KHz$

until $f_1 = \frac{R_f}{R_1} f_0 = 16\ KHz$.

$$\therefore V_{no2(rms)}^2 = (60 \times 10^{-9})^2 \frac{\pi \times 8K}{2} + (120 \times 10^{-9})^2 \frac{\pi}{2}(1M - 16K)$$

$$\simeq \underline{(25.9\ \mu V)^2}$$

The total output noise is estimated to be:

$$V_{no(rms)}^2 = V_{no1(rms)}^2 + V_{no2(rms)}^2 = (34.2\ \mu V)^2$$

$$\Rightarrow \underline{V_{no(rms)} = 34.2\ \mu V\ rms}$$

4.8) Since the opamp is assumed to be ideal and noiseless, we have $V_x = 0$ at all times.

$$\therefore V_{nx}^2(f) = 0 \Rightarrow \underline{V_{nx}^2(rms) = 0}$$

This noise is smaller than $\frac{KT}{1pF}$ since the voltage across the capacitor is fixed by another device (i.e. opamp).

For the noise at V_o, we have:

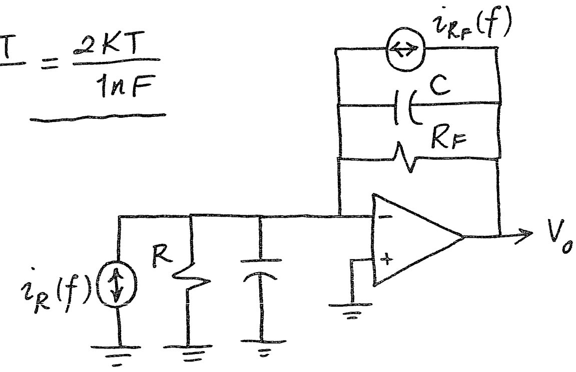
$$V_{no}^2(f) = (i_R^2(f) + i_{R_F}^2(f)) \frac{R_F^2}{1+\omega^2 R_F^2 C^2}$$

$$= \left(\frac{4KT}{R} + \frac{4KT}{R_F}\right) R_F^2 \times \frac{\pi}{2} \frac{1}{2\pi R_F C}$$

(cont.)

4.8) (cont.)

$$\therefore \quad V_{no}^2(f) = \frac{2KT}{C} = \underline{\frac{2KT}{1nF}}$$



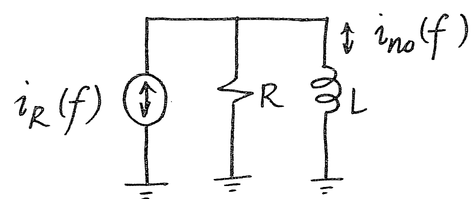
The noise at V_o is twice as much as $\frac{KT}{1nF}$.

The reason for this increase is the additional noise current source $i_R(f)$. Since the voltage across this current source is fixed (i.e. it is equal to zero), it does not affect the ω_{3dB} of the filter and acts as an ideal noise current source!

4.9)

$$I_{no}^2(f) = \left|\frac{R}{R+j\omega L}\right|^2 I_R^2(f)$$

$$= \frac{1}{1+\frac{L^2}{R^2}\omega^2}\frac{4KT}{R}$$



$$I_{no(rms)}^2 = \int_0^\infty \frac{4KT/R}{1+\frac{L^2}{R^2}\omega^2}df$$

$$= \frac{4KT}{R}\frac{1}{2\pi}\frac{R}{L}\int_0^\infty \frac{d\omega}{1+\omega^2} = \underline{\frac{KT}{L}}$$

4.10) a) For circuit (I):

$$\frac{V_o}{V_i} = \frac{-1}{1 + j\omega(7K)(80n)} \qquad (1)$$

For circuit (II):

$$\left(\frac{V_o}{2}\left(j\omega(80n)\right) + \frac{V_i}{14K}\right) 14K = -V_o$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{-1}{1 + j\omega(80n)(7K)} \qquad (2)$$

(1) & (2) show identical transfer functions.

b) $V_{no}^2(0) = [I_{n1}^2(f) + I_{nf}^2(f)] R_f^2 = (15.2 \frac{nV}{\sqrt{Hz}})^2$

