SOLUTIONS. Exam August 21, 2008
TSEI05 Analog and Discrete-time Integrated Circuits.

Exercise 1.

M1: Figure 1 gives: $V_{GS1} = V_{in} = 3$ V och $V_{DS1} = V_{out} = 0.03$ V

$V_{DS1} << V_{GS1} - V_{tn} = 2.5$ V, which means that transistor **M1** works in the *linear* region.

Enclosed formulas then give:

$$I_{D1} = \frac{\mu_{0n} C_{ox}}{2} \left(\frac{W}{L}\right)_1 \left(2(V_{GS1} - V_{Tn}) - V_{DS1}\right) V_{DS1} \tag{1}$$

With $I_{D1} = I_{D2} = 20$ nA relation (1) gives:

$$20 \cdot 10^{-9} = 10 \cdot 10^{-9} \cdot \left(\frac{W}{10^{-6}}\right)_1 (2 \cdot 2.5 - 0.03)\, 0.03 \Rightarrow W_1 \approx 13.4\mu\text{m}$$

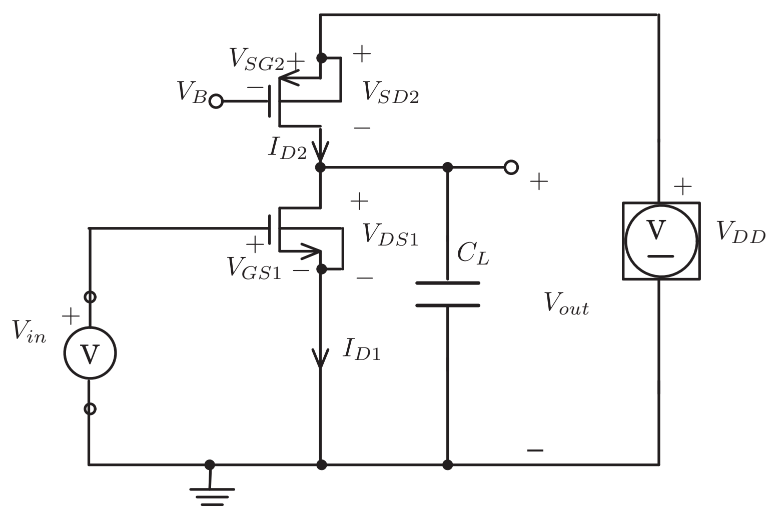

Figure 1: Inverter. Large signal analysis.

M2: From **Figure 1**: $V_{SG2} = V_{DD} - V_B = 2$ V and $V_{SD2} = V_{DD} - V_{out} = 2.97$ V

$V_{SD2} > V_{SG2} - V_{tp} = V_{eff2} = 1.4$ V, which means that transistor **M2** works in the *saturated* region.

Enclosed formulas than give:

$$I_{D2} = \frac{\mu_{0p} C_{ox}}{2} \left(\frac{W}{L}\right)_2 \left((V_{SG2} - V_{Tp})^2\right) \left(1 + \lambda_p (V_{SD2} - V_{eff2})\right) \tag{2}$$

$I_{D1} = I_{D2} = 20$ nA inserted in (2):

$$20 \cdot 10^{-9} = 3 \cdot 10^{-9} \left(\frac{W}{10^{-6}}\right)_2 1.4^2 (1 + 0.05 \cdot (2.97 - 1.4)) \Rightarrow W_2 \approx 3.19\mu\text{m}$$

Answer: $W_1 \approx 13.4$ μm och $W_2 \approx 3.19$ μm

Exercise 2.

a) **Figure 2 a)** shows a complete small signal equivalent. As the DC voltage source (V_{DD}) is ideal it will be replaced by a short circuit in the small signal equivalent circuit.

