Lesson 2

Lesson Exercises: **B1.8, B1.9, B3.1-B3.3, B3.7, 3**

Recommended Exercises: *B1.6, B1.7, B1.10, B3.4, B3.6, B3.8-B3.10*

Theoretical Issues: **Large Signal Models, Small Signal Models, Small Signal Capacitors, Simple Current Mirror, Common Source Amplifier, Common Drain Amplifier, Common Gate Amplifier**

Exercise B1.6

From p. 56-60.

Active: $I_D = \frac{\beta}{2}(V_{GS}-V_T)^2(1+\lambda(V_{DS}-V_{eff})) = \beta(V_{GS}-V_T)^2$ since $V_{DS} = V_{eff}$

Triode: $I_D = \beta\left((V_{GS}-V_T)V_{DS}-\frac{V_{DS}^2}{2}\right)$. But $V_{eff} = V_{DS}$ =>

$$I_D = \beta\left((V_{GS}-V_T)(V_{GS}-V_T)-\frac{(V_{GS}-V_T)^2}{2}\right) = \frac{\beta}{2}(V_{GS}-V_T)^2$$

Exercise B1.7

From p. 56-60.

$$I_D = \frac{\beta}{2}(V_{GS}-V_T)^2(1+\lambda(V_{DS}-V_{eff}))$$

Find λ:

$$\lambda = \frac{k_{rds}}{2L\sqrt{V_{DS}-V_{eff}+\Phi_o}}$$ where

$$k_{rds} = \sqrt{\frac{2K_s\varepsilon_o}{qN_A}} = \sqrt{\frac{2\cdot 11.8\cdot 8.85\cdot 10^{-12}}{1.602\cdot 10^{-19}\cdot 10^{22}}} = 3.612\cdot 10^{-7}\text{m}/\sqrt{V}$$

$$V_{DS} = V_{eff} \Rightarrow \lambda = \frac{3.612\cdot 10^{-7}}{2\cdot 1.5\cdot 10^{-6}\sqrt{0+0.9}} = 0.127V^{-1}$$

$$I_D\Big|_{V_{DS}=V_{eff}} = \frac{92\mu}{2}\cdot\frac{50}{1.5}\cdot(1.1-0.8)^2\cdot(1+\lambda\cdot 0) = 138\mu A$$

$$I_D\Big|_{V_{DS}=V_{eff}+0.3} = \frac{92\mu}{2}\cdot\frac{50}{1.5}\cdot(1.1-0.8)^2\cdot(1+\lambda\cdot 0.3) = 143\mu A$$

Exercise B1.8

$$I_D\Big|_{V_{DS}=V_{eff}} = 20\mu A,\ I_D\Big|_{V_{DS}=V_{eff}+0.5} = 23\mu A$$

$$(1+\lambda\cdot 0.5)I_D\Big|_{V_{DS}=V_{eff}} = I_D\Big|_{V_{DS}=V_{eff}+0.5} \Rightarrow \lambda = 2\left(\frac{23\mu}{20\mu}-1\right) = 0.3V^{-1}$$

$$r_{ds} \approx \frac{\Delta V}{\Delta I} = \frac{0.5}{3\mu} = 167k\Omega$$

Exercise B1.9

Use p. 56-60 in the book. In the following we will ignore the increase in threshold voltage caused by the non-zero bulk-source voltage, since the transistor will otherwise be cut-off. (This is probably a mistake by the person who made this problem).

$$\lambda\Big|_{V_{eff}=V_{DS}} = \frac{k_{rds}}{2L\sqrt{\Phi_o}} = \frac{3.612\cdot 10^{-7}}{2\cdot 1.2\cdot 10^{-6}\sqrt{0.9}} = 0.159V^{-1}$$

$$I_D = \frac{\beta}{2}(V_{GS}-V_T)^2 = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS}-V_T)^2 = \frac{92\cdot 10^{-6}}{2}\frac{10}{1.2}\cdot(0.3)^2 = 34.5uA$$

$$r_{ds} = \frac{1}{\lambda I_D} = \frac{1}{0.159\cdot 34.5u} = 183k\Omega$$

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = \sqrt{2\cdot 92\cdot 10^{-6}\frac{10}{1.2}34.5\mu} = 230\mu A/V$$

$$g_s = \frac{\gamma g_m}{2\sqrt{V_{SB}-2|\phi_F|}} = \frac{0.5\cdot 230\mu}{2\sqrt{1-2\cdot 0.34}} = 44\mu A/V$$

Exercise B1.10

p. 33-35, 59.