$\therefore V_{no\,(rms)}^2 = V_{no}^2(0) \frac{1}{4R_F C_F} = \underline{(0.32\ \mu V)^2}$

c) the transfer function for current source entering the negative terminal of opamp 1 is:

$$\frac{V_o}{I_{i1}} = \frac{-R_F}{1 + j\omega \frac{C_F R_F}{2}}$$

$$\Rightarrow V_{no1}^2(f) = [I_{n1}^2(f) + I_{nf}^2(f)] \frac{R_F^2}{1 + \omega^2 \frac{C_F^2 R_F^2}{4}}$$

$$V_{no1}^2(0) = [I_{n1}^2(f) + I_{nf}^2(f)] R_F^2 = (21.5 \frac{nV}{\sqrt{Hz}})^2$$

$$V_{no1(rms)}^2 = V_{no1}^2(0) \frac{1}{4 \frac{R_F C_F}{2}} = \underline{(0.45\ \mu V)^2}$$

the transfer function for the current source entering the negative terminal of opamp 2 is:

$$\frac{V_o}{I_{i2}} = \frac{-R_2}{1 + \frac{1}{j\omega C_F R_F/2}} = \frac{-j\omega R_2 R_F C_F/2}{1 + j\omega C_F R_F/2}$$

(cont.)

4.10) (cont.) This is the transfer function of a high-pass filter, which means, ideally, the total output noise due to any current noise at the negative terminal of opamp 2 is ∞.

4.11) $$\frac{V_o}{V_i} = \frac{-R_F/R_1}{1+j\omega C_F R_F}.$$

C_F is reduced by a factor of 1000. Therefore, to keep the same transfer function, R_F & R_1 must be increased by the same factor. The parameters of the new circuit are:

$R_F = 7M\Omega$, $R_1 = 7M\Omega$, $C_F = 80\ pF$

$$\Rightarrow V_{no}^2(0) = [I_{n1}^2(f) + I_{nf}^2(f)]\, R_F^2 = 8KTR_F$$

$$V_{no}^2(rms) = 8KTR_F \frac{1}{4R_F C_F} = \frac{2KT}{C_F}$$

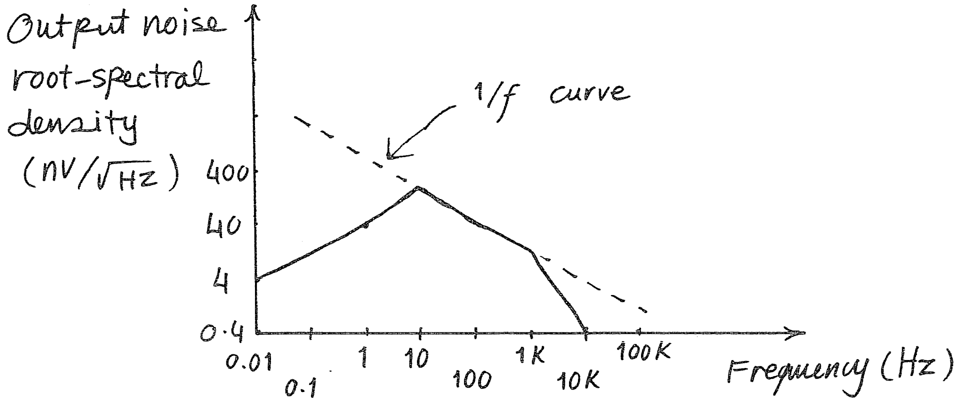
$\therefore$ the total noise power is just a function of C_F (for the case $R_F = R_1$) and increases by decreasing C_F:

$$V_{no}^2(rms) = \underline{(10.2\ \mu V)^2}$$

4.12) The $\frac{1}{f}$ tangent line touches the spectral density of Fig. 4.3 at 1000 Hz. The noise power can be estimated as:

$$V_n^2(rms) = 10\frac{(\mu V)^2}{Hz} \times \frac{\pi \times 1000}{2} = \underline{(125\ \mu V)^2}$$

4.13)



Estimate output noise by integrating from 1 to 10KHz

$$V_{no(rms)}^2 \cong \int_1^{10} 40^2 f\,df + \int_{10}^{1k} \frac{400^2}{f}\,df + \int_{1k}^{10k} \frac{0.4^2 (10k)^2}{f^2}\,df$$

$$= 7.92\times10^4 (nV)^2 + 7.37\times10^5 (nV)^2 + 1.44\times10^4 (nV)^2$$

$$= 8.31\times10^5 (nV)^2 \cong (1\,\mu V)^2 \quad \text{Roughly } 1\,\mu V \text{ RMS}$$

