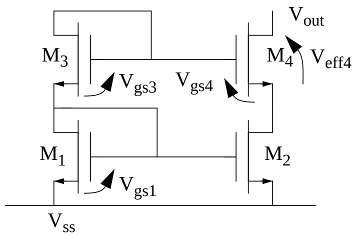
Lesson 3

Lesson Exercises: **B3.12, B3.13, B4.5, B4.6, B4.8**

Recommended Exercises: *B3.14, B3.15*

Theoretical Issues: **Improved Current Mirrors, Cascode Amplifier, Noise.**

Exercise B3.12



All transistors must be in the active region:

$$V_{out,\,min} = V_{ss} + V_{gs1} + V_{gs3} - V_{gs4} + V_{eff}$$

Assuming $V_{ss} = 0$ and all V_{gs} to be equal:

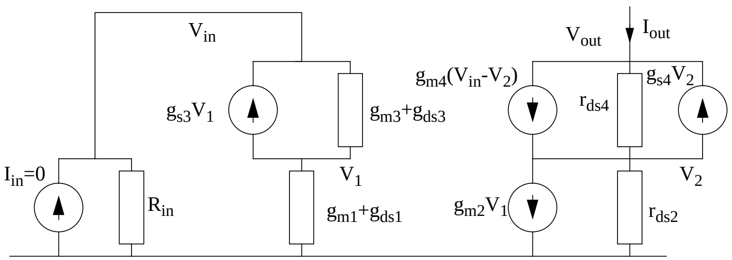
$$V_{out,\,min} = 2V_{gs} - V_t$$

$$V_{gs} = \sqrt{\frac{2I_D}{\mu_n C_{ox}(W/L)}} + V_t = 1.064\,\mathrm{V}$$

$$V_{out,\,min} = 1.33\,\mathrm{V}$$

Exercise B3.13

Small Signal Model



When calculating the output resistance, variations in the output voltage and the output current are considered. All constant voltages and current that are not affected by variations in output current or voltage are set to 0. This is the case for I_{in} and since no currents or voltages in the

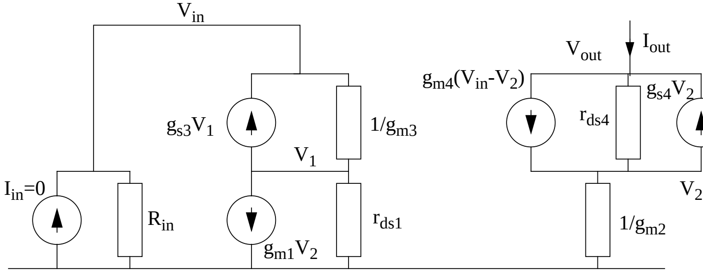
input circuit is affected by the output circuit we have $V_1 = V_{in} = 0$. The resulting nodal equations are thus

$$\begin{cases} I_{out} = -g_{m4}\cdot V_2 - g_{s4}V_2 + (V_{out} - V_2)g_{ds4} \\ -g_{m4}\cdot V_2 - g_{s4}V_2 + (V_{out} - V_2)g_{ds4} = V_2 g_{ds2} \end{cases} \Rightarrow$$

$$r_{out} = \frac{V_{out}}{I_{out}} = \frac{g_{ds2} + g_{ds4} + g_{s4} + g_{m4}}{g_{ds2}g_{ds4}} \approx g_{m4} r_{ds2} r_{ds4}$$

Exercise B3.15

Small Signal Model (It has been assumed that $g_{m3} \gg g_{ds3}$ and $g_{m2} \gg g_{ds2}$.



If we compare this circuit to the one in 3.13 we see that in this case transistor M1 has its gate connected to the output circuit. Thus we can **not** assume that V_{in} and V_1 are 0. The input source is not affected by the output circuit and can be set to 0, i.e. $I_{in} = 0$. The resulting nodal equations are thus

$$\begin{cases} I_{out} = g_{m4}\cdot(V_{in} - V_2) - g_{s4}V_2 + (V_{out} - V_2)g_{ds4} \\ g_{m4}(V_{in} - V_2) - g_{s4}V_2 + (V_{out} - V_2)g_{ds4} = V_2 g_{m2} \\ g_{m1}V_2 + V_1 g_{ds1} + g_{s3}V_1 + (V_1 - V_{in})g_{m3} = 0 \\ (V_1 - V_{in})g_{m3} + g_{s3}V_1 = V_{in}G_{in} \end{cases} \Rightarrow$$

$$r_{out} = \frac{V_{out}}{I_{out}} = \quad \text{(where } g_4 = g_{m4} + g_{s4} + g_{ds4} \approx g_{m4} + g_{s4}\text{)} =$$

$$\frac{(g_{m1}g_{m4}g_{s3} + g_{m3}(g_{gs1}(g_4 + g_{m2}) + g_{m1}g_{m4})) + G_{in}(g_{ds1} + g_{m3} + g_{s3})(g_4 + g_{m2})}{g_{ds4}g_{m2}(g_{ds1}(G_{in} + g_{m3}) + G_{in}(g_{m3} + g_{s3}))}$$

$$\approx \frac{g_{m1}g_{m4}(g_{m3} + g_{s3})}{g_{ds4}g_{m2}(g_{ds1}g_{m3} + G_{in}(g_{m3} + g_{s3}))} \text{ if } G_{in} \ll g_m$$

