Tutorial 3: Mixer Solutions

Problem 1

Consider the active mixer shown in the figure below where the LO has abrupt edges and a 50% duty cycle. Also, channel-length modulation and body effect are negligible. The load resistors exhibit mismatch, but the circuit is otherwise symmetric. Assume $M1$ carries a bias current of I_{SS}. Determine the output offset voltage.

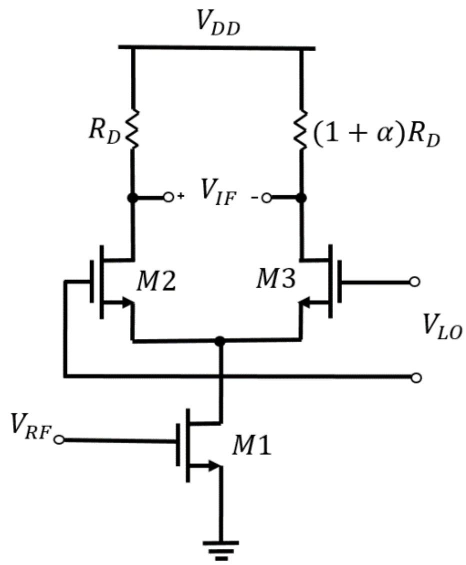

Fig. 1.1 Active mixer with load mismatch

Solution:

$V_{IF}(t)$ is expressed as

$$V_{IF}(t) = V_{IF}^{+}(t) - V_{IF}^{-}(t)$$

$$V_{IF}(t) = i_{IF}^{+}(t) \cdot R_D - i_{IF}^{-}(t) \cdot (1+\alpha)R_D$$

$$\Rightarrow V_{IF}(t) = \left(i_{IF}^{+}(t) - i_{IF}^{-}(t)\right)R_D - i_{IF}^{-}(t)\alpha\, R_D \tag{1.1}$$

Notice from (1.1) that the offset in $V_{IF}(t)$ corresponds to the second term, $i_{IF}^{-}(t) \cdot \alpha\, R_D$. Therefore, elaborating for $i_{IF}^{-}(t)$, which corresponds to the product between $i_{RF}(t)$ and a train of rectangular pulses with amplitude 1 and period T_{LO}. We have,

$$i_{IF}^{-}(t) = g_{m1} i_{RF}(t) \cdot \left[\frac{1}{2} + \sum_{k=1}^{\infty} \operatorname{sinc}\left(\frac{k\pi}{2}\right) \cos\left(k\omega_{LO}t - \frac{3k\pi}{2}\right)\right] \tag{1.2}$$

Since, we are interested in ω_{LO}, we work only with k = 1 in (1.2), which simplifies to

$$i_{IF}^{-}(t) = i_{RF}(t) \cdot \left[\frac{1}{2} - \frac{2}{\pi}\sin(\omega_{LO}t)\right] \tag{1.3}$$

On the other hand, i_{RF} can be expressed as a complex envelope signal with a dc component by

$$i_{RF}(t) = I_{DC} + g_{m1}V_{RF}(t)$$

$$\Rightarrow i_{RF}(t) = I_{DC} + g_{m1}\mathrm{a(t)}\cos[\omega_{RF}t + \theta(t)] \quad (1.4)$$

Substituting (1.4) into (1.3) and solving for $i_{IF}^{-}(t)$**,** we have

$$i_{IF}^{-}(t) = \frac{I_{DC}}{2} + \frac{g_{m1}V_{RF}(t)}{2} - \frac{2I_{DC}\sin(\omega_{LO}t)}{\pi} - \frac{2\,g_{m1}V_{RF}(t)\sin(\omega_{LO}t)}{\pi} \quad (1.5)$$

Applying a low-pass filter to (1.5), we obtain for $i_{IF}^{-}(t)$,

$$\Rightarrow i_{IF}^{-}(t) = \frac{I_{DC}}{2} - \frac{g_{m1}\mathrm{a(t)}}{\pi}\left[\cos\big((\omega_{RF} - \omega_{LO})t + \theta(t)\big)\right] \quad (1.6)$$