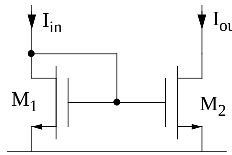
$$C_{gs} = 2/3\cdot WLC_{ox}+C_{ov}W = \frac{2}{3}50\cdot 1.2\cdot 1.9\cdot 10^{-3}+2\cdot 10^{-4}\cdot 50 = 86fF$$

$$C_{gd} = C_{ov}W = 2\cdot 10^{-4}\cdot 50 = 10fF$$

$$C_{db} = A_d\cdot C_j+P_d\cdot C_{jsw} = 200\cdot 2.4\cdot 10^{-4}+58\cdot 2\cdot 10^{-4} = 60fF$$

$$C_{sb} = C_j(A_s+WL)+C_{jsw}P_s = 2.4\cdot 10^{-4}(200+50\cdot 1.2)+2\cdot 10^{-4}\cdot 58 = 74fF$$

Exercise B3.1



$$I_{in} = 80uA$$

$$V_{eff1} = \sqrt{\frac{2I_{D1}}{\beta_1}},\ I_{out} = I_{D2} = V_{eff2}^2\frac{\beta_2}{2} = V_{eff1}^2\frac{\beta_2}{2} = \frac{2I_{D1}}{\beta_1}\frac{\beta_2}{2} = \frac{(W/L)_2}{(W/L)_1}I_{D1} =>$$

$$I_{D2} = \frac{W_2}{W_1} I_{D1} = \frac{25}{100} 80\mu = 20\mu A,$$

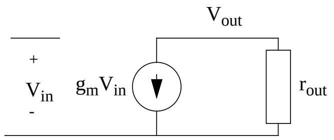
$$r_{out} = r_{ds2} = 8000 \cdot \frac{L}{I_{D2}} = 8000 \cdot \frac{1.6}{20\mu} = 640 k\Omega$$

Transistor M_2 must be in the active region:

$$V_{out} = V_{DS2} > V_{eff2} = \sqrt{\frac{2I_{D2}}{\beta_2}} = \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}(W/L)_2}} = \sqrt{\frac{2 \cdot 20\mu}{92 \cdot 10^{-6} \cdot (25/1.6)}} = 170 mV$$

Exercise B3.2

Small Signal Model:



$$A_v = \frac{V_{out}}{V_{in}} = -g_m r_{out} = -g_m (r_{ds1} \| r_{dss2})$$

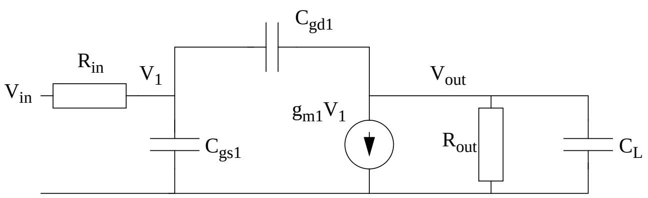
$$r_{ds1} = \frac{8000L}{I_{bias}}, \; r_{ds2} = \frac{12000L}{I_{bias}} \Rightarrow r_{out} = \frac{(96 \cdot 10^6)L^2}{20000 L I_{bias}} = \frac{4800L}{I_{bias}}$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_{bias}} \Rightarrow A_v = -4800 \sqrt{\frac{2\mu_n C_{ox} WL}{I_{bias}}}$$

Conclusion: Large gain => Small I_{bias} and Large transistor area.

Exercise B3.3

Small Signal Model:



Nodal Equations:

$$\begin{cases} \frac{(V_{in} - V_1)}{R_{in}} - V_1 s C_{gs1} - (V_1 - V_{out}) s C_{gd1} = 0 \\ (V_1 - V_{out}) s C_{gd1} - g_{m1} V_1 - V_{out} R_{out} - V_{out} s C_L = 0 \end{cases}$$

Solving for V_{out} gives:

$$A_v(s) = \frac{-g_{m1} R_{out}\left(1 - \frac{s}{g_{m1}/C_{gd1}}\right)}{1 + as + bs^2}$$

where $a = R_{in}(C_{gs1} + C_{gd1}(1 + g_{m1} R_{out})) + R_{out}(C_{gd1} + C_L)$ and

$b = R_{in} R_{out}(C_{gd1} C_{gs1} + C_{gd1} C_L + C_{gs1} C_L)$.