Assuming $G_{in} \approx g_{ds1}$, $g_{m4} \approx g_{m2}$ and $g_{s3} = 0$:

$$r_{out} = \frac{g_{m1}}{g_{ds4}g_{ds1}2} = \frac{g_{m1}r_{ds1}r_{ds4}}{2}$$

Assuming $G_{in} \approx 0$, $g_{m4} \approx g_{m2}$ and $g_{s3} = 0$:

$$r_{out} \approx \frac{g_{m1}}{g_{ds4} g_{ds1}} = g_{m1} r_{ds1} r_{ds4}$$

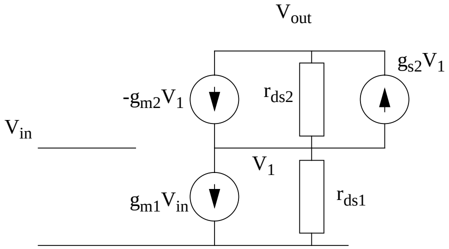
Conclusion: If the input source resistance is small ($R_{in} \approx r_{ds}$) the output resistance is half the output resistance of the cascode mirror while for a large source resistance the output resistance is approximately equal to the cascode case. The output resistance is thus dependent on the input load resistance.

Exercise B3.15

Same as 3.15 but $g_{s3} = g_{s4} = 0$. See above

Exercise 5

Small Signal Model

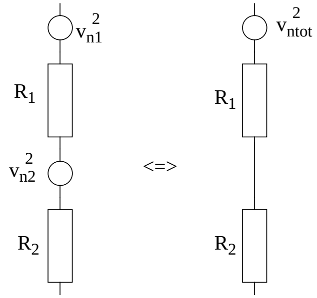


Nodal Equations:

$$\begin{cases} g_{m1} V_{in} + V_1 g_{ds1} + g_{m2} V_1 + (V_1 - V_{out}) g_{ds2} + g_{s2} V_1 = 0 \\ g_{m2} V_1 + (V_1 - V_{out}) g_{ds2} + g_{s2} V_1 = 0 \end{cases} \Rightarrow$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{(g_{m2} + g_{s2} + g_{ds2}) g_{m1}}{g_{ds1} g_{ds2}} \approx g_{m2} r_{ds2} r_{ds1}$$

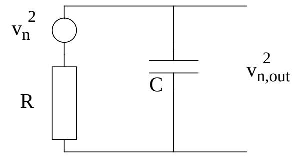
Exercise B4.5



If the noise sources are assumed to be uncorrelated we can add the spectral densities of the sources:

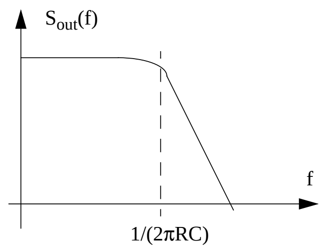
$$S_{tot}(f) = S_1(f) + S_2(f) = 4kTR_1 + 4kTR_2 = 4kT(R_1 + R_2)$$

Exercise B4.6



$$H(s) = \frac{V_{n,out}(s)}{V_n(s)} = \frac{1}{1 + RCs}$$

$$S_{ut}(f) = |H(j\omega)|^2 S_{in}(f) = \frac{1}{\sqrt{1 + (\omega RC)^2}} 4kTR$$

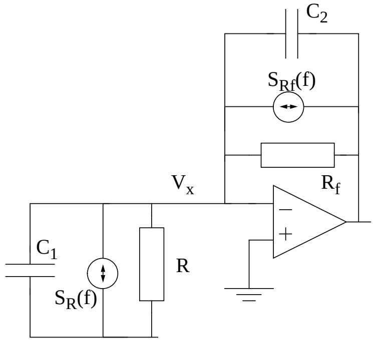


By using the noise bandwidth $NBW = \frac{\pi}{2} f_{-3dB} = \frac{\omega_{-3dB}}{4}$ we have

$$v_{n,out}^2 = S_{out}(f) \cdot NBW = 4kTR \cdot \frac{1}{4RC} = \frac{kT}{C}$$

Conclusion: The noise in a RC circuit is independent of the resistor value.

Exercise B4.8



Ideal opamp => $V_x = 0$. We have contributions from two noise sources. The transfer function from input current to output voltage is given by:

$$H(s) = \frac{V_{out}}{I_{in}} = \frac{R_f}{1 + R_f C_2 s}$$

Spectral density:

$$S_{vo}(f) = (S_{iRf}(f) + S_{iR}(f))\frac{R_f^2}{1 + (R_f C_2 \omega)^2}$$

The total noise is given by:

$$v_{no}^2 = \left(\frac{4kT}{R_f} + \frac{4kT}{R}\right)R_f^2 \frac{1}{4 R_f C_2} = \frac{2kT}{C_2}$$