Now, calculating for the offset voltage, we have

$$V_{IF\_offset}(t) = \frac{I_{DC}}{2}\alpha\, R_D - \frac{g_{m1}\mathrm{a(t)}}{\pi}\left[\cos\big((\omega_{RF} - \omega_{LO})t + \theta(t)\big)\right]\alpha\, R_D \;\blacksquare \quad (1.7)$$

The first term in (1.7) is the DC_offset and it does not cancel. Notice that the larger the mismatch, the larger the DC_offset is. Then, the second term in (1.7) degrades the voltage gain at ω_{IF}, so that

$$A_V = \frac{V_{IF}(t)}{V_{RF}(t)} = g_{m1}R_D(2-\alpha) \;\blacksquare \quad (1.8)$$

Problem 2

Shown below is the front-end of a 1.8-GHz receiver. The LO frequency is chosen to be 900 MHz and the load inductors and capacitances resonate with a quality factor Q at IF. Assume $M1$ is biased at a current $I1$, and the mixer and LO are perfectly symmetric. Also assume $M2$ and $M3$ are ideal switches (they switch abruptly and completely). Compute (a) the measured level of the 900-MHz at the output in the absence of an RF signal, (b) the LO-IF feedthrough with the presence only of the gate-drain capacitance C_{GD}. Neglect gate-source and gate-bulk capacitance.

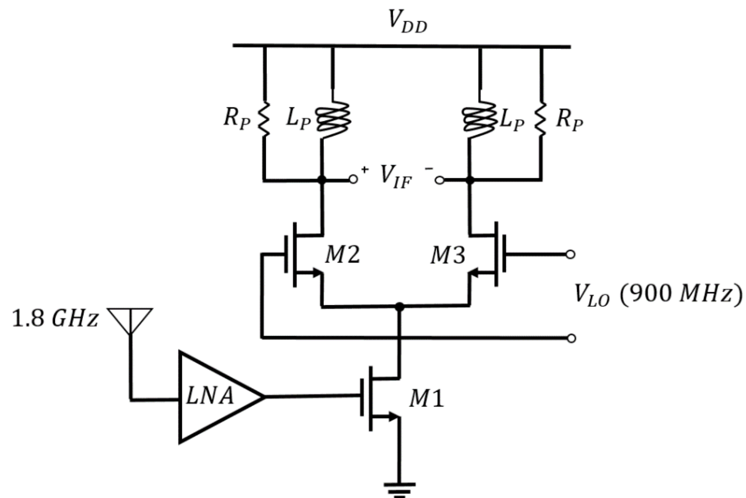

Fig. 2.1 Receiver front-end

Solution:

a) The measured level of the 900-MHz at the output in the absence of an RF signal.

$$V_{LO}^{+}(t) = \frac{1}{2} + \frac{2}{\pi}\sin(\omega_{LO}t) + \frac{2}{3\pi}\sin(3\omega_{LO}t) + \frac{2}{5\pi}\sin(5\omega_{LO}t) + \cdots$$

$$V_{LO}^{-}(t) = \frac{1}{2} - \frac{2}{\pi}\sin(\omega_{LO}t) - \frac{2}{3\pi}\sin(3\omega_{LO}t) - \frac{2}{5\pi}\sin(5\omega_{LO}t) + \cdots$$

$$i_{RF}(t) = I_1 + I_{RF}\cos\omega_{RF}t$$

No RF signal: $I_{RF} = 0 \Rightarrow i_{RF}(t) = I_1$. The output current at IF is given by:

$$i_{IF}^{+}(t) = V_{LO}^{+}(t) \times i_{RF}(t) = \left[\frac{1}{2} + \frac{2}{\pi}\sin(\omega_{LO}t) + \frac{2}{3\pi}\sin(3\omega_{LO}t) + \frac{2}{5\pi}\sin(5\omega_{LO}t) + \cdots\right] \times I_1$$