4.14) A : Graphical Approach

Region 1 : $0.01 < f < 1 \quad V_n^2(f) = \left| \frac{10n}{\sqrt{f}} \frac{V}{\sqrt{Hz}} \right|^2 = \frac{10^{-16}}{f} \frac{V^2}{Hz}$

$$\Rightarrow V_{n1}^2 = \int_{0.01}^{1} V_n^2(f)\,df = 4.6\times10^{-16}\,V^2$$

Region 2 : $1 < f < 100 \quad V_n^2(f) = 10^{-16} \frac{V^2}{Hz}$

$$\Rightarrow V_{n2}^2 = 99\times10^{-16}\,V^2$$

Region 3 : $100 < f < 1K \quad V_n^2(f) = \frac{10^{-12}}{f^2} \frac{V^2}{Hz}$

$$\Rightarrow V_{n3}^2 = \int_{100}^{1K} \frac{10^{-12}}{f^2}\,df = 90\times10^{-16}\,V^2$$

Region 4 : $1K < f < \infty \quad V_{n4}^2 = 10^{-18} \times \frac{\pi}{2} \times 9000 = 14.1\times10^{-15}\,V^2$

$$\therefore V_{n(rms)}^2 = V_{n1}^2 + V_{n2}^2 + V_{n3}^2 + V_{n4}^2 = (0.18\,\mu V)^2$$

(cont.)

4.14) (cont.) B: Using $\frac{1}{f}$ tangent line

The $\frac{1}{f}$ line touches the curve at 100 Hz & 10 KHz simultaneously. If we approximate the noise at 100 Hz and 10 KHz with two lowpass-filtered noise, we will have:

$$V_n^2 = 10^{-16} \times \frac{\pi}{2} \times 100 + 10^{-18} \times \frac{\pi}{2} \times 9000 = \underline{(0.17\,\mu V)^2}$$

4.15)

$$A(s) = \frac{A_0}{\left(1+\frac{s}{2\pi f_0}\right)^2} \Rightarrow A(j\omega) = \frac{A_0}{\left(1+j\frac{f}{f_0}\right)^2}$$

$$\Rightarrow \text{equivalent noise bandwidth} = \frac{1}{A_0^2}\int_0^\infty |A(j\omega)|^2 df$$

$$= f_0 \int_0^\infty \frac{df}{(1+f^2)^2} = \underline{f_0 \frac{\pi}{4}}$$

4.16) The equivalent circuits for noise calculations are shown below:

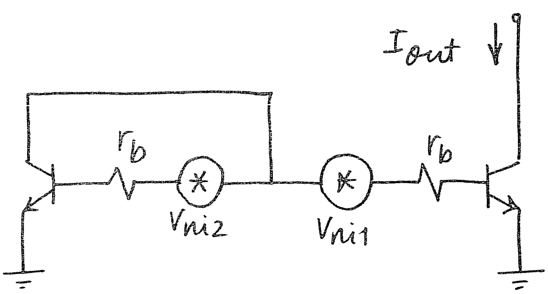


Fig. P4.16 (a)

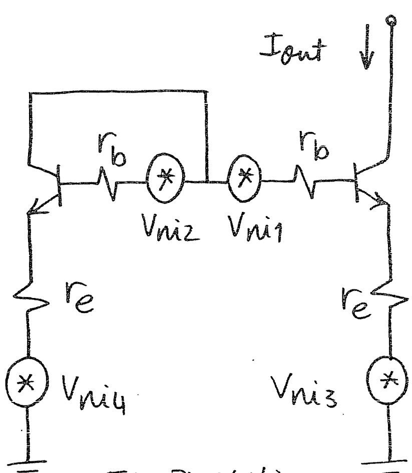


Fig. P4.16 (b)

(cont.)

4.16) (cont.) SNR for the current mirror of Fig. P4.16(a):

Using superposition for V_{ni1} & V_{ni2}, and approximating $\beta + 1 \simeq \beta = 100$, we have:

$$I_{no}^2(f) = [V_{ni1}^2(f) + V_{ni2}^2(f)] \frac{\beta^2}{(r_b + r_\pi)^2}$$

where $V_{ni1}^2(f) = V_{ni2}^2(f) = 4KTr_b = 5.46 \frac{(nV)^2}{Hz}$

and $r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} = 2.6\,K\Omega$

$\therefore\ I_{no}^2(f) = 0.0127 \frac{(nA)^2}{Hz}$

$\therefore\ I_{no}^2(rms) = 50\,MHz \times I_{no}^2(f) = (0.8\,\mu A)^2$

$\therefore\ SNR = 20 \log \frac{100\mu}{0.8\mu} = \underline{41.94\ dB}$

SNR for the current mirror of Fig. P4.16(b):

