Lecture 7, ATIK

Continuous-time filters 2
Discrete-time filters
What did we do last time?

Switched capacitor circuits with nonideal effects in mind

    What should we look out for?
    What is the impact on system performance, like filters.

Continuous-time filters

    The way forward, and the background to generate filters.
    OTA/Gm-C and active-RC

Revisited today
What will we do today?

Continuous-time filters
- Some slides from last lecture
- Scaling
- Sensitivity
- Noise

Discrete-time filters
- Simulation of the continuous-time filters
- Discrete-time accumulators
- LDI transform
- Bilinear transform
Ladder networks

Use an analog reference filter, values are given by table or MATLAB-ish.

Typically doubly-resistive terminated for maximum power transfer.

What is the DC gain?
State-space representation

Note the voltages and currents through the ladder network.

Introduce a helping "R" to create voltages

Input current

\[ I_0 = \frac{V_0 - V_1}{R_i} \Rightarrow R \cdot I_0 = \frac{R}{R_i} \cdot (V_0 - V_1) \]

Inductor current

\[ I_2 = \frac{V_1 - V_3}{s L_2} \Rightarrow R \cdot I_2 = \frac{R}{s L_2} \cdot (V_1 - V_3) \]

Kirchhoff

\[ R \cdot I_0 = V_1 \cdot s C_3 + R \cdot I_2, \text{ etc.} \]

Combine all equations into a flow graph
Active-RC implementation

Replace integrators with corresponding active components

Identify the paths and set the values accordingly
Transconductance-C filters

(Operational) transconductance amplifier with capacitive load

\[ V_{out}(s) \cdot sC_L = I_{out}(s) = -g_{m1} \cdot V_{in}(s) \Rightarrow \frac{V_{out}(s)}{V_{in}(s)} = \frac{-g_{m1}}{sC_L} \]

How do we sum several inputs for the Gm-C?

How do we create a gain circuit with Gm-C?
Transconductance-C filters, cont'd

A first-order pole

Requires two transconductors!

\[-g_{m1} \cdot V_{in}(s) - V_{out}(s) \cdot s \cdot C_L - g_{m2} \cdot V_{out}(s) = 0\]

\[
V_{out}(s) = \frac{-g_{m1}}{g_{m2} + s \cdot C_L} = \frac{-g_{m1}/g_{m2}}{1 + \frac{s}{g_{m2}/C_L}}
\]
Transconductance-C filters, cont'd

Leapfrog: replacement of active-RC integrators with Gm-C elements.

Requires more active elements, but easier to tune.
Continuous-time filters, scaling

For optimal performance, we need to scale our filters

- Minimize distortion, i.e., minimize your swing
- Maximize SNR, i.e., maximize your swing

What norm to use in order to scale?

- Different nodes might peak at different frequencies!
Continuous-time filters, scaling example

1) Originate from flow graph and introduce scaling coefficients.
2) All inputs scaled with K and all outputs scaled with 1/K
3) Take them "one-by-one" from input towards the output
Continuous-time filters, sensitivity

General

The transfer function is a function of all values, $A_0$, $R$, $C$, etc.

We can define the sensitivity as

$$S_i^H = \frac{q_i}{H} \frac{\partial H}{\partial q_i}$$

Leapfrog filters are less sensitive than second-order sections.
Continuous-time filters, sensitivity example

First-order pole with active RC

\[ H(s) = \frac{-R_8/R_4}{1 + \frac{s}{1/C_2 R_8}} \]

Variation as function of RC

\[ S_C^H = \frac{C_2}{H} \cdot \frac{\partial H}{\partial C_2} = \frac{C_2}{H} \cdot H \cdot \frac{-1}{1 + \frac{s}{1/C_2 R_8}} \cdot s R_8 \]

such that

\[ S_C^H = \frac{-s C_2 R_8}{1 + s C_2 R_8} \]

What does this imply?
Continuous-time filters, noise

All OP amp noise can be reverted back to their inputs

Scaled with the transfer functions to the output
Use the superfunction

All resistor noise is converted to output too
Going to discrete-time domain

A general transfer function is given by

\[ H(z) = \frac{Y(z)}{X(z)} = \frac{a_0 + a_1 z + a_2 z^2 + \ldots}{b_0 + b_1 z + b_2 z^2 + \ldots} \]

Identify integrators in the expression and rewrite

\[ A(z) = \frac{\alpha}{z - 1} \]

(Notice that the approach is a bit different to "digital" IIR/FIR filter design)

For filter purposes we will originate from the continuous-time filter specification

eventually determines our a,b values.
The lossless discrete integrator transform

Maps just part of the frequency range

\[ s = s_0 \cdot [z^{0.5} - z^{-0.5}] = s_0 \cdot \frac{1 - z^{-1}}{z^{-0.5}} \]

with the scaling factor according to

\[ \omega_a = 2s_0 \sin \left( \frac{\omega T}{2} \right) \]

which maps the analog frequencies with the discrete-time angles.