$$i_{IF}^{+}(t) = \frac{I_1}{2} + \frac{2I_1}{\pi}\sin\omega_{LO}t$$

$$i_{IF}^{-}(t) = V_{LO}^{-}(t) \times i_{RF}(t) = \left[\frac{1}{2} - \frac{2}{\pi}\sin(\omega_{LO}t) - \frac{2}{3\pi}\sin(3\omega_{LO}t) - \frac{2}{5\pi}\sin(5\omega_{LO}t) - \cdots\right] \times I_1$$

$$i_{IF}^{-}(t) = \frac{I_1}{2} - \frac{2I_1}{\pi}\sin\omega_{LO}t$$

$$i_{IF}(t) = i_{IF}^{+}(t) - i_{IF}^{-}(t) = \frac{4I_1}{\pi}\sin\omega_{LO}t$$

$$V_{IF}(t) = i_{IF}(t) \times R_P = \frac{4I_1R_P}{\pi}\sin\omega_{LO}t \ \blacksquare$$

b) Working with one of the single-ended sections and considering only the parasitic gate-drain capacitance C_{GD}, the LO-IF feedthrough can be derived from the circuit shown in Fig. 2.2. Notice that only V_{LO}^{+} is operating.

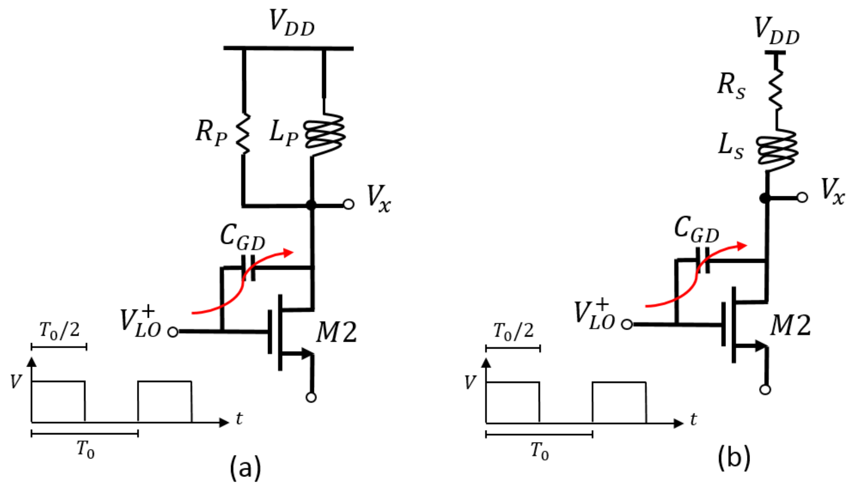

Fig. 2.2 (a) Single-ended with LO-IF feedthrough (b) transformation from parallel to series

From Fig. 2.2 (b), the presence of V_{LO}^{+} at node Vx can be derived by the following expressions,

$$\frac{V_{LO}^{+} - Vx}{Z_C} = \frac{Vx - V_{DD}}{Z_L + R_s}$$

$$\Rightarrow \ Vx\left[\frac{Z_C + Z_L + R_s}{(Z_L + R_s)Z_C}\right] = \frac{V_{LO}^{+}}{Z_C} + \frac{V_{DD}}{Z_L + R_s}$$

$$\Rightarrow Vx = \frac{V_{LO}^{+}(Z_L + R_s)}{Z_C + Z_L + R_s} + \frac{V_{DD}Z_C}{Z_C + Z_L + R_s} \qquad (2.1)$$

The voltage V_{LO}^{+} is considered to be a train of rectangular pulses. Its representation in the time domain can be obtained from the Fourier series. Following the analysis in problem 1, but this time considering $\alpha = 0.5$ and $V = 1$, we have

$$V_{LO}^{+} = \frac{1}{2} + \sum_{k=1}^{\infty}\operatorname{sinc}\left(\frac{k\pi}{2}\right)\cos\left(k\omega_{LO}t - {}^{k\pi}/_{2}\right) \qquad (2.2)$$