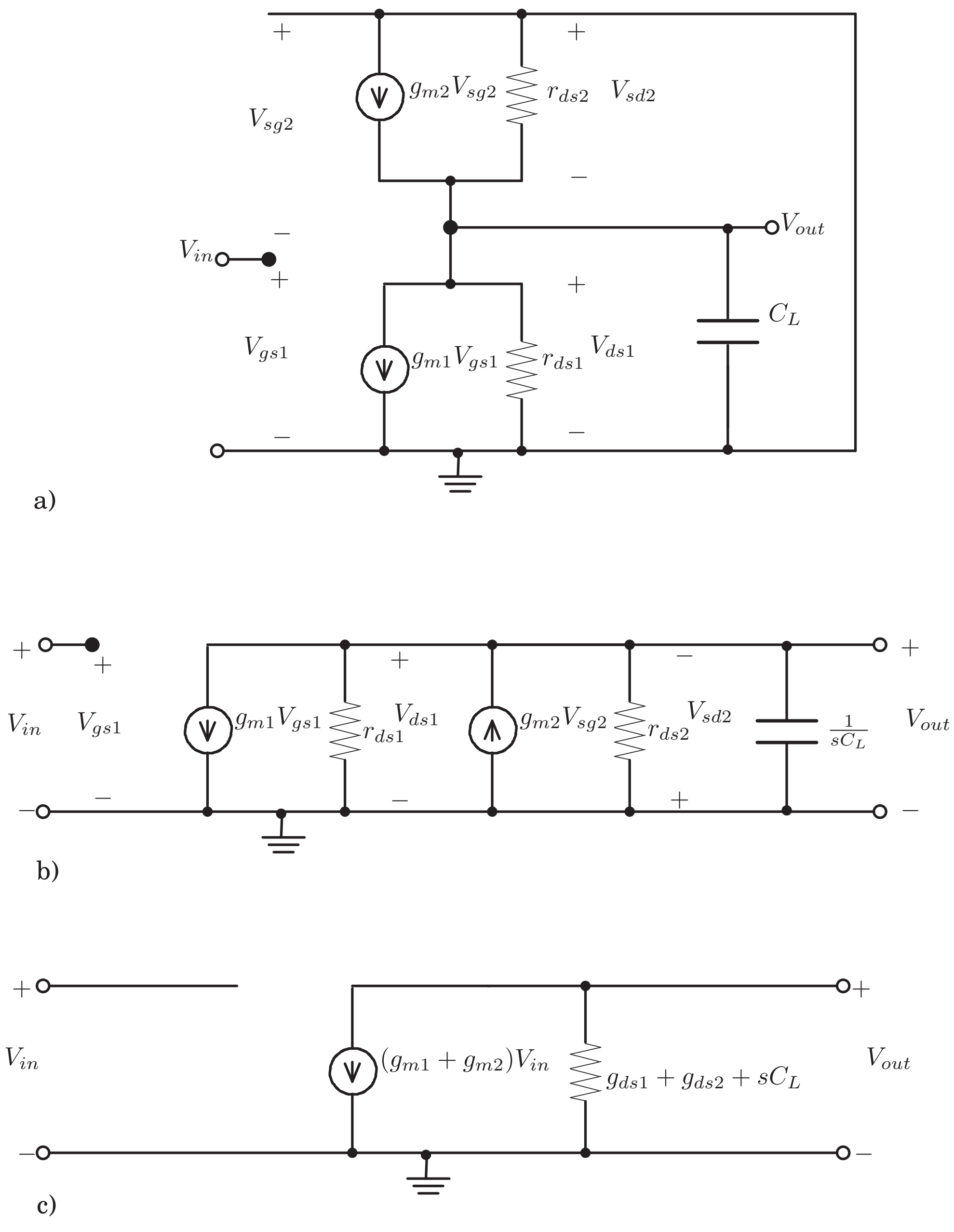

Figure 2: Complete small signal equivalent.

b) **Figure 2b)** is obtained by rewriting **figure 2a)**. Note that $V_{sg2} = -V_{gs1}$.