$$I_{no}^2(f) = [V_{ni1}^2(f) + V_{ni2}^2(f) + V_{ni3}^2(f) + V_{ni4}^2(f)] \frac{\beta^2}{(r_b + r_\pi + \beta r_e)^2}$$

where $V_{ni3}^2(f) = V_{ni4}^2(f) = 4KTr_e = 3.31 \frac{(nV)^2}{Hz}$

$V_{ni1}^2(f) = V_{ni2}^2(f) = 4KTr_b = 5.46 \frac{(nV)^2}{Hz}$

$r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} = 2.6\,K\Omega$

$\therefore\ I_{no}^2(f) = 0.0003336 \frac{(nA)^2}{Hz}$

$\therefore\ I_{no}^2(rms) = 50\,MHz \times I_{no}^2(f) = (0.13\,\mu A)^2$

$\therefore\ SNR = 20 \log \frac{100\mu}{0.13\mu} = \underline{57.7\,dB}$

4.17) Using (4.61): $V_{no1}^2(0) = (I_{n1}^2(0) + I_{nf}^2(0) + I_{n-}^2(0))\, R_F^2$

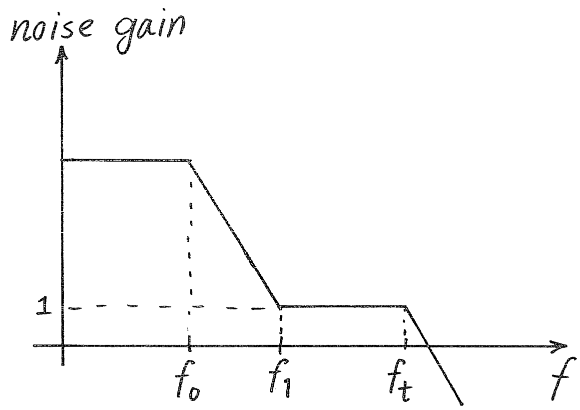
where $I_{nf}^2(f) = \frac{4KT}{R_F} = 1.035\left(\frac{pA}{\sqrt{Hz}}\right)^2$

$\therefore\ V_{no1}^2(0) = (2 \times 0.6^2 + 1.035)(16K)^2\left(\frac{pV}{\sqrt{Hz}}\right)^2 = \left(21.2\ \frac{nV}{\sqrt{Hz}}\right)^2$

$V_{no1}^2(rms) = V_{no1}^2(0) \times \frac{1}{4R_F C_F} = (2.65\ \mu V)^2$

Using (4.62): $V_{no2}^2(0) = V_n^2(f)\left(1 + \frac{R_F}{R_1}\right)^2 = \left(220\ \frac{nV}{\sqrt{Hz}}\right)^2$

this noise is lowpass filtered at $f_0 = \frac{1}{2\pi R_F C_F}$ until $f_1 = \frac{R_F}{R_1} f_0$ where the noise gain reaches unity.



$V_{no2}^2(rms) = (220 \times 10^{-9})^2 \frac{1}{4R_F C_F} + (20 \times 10^{-9})^2 \frac{\pi}{2}(f_t - f_1)$

$= (44.16\ \mu V)^2$

Using (4.64): $\underline{V_{no}^2(rms) = (44.24\ \mu V)^2}$

SNR for a 100 mV signal $= 20 \log \frac{100\,m}{44.24\,\mu}$

$\underline{= 67.08\ dB}$

4.18) Using (4.97):

$$V_{neq}^2(f) = \left(\frac{16}{3}\right) KT \frac{1}{g_{m1}} + \frac{16}{3} KT \left(\frac{g_{m3}}{g_{m1}}\right)^2 \frac{1}{g_{m3}}$$

$$= \underline{\left(5.75 \frac{nV}{\sqrt{Hz}}\right)^2}$$

Since $g_m \propto \sqrt{I_{BIAS}}$, if I_{BIAS} is doubled, g_{m1} & g_{m3} will be increased by $\sqrt{2}$. Therefore, $V_{neq}^2(f)$ will be reduced by $\sqrt{2}$, and we have:

$$V_{neq}^2(f) = \frac{1}{\sqrt{2}} \left(5.75 \frac{nV}{\sqrt{Hz}}\right)^2 = \underline{\left(4.84 \frac{nV}{\sqrt{Hz}}\right)^2}$$