Since we are interested in the frequency component at ω_{LO}, we work with $k = 1$. Substituting (2.2) into (2.1), the LO-IF feedthrough can be expressed as,

$$Vx = \frac{(Z_L + R_s)}{Z_C + Z_L + R_s}\left[\frac{1}{2} + \frac{2}{\pi}\cos\left(\omega_{LO}t - {}^{\pi}/_{2}\right)\right] + \frac{V_{DD}Z_C}{Z_C + Z_L + R_s}$$

$$= \frac{(Z_L + R_s)}{Z_C + Z_L + R_s}\left[\frac{1}{2} + \frac{2}{\pi}\sin(\omega_{LO}t)\right] + \frac{V_{DD}Z_C}{Z_C + Z_L + R_s} \quad (2.3)$$

Carrying out the same analysis, but for V_{LO}^{-} with $k = 1$, we have

$$V_{LO}^{-}|_{k=1} = \frac{1}{2} - \frac{2}{\pi}\sin(\omega_{LO}t) \quad (2.4)$$

Therefore, the LO-IF feedthrough in the other section of the circuit, Vy equals

$$Vy = \frac{(Z_L + R_s)}{Z_C + Z_L + R_s}\left[\frac{1}{2} - \frac{2}{\pi}\sin(\omega_{LO}t)\right] + \frac{V_{DD}Z_C}{Z_C + Z_L + R_s} \quad (2.5)$$

Finally, the differential voltage $Vx - Vy$, corresponding to the total contribution of the LO-IF feedthrough is expressed as

$$Vx - Vy = \frac{4}{\pi}\frac{(Z_L + R_s)}{(Z_C + Z_L + R_s)}\sin(\omega_{LO}t) \quad (2.6)$$

Notice that the DC component is cancelled out

Problem 3

The circuit shown below is a dual-gate mixer used in traditional microwave design. Assume abrupt edges and a 50% duty cycle for the LO and neglect channel-length modulation and body effect.

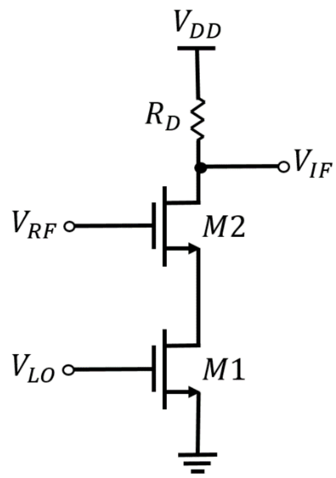

Fig. 3.1 Dual-gate mixer

a) Assume that $M1$ is an ideal switch. Determine the frequency components which appear at the mixer IF port.
b) Assume when $M1$ is on, it has an on-resistance of R_{on1}. Compute the voltage conversion gain of the circuit. Assume $M2$ does not enter the triode region and denote its transconductance by g_{m2}.
c) Assume when $M1$ is an ideal switch. Compute the voltage conversion gain of the circuit.

Solution:

a) The current appearing in the transistor $M2$ due to the input voltage V_{RF} can be expressed as

$$i_{RF}(t) = I_{DC} + g_{m2}V_{RF}(t) \quad (3.1)$$

Due to the switching action of $M1$, the resultant current at the output corresponds to the product between i_{RF} and a rectangular signal with 50% duty cycle. This can be expressed as follows

$$i_{out}(t) = [I_{DC} + g_{m2}V_{RF}(t)] \times \left[\frac{1}{2} + \sum_{k=1}^{\infty} \operatorname{sinc}\left(\frac{k\pi}{2}\right) \cos\left(k\omega_{LO}t - {}^{k\pi}/_{2}\right)\right] \quad (3.2)$$