The equivalent circuit in **Figure 2c)** (the asked for equivalent) is obtained from **Figure 2b)** by:

1. Note that resistors $1/g_{ds1}$, $1/g_{ds2}$ and the capacitor $\frac{1}{sC_L}$ are parallel, then the total admittans obtains by adding the admittanses g_{ds1}, g_{ds2} and sC_L.
2. Observe that $V_{sg2} = -V_{gs1} = -V_{in}$ which yields that the current source $g_{m2}V_{sg2}$ in **Figure 2b)** can be changed to a current source $g_{m2}V_{in}$ with opposite direction. This current source is parallel to the current source $g_{m1}V_{gs1}$. Thus they can be added to one current source $(g_{m1} + g_{m2})V_{in}$.

Figure 2c yields:

$$V_{out} = -\frac{(g_{m1} + g_{m2})V_{in}}{g_{ds1} + g_{ds2} + sC_L} \tag{3}$$

$$(3) \Rightarrow \frac{V_{out}}{V_{in}} = -\frac{g_{m1} + g_{m2}}{g_{ds1} + g_{ds2} + sC_L}$$

c) $s = j\omega$ yields the transfer function $H(\omega)$:

$$H(\omega) = -\frac{g_{m1} + g_{m2}}{g_{ds1} + g_{ds2} + j\omega C_L} \Rightarrow \mid H(\omega) \mid = \frac{g_{m1} + g_{m2}}{\left((g_{ds1} + g_{ds2})^2 + (\omega C_L)^2\right)^{1/2}} \tag{4}$$

Unity-gain frequency is that angular frequency ω_u when $\mid H(\omega) \mid = 1$.

$$\mid H(\omega) \mid = 1 \overset{(4)}{\Rightarrow} (g_{ds1}+g_{ds2})^2+(\omega C_L)^2 = (g_{m1}+g_{m2})^2 \Rightarrow \omega_u = \frac{\left((g_{m1} + g_{m2})^2 - (g_{ds1} + g_{ds2})^2\right)^{1/2}}{C_L}$$

$$\textbf{Answer: } \omega_u = \frac{\left((g_{m1} + g_{m2})^2 - (g_{ds1} + g_{ds2})^2\right)^{1/2}}{C_L} \approx \frac{g_{m1} + g_{m2}}{C_L}$$

Exercise 3.

a)

Phase I:

- $V_{INP} = +E, V_{INN} = 0 \Rightarrow$ M1 conducts and M3 blocks.
- M3 blocks $\Rightarrow I_4 = 0$
  M1 conducts $\Rightarrow I_1 = I_2 = I_0$
- M2 and M4 constitute a current mirror and as M2 and M4 are identical $I_3 = I_1 = I_0$
- M3 blocks $(I_4 = 0) \Rightarrow I_5 = I_3 = I_0$

Phase II:

- $V_{INP} = 0, V_{INN} = +E \Rightarrow$ M1 blocks and M3 conducts.
- M1 blocks $\Rightarrow I_6 = I_7 = 0$
- M2 and M4 constitute a current mirror $\Rightarrow I_8 = I_6 = 0$
- M3 conducts $\Rightarrow I_9 = I_0$
- $I_8 = 0$ and $I_9 = I_0 \Rightarrow I_{10} = -I_9 = -I_0$

b)

$$\text{Definition: Slew-Rate (SR) =max}\frac{dv_{out}(t)}{dt}$$

For capacitor C_L we have that:

$$i_{CL}(t) = C_L\frac{dv_{CL}(t)}{dt} = C_L\frac{dv_{out}(t)}{dt} \Rightarrow \frac{dv_{out}}{dt} = \frac{i_{CL}(t)}{C_L}$$

Thus the maximum value of $\frac{dv_{out}(t)}{dt}$ obtains when $i_{CL}(t)$ has it maximum value, which according to a) is I_0.

$$\textbf{Answer: } \text{Slew-Rate} = \frac{I_0}{C_L}$$

Exercise 4.

Figure 3 a) shows a small signal equivalent and **Figure 3 b)** a redrawn version, where we have used the fact that $V_{gs2} = 0$ (giving $g_{m2}V_{gs2} = 0$) and that g_{ds1}, g_{ds2} and $\frac{1}{sC_L}$ are parallel.