Assuming widely separated poles the dominant pole is given by

$$|p_1| = \frac{1}{a}$$

If the signal source resistance (R_{in}) is small the second term in a dominates and we have

$$|p_1| = \frac{1}{a} \approx \frac{1}{R_{out}(C_{gd1} + C_L)} \approx \frac{1}{R_{out} C_L} = \frac{1}{(r_{ds1} \| r_{dss2}) C_L} = \frac{I_{bias}}{4800 L C_L}$$

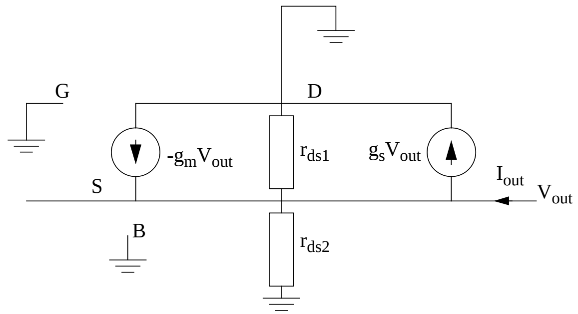
Conclusion: Large Bandwidth => Large bias current and small length.

Exercise B3.4

See above.

Exercise B3.6

Small Signal Model:



The output resistance is derived for zero input signal. Nodal equations:

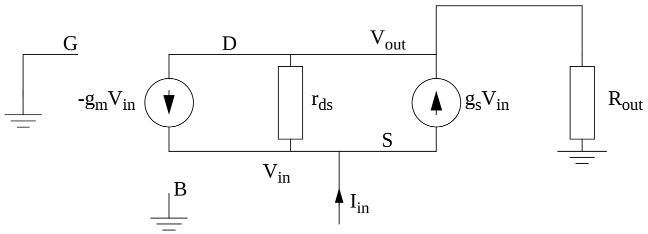
$$I_{out} = g_s V_{out} + \frac{V_{out}}{r_{ds1}} + \frac{V_{out}}{r_{ds2}} + g_m V_{out} \Rightarrow r_{out} = \frac{V_{out}}{I_{out}} = \frac{1}{g_m + g_s + \frac{1}{r_{ds1}} + \frac{1}{r_{ds2}}} \approx \frac{1}{g_m + g_s}$$

Exercise B3.7

Same problem as 3.6 with $g_{ds1} = g_{ds2} = 0$.

Exercise 3

Small Signal Model:



When using y- or z- parameters to derive the input impedance, either the output current or the output voltage should be set to zero. When analysing circuits where the input resistance depends on the load impedance (the CG stage is such a circuit) it is more convenient to include the load in the analysis and leaving the output open. This has be done in the above small signal model and gives the following nodal equations:

$$\begin{cases} I_{in} = V_{in}g_m + (V_{in} - V_{out})g_{ds} + g_s V_{in} \\ V_{in}g_m + (V_{in} - V_{out})g_{ds} + g_s V_{in} = V_{out}G_{out} \end{cases} \Rightarrow r_{in} = \frac{V_{in}}{I_{in}} =$$

$$= \frac{\left(1 + \frac{g_{ds}}{G_{out}}\right)}{g_{ds} + g_s + g_m} \approx \frac{1}{g_m}\left(1 + \frac{R_{out}}{r_{ds}}\right)$$

Exercise B3.8

p. 156-160. Referring to fig. 3.34-6 in the book:

a)

$$g_{m1} = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_{D1}} = \sqrt{2 \cdot 92 \cdot 10^{-6} \cdot \frac{100}{1.6}100 \cdot 10^{-6}} = 1.06\frac{mA}{V}$$

$$r_{ds1} = r_{ds2} = \frac{8000L}{I_{D1}} = \frac{8000 \cdot 1.6}{0.1} = 128k\Omega$$

$$g_{s1} = \frac{\gamma g_{m1}}{2\sqrt{V_{SB} - 2|\phi_F|}} = \frac{0.5 \cdot 1.06m}{2\sqrt{2 + 0.7}} = 0.16\frac{mA}{V}$$

$$G_{in} = \frac{1}{R_{in}} = \frac{1}{180k} = 5.56 \cdot 10^{-6}\text{S},\ G_{s1} = g_{s1} + g_{ds1} + g_{ds2} = 0.176\frac{mA}{V}$$

$$C_s = C_L + C_{sb1} = 0.54pF,\ C_{in'} = C_{in} + C_{gd1} = 45fF,\ C_{gs1} = 0.2pF$$

$$\omega_0 = \sqrt{\frac{G_{in}(g_{m1} + G_{s1})}{C_{gs1}C_s + C_{in'}(C_{gs1} + C_s)}} = 2\pi \cdot 35MHz$$

$$Q = \sqrt{\frac{G_{in}(g_{m1} + G_{s1})(C_{gs1}C_s + C_{in'}(C_{gs1} + C_s))}{G_{in}C_s + C_{in'}(C_{gs1} + C_s) + C_{gs1}C_{s1}}} = 0.332$$

$$z_1 = \frac{g_{m1}}{C_{gs1}} = 2\pi \cdot 844MHz$$

b) All parameters are the same except: $G_{s1} = g_{ds1} + g_{ds2} = 15.6\mu$ A/V:

$$\omega_0 = 2\pi \cdot 33MHz$$

$$Q = 0.57$$

$$z_1 = 2\pi \cdot 844MHz$$

Conclusion: Q should be smaller than 0.5 to avoid overshoot in the step response. This means that if we eliminate the body effect by connecting the bulk of M_1 the source of M_1 we will have a small overshoot which is not desirable.