Besides, V_{RF} can be expressed as a complex envelope by

$$V_{RF}(t) = \mathrm{a(t)} \cdot \cos[\omega_{RF}t + \theta(t)] \quad (3.3)$$

Substituting (3.3) into (3.2) we have

$$i_{out}(t) = [I_{DC} + g_{m2}\mathrm{a(t)}\cos[\omega_{RF}t + \theta(t)]] \cdot \left[\sum_{k=1}^{\infty} \frac{1}{2} + \operatorname{sinc}\left(\frac{k\pi}{2}\right) \cos\left(k\omega_{LO}t - {}^{k\pi}/_{2}\right)\right] \quad (3.4)$$

From (3.4) we can obtain all frequency components at the mixer's output for different k values. Working with $k = 1$, then (3.4) can be expressed as

$$i_{out}(t) = [I_{DC} + g_{m2}\mathrm{a(t)}\cos[\omega_{RF}t + \theta(t)]] \cdot \left[\frac{1}{2} + \frac{2}{\pi}\sin(\omega_{LO}t)\right]$$

$$\Rightarrow i_{out}(t) = \underbrace{\frac{I_{DC}}{2}}_{DC\ Component} + \underbrace{\frac{2I_{DC}}{\pi}\sin(\omega_{LO}t)}_{LO\ Mixing\ Product} + \underbrace{\frac{g_{m2}\mathrm{a(t)}}{2}\cos[\omega_{RF}t + \theta(t)]}_{RF\ Mixing\ Product}$$

$$+ \frac{g_{m2}\mathrm{a(t)}}{\pi}\left[\underbrace{\sin\big((\omega_{RF} - \omega_{LO})t + \theta(t)\big)}_{IF} + \underbrace{\sin\big((\omega_{RF} + \omega_{LO})t + \theta(t)\big)}_{HF\ (to\ be\ filtered)}\right] \blacksquare \quad (3.5)$$

b) The output voltage V_{out} due to the action of V_{RF} and the presence of on-resistance R_{on}can be derived through the small-signal model shown in Fig. 3.2.

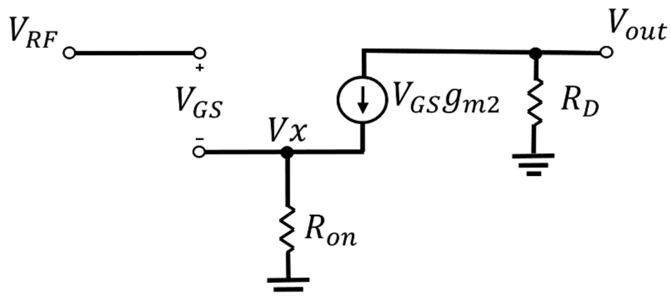

Fig. 3.2 Small-signal model with on-resistance R_{on}

Initially, solving for V_{GS}, we have

$$V_{GS} = V_{RF} - Vx$$
$$= V_{RF} - V_{GS}\, g_{m2}R_{on}$$
$$\Rightarrow V_{GS}(1 + g_{m2}R_{on}) = V_{RF}$$
$$\Rightarrow V_{GS} = \frac{V_{RF}}{1 + g_{m2}R_{on}} \quad (3.6)$$

On the other hand, V_{out} can be expressed as

$$V_{out} = -g_{m2}V_{GS}R_D \quad (3.7)$$

Substituting (3.6) into (3.7), we have

$$V_{out} = \frac{-g_{m2}R_D}{1 + g_{m2}R_{on}}V_{RF} = \alpha V_{RF} \quad (3.8)$$

To obtain the conversion gain, we have to find the IF component. This can be found by the product of V_{out} and the rectangular LO signal with $k = 1$ and using bandpass filter centered at ω_{IF}. This can be expressed as

$$V_{IF} = BPF\left\{\alpha V_{RF}(t) \times \left[\frac{1}{2} + \frac{2}{\pi}\cos\left(\omega_{LO}t - {}^{\pi}/_{2}\right)\right]\right\}$$

$$= BPF\left\{dc + \frac{\mathrm{a(t)}\alpha}{\pi} \cdot \cos\left[\underbrace{(\omega_{RF} - \omega_{LO})}_{\omega_{IF}} t + \theta(t) - {}^{\pi}/_{2}\right] + HF\ comp\right\}$$

$$= \frac{\alpha \mathrm{a(t)}}{\pi} \cdot \cos\left[\underbrace{(\omega_{RF} - \omega_{LO})}_{\omega_{IF}} t + \theta(t) - {}^{\pi}/_{2}\right] \quad (3.9)$$