We have already decided to use high oversampling ratio, so the LDI transform is sufficiently OK.
The bilinear transform

Maps the "whole" analog LHP on the unit disk

\[ s = s_0 \cdot \frac{z - 1}{z + 1} \]

with the scaling factor according to

\[ \omega_a = s_0 \tan \left( \frac{\omega T}{2} \right) \]

which maps the analog frequencies with the discrete-time angles.

Bilinear transform is accurate,
but sensitivity requires us to select a low scaling factor.
Accumulators of different flavours

See the handouts for state-space realization and implementations

http://www.es.isy.liu.se/courses/ANIK/download/filterRef/ANTI
  K_0NNN_LN_leapfrogFiltersOH1_A.pdf

http://www.es.isy.liu.se/courses/ANIK/download/filterRef/ANTI
  K_0NNN_LN_leapfrogSynthesisExtra1_A.pdf

http://www.es.isy.liu.se/courses/ANIK/download/filterRef/ANTI
  K_0NNN_LN_leapfrogSynthesisExtra2_A.pdf

http://www.es.isy.liu.se/courses/ANIK/download/switcapRef/ANTI
  TIK_0NNN_LN_switcapHandout_B.pdf
The steps forward

1) Analog reference filter
2) Frequency scaling of reference filter
3) Signal flow graph
4) Replacing integrators with discrete-time counterparts
5) Eliminate the $z^{(1/2)}$
6) Introduce SC network
7) Map the values
8) Scale

This you will do in the lab!
Step 1, Analog reference filter

Use a reference filter to get the order and component values

Example: \( C_1 = C_3 = 0.5 \) and \( L_2 = 1.3333 \), \( R_L = 1000 \), \( \omega_0 = 1000 \)

Optimum wrt. sensitivity (exercises in lessons)
Step 2, Denormalization of reference filter

R0 is the termination load

\[ R_n = \frac{R}{R_0}, \quad L_n = \frac{\omega_0 L}{R_0}, \quad C_n = \omega_0 C R_0 \]

Example values gives

\[ R = 1000 \]
\[ L = 1.3333 \]
\[ C_1 = C_3 = 0.5 \cdot 10^{-6} \]

In the lab, a MATLAB script is given
Step 3, Signal flow graph

Example third-order leapfrog

\[ V_0 = V_{in} \]
\[ X_1 \]
\[ X_2 \]
\[ X_3 = V_{out} \]
\[ \frac{R}{R_i} \]
\[ \frac{-1}{sRC_3} \]
\[ \frac{R}{sL_2} \]
\[ \frac{-1}{sRC_1} \]
\[ \frac{R}{R_L} \]
Step 4, Replacing integrators with discrete-time counterparts (LDI)

\[ V_0 = V_{in} \]

\[ R \frac{R}{R_i} \]

\[ X_1 \]

\[ \frac{-1}{s_0 RC_3} \cdot \frac{z^{-0.5}}{1-z^{-1}} \]

\[ X_2 \]

\[ \frac{R}{s_0 L_2} \cdot \frac{z^{-0.5}}{1-z^{-1}} \]

\[ X_3 = V_{out} \]

\[ \frac{-1}{s_0 RC_1} \cdot \frac{z^{-0.5}}{1-z^{-1}} \]

\[ R \frac{R}{R_L} \]
Step 5, Eliminate the $z^{(1/2)}$
Consider the product
\[
R \cdot z^{-0.5} = \frac{R}{z^{0.5}} = \frac{R}{\cos \frac{\omega T}{2} + j \sin \frac{\omega T}{2}} = \sqrt{1 - \left(\frac{\omega_a}{2s_0}\right)^2} + j \frac{\omega_a}{2s_0}
\]

Compare with a capacitor in parallel with a resistor:

Remember that originally this resistance was in parallel with capacitor and we can reduce the C value.

The change in resistance can be ignored since the value is much smaller than unity.
Step 5, Eliminate the $z^{(1/2)}$, cont'd

Sample on counter phase.
Step 6, Introduce SC network
Step 7, Map the values

Consider the transfer functions from node to node

Example: $V_{in}$ to $X_1$:

$$X_1 = \frac{1}{s_0 R_i C_3'} \text{ vs. } \frac{a_0}{c_1} \text{ vs. } \frac{a_1}{c_1}$$

e tc.

More variables than equations and you need to fix values:

Typically choose all the opamp capacitors equal

All others: "as equal as possible"
Step 8, Scale

Follows exactly the same principles as for continuous-time filters.

Introduce scaling and make sure all entry points are scaled with $K$ and all exit points are scaled with $1/K$.

Left as exercise and laboratory work.

Remember that reference filter has a DC gain of 0.5!

Remember that gain is dependent on sample frequency!
Conclusions SC filters

Today computer-aided, of course.

Since it is computer aided - choose the leapfrog filters for optimum sensitivity!

Use fixed templates and plug in your values

Remember to use reasonable values

i.e., in an integrated circuit, the sizes should be in pF region. This also implies high cut-off frequencies!
What did we do today?

Continuous-time filters
   Wrap-up and some more conclusions

Discrete-time filters
   Simulation of the continuous-time filters
   Discrete-time accumulators
   LDI transform
   Bilinear transform
What will we do next time?

Data converters
  Fundamentals

DACs
Lecture 8, ATIK

Data converters 1