Exercise B3.9

p. 161-63.

a)

$$C_1 = \frac{g_{m1}C_{gs1}C_s}{(g_{m1} + G_{s1})(C_{gs1} + C_s)} = \frac{(1.06m)(0.2p)(0.54p)}{(1.06m + 0.176m)(0.2p + 0.54p)} = 0.125pF$$

$$R_1 = \frac{(C_{gs1} + C_s)^2}{C_{gs1}C_s g_{m1}} = \frac{(0.74p)^2}{0.2p0.54p1.06m} = 4780\Omega$$

$$|p_1| \approx \frac{G_{in}}{C_{gs1} + C_{in'}} = 2\pi \cdot 3.61MHz,\ |p_2| \approx \frac{G_{s1} + g_{m1}}{C_{gs1} + C_L} = 2\pi \cdot 281MHz$$

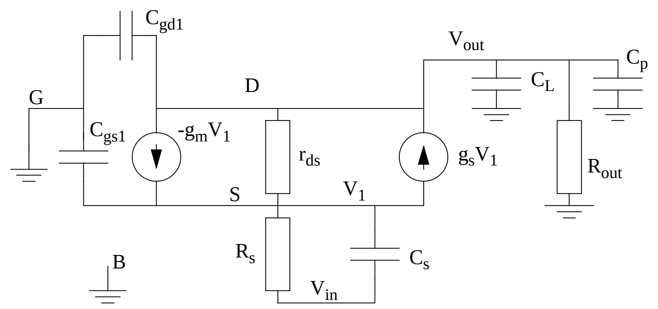
b)

$G_{s1} = g_{ds1} + g_{ds2} = 15.6\mu$ A/V => $C_1 = 0.144pF$, $R_1 = 4780\Omega$,

$|p_1| = 2\pi \cdot 3.61MHz$, $|p_2| = 2\pi \cdot 244MHz$

Exercise B3.10

Small Signal Model



Total capacitance at the output node:

$$C_{out} = C_p + C_L + C_{gd1} = C_L + C_{gd1} + C_{db2} + C_{gd2} + C_{db1} = 1.07\,\text{pF}$$

Output resistance: $R_{out} = r_{ds2} = 12000L/I_{D2} = 192k\Omega$

Nodal Equations:

$$\begin{cases} \{V_{in} - V_1\}(sC_s + G_s) = V_1 sC_{gs1} + (g_m + g_s)V_1 + (V_1 - V_{out})g_{ds1} \\ (g_m + g_s)V_1 + (V_1 - V_{out})g_{ds1} = V_{out}(sC_{out} + G_{out}) \end{cases}$$

Solving for V_{out} gives:

$$\frac{V_{out}}{V_{in}} = \frac{A_0(1 + cs)}{1 + as + bs^2}$$

where

$$A_0 = \frac{(g_m + g_{ds1} + g_s)G_s}{G_{out}(g_{ds1} + g_m + g_s) + (g_{ds1} + G_{out})G_s} \approx \begin{cases} G_s/G_{out} \text{ if } G_s \text{is small} \\ \dfrac{(g_m + g_s)}{(g_{ds1} + G_{out})} \text{ if } G_s \text{is large} \end{cases}$$

$$a = \frac{(C_{gs1} + C_s)(g_{ds1} + G_{out}) + C_{out}(g_m + g_{ds1} + g_s + G_s)}{G_{out}(g_{ds1} + g_m + g_s) + (g_{ds1} + G_{out})G_s} \approx \frac{C_{out}}{G_{out}} = C_{out}R_{out} \text{ if}$$

$G_s \ll g_m$

$$b = \frac{C_{out}(C_{gs1} + C_s)}{G_{out}(g_{ds1} + g_m + g_s) + (g_{ds1} + G_{out})G_s}, \; c = \frac{C_s}{G_s} = R_s C_s$$

Assuming widely separated poles => Dominant pole:

$$|p_1| = \frac{1}{a} = \frac{1}{C_{out}R_{out}} = \frac{1}{192k \cdot 1.07p} = 2\pi \cdot 0.77\,\text{MHz}$$