Then, the voltage conversion gain is equal to

$$A_V = \frac{V_{IF}(t)}{V_{RF}(t)} \quad (3.10)$$

Substituting (3.3) and (3.9) into (3.10), we have

$$A_V = \frac{\alpha \mathrm{a(t)}}{\mathrm{a(t)}\pi} = \frac{\alpha}{\pi} = \frac{-g_{m2}R_D}{\pi(1 + g_{m2}R_{on})} \ \blacksquare \quad (3.11)$$

c) When $M1$ operates as an ideal switch the on-resistance R_{on} equals zero, and the voltage conversion gain can be expressed as

$$A_V = \frac{-g_{m2}R_D}{\pi} \ \blacksquare \quad (3.12)$$

Appendix: Fourier series representation of rectangular pulses

A train of rectangular pulses as shown in Fig A.1.

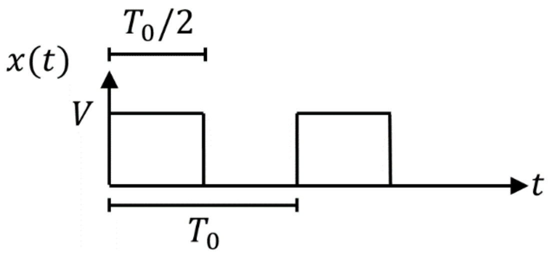

Fig. A.1. Time-domain representation of rectangular pulses

Its representation in the time domain can be obtained from the Fourier series as

$$x(t) = C_0 + \sum_{k=1}^{\infty} 2C_k e^{jk\omega_0 t}$$

where C_k is the complex coefficient expressed by

$$C_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\omega_{LO}t}\,dt$$

Now, solving for C_k

$$C_k = \frac{V}{T_0}\int_0^{T_0/2} e^{-jk\omega_{LO}t}dt = \frac{jV}{2\pi k}\left(e^{-j\pi k} - 1\right)$$

$$= \frac{jV}{2\pi k}\left[\left[\cos\left(\frac{k\pi}{2}\right) - j\sin\left(\frac{k\pi}{2}\right)\right]^2 - 1\right]$$

$$= \frac{jV}{2\pi k}\left[\cos^2\left(\frac{k\pi}{2}\right) - 2j\cos\left(\frac{k\pi}{2}\right)\sin\left(\frac{k\pi}{2}\right) - \sin^2\left(\frac{k\pi}{2}\right) - 1\right]$$

$$= \frac{jV}{2\pi k}\left[-2j\cos\left(\frac{k\pi}{2}\right)\sin\left(\frac{k\pi}{2}\right) - 2\sin^2\left(\frac{k\pi}{2}\right)\right]$$

$$= \frac{V}{\pi k}\sin\left(\frac{k\pi}{2}\right)\left[\cos\left(\frac{k\pi}{2}\right) - j\sin\left(\frac{k\pi}{2}\right)\right]$$

$$= \frac{V}{\pi k}\sin\left(\frac{k\pi}{2}\right)e^{-j\frac{k\pi}{2}} = \frac{V}{2}\mathrm{sinc}\left(\frac{k\pi}{2}\right)e^{-j\frac{k\pi}{2}} \blacksquare$$

Now, for $x(t)$ with $C_0 = V/2$, we have

$$x(t) = \frac{V}{2} + \sum_{k=1}^{\infty} V\mathrm{sinc}\left(\frac{k\pi}{2}\right)\cos\left(k\omega_0 t - k\pi/2\right)$$