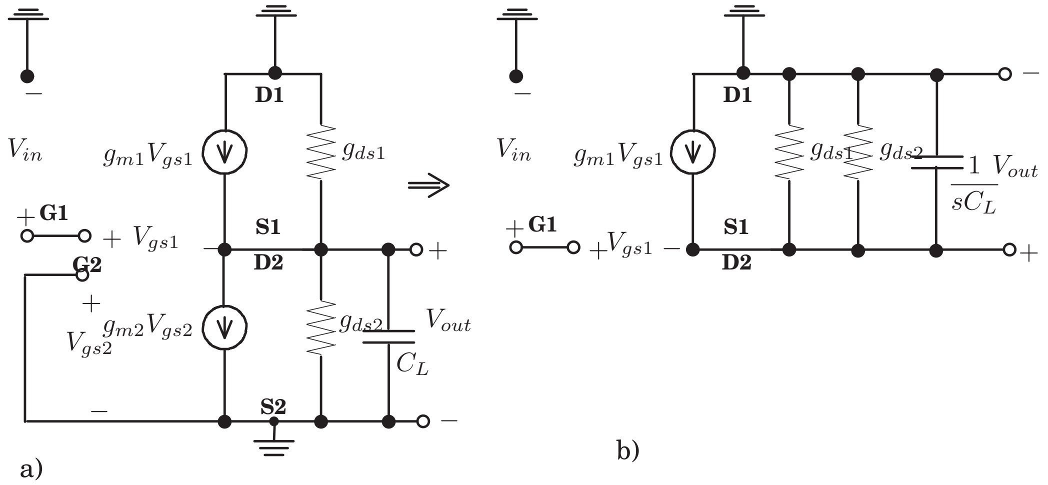

Figure 3: Small signal equivalent circuit.

Determine V_{out}/V_{in}**:**
Figure 3 b) gives:

$$V_{gs1} = V_{in} - V_{out} \tag{5}$$

and

$$V_{out} = g_{m1}V_{gs1} \cdot \frac{1}{g_{ds1} + g_{ds2} + sC_L} \tag{6}$$

(5) inserted in (6) gives:

$$V_{out} = g_{m1}(V_{in} - V_{out}) \cdot \frac{1}{g_{ds1} + g_{ds2} + sC_L} \tag{7}$$

(7) gives the transfer function:

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{\frac{g_{m1}}{g_{ds1}+g_{ds2}+sC_L}}{1 + \frac{g_{m1}}{g_{ds1}+g_{ds2}+sC_L}} = \frac{g_{m1}}{g_{m1} + g_{ds1} + g_{ds2} + sC_L} \tag{8}$$

As $g_m >> g_{ds}$ following approximation of $H(s)$ obtains:

$$H(s) \approx \frac{g_{m1}}{g_{m1} + sC_L} = \frac{1}{1 + \frac{sC_L}{g_{m1}}} \tag{9}$$

Determine R_{out}**:**
Set the input signal $V_{in} = 0$ and introduce the noisy voltage source V_{Th} with spectral density $R_{Th}(f) = \frac{8kT}{3} \cdot \frac{1}{g_{m1}}$ (from enclosed formulas) between **G1** and ground. See **Figure 4**. Note that V_{Th} will have the same position as V_{in} in **Figure 3 b)**, i.e. between **G1** and ground. Which means that eqn. (9) also gives the relation between V_{Th} and V_{out}.

The relation $R_{out}(f) = |H(f)|^2 R_{in}(f)$ gives (introduce $s = j2\pi f$ in $H(s)$):

$$R_{out}(f) = \left(\frac{1}{\sqrt{1 + (\frac{2\pi f C_L}{g_{m1}})^2}}\right)^2 R_{in}(f) = \frac{1}{1 + (\frac{2\pi f C_L}{g_{m1}})^2} \cdot \frac{8kT}{3} \cdot \frac{1}{g_{m1}} \tag{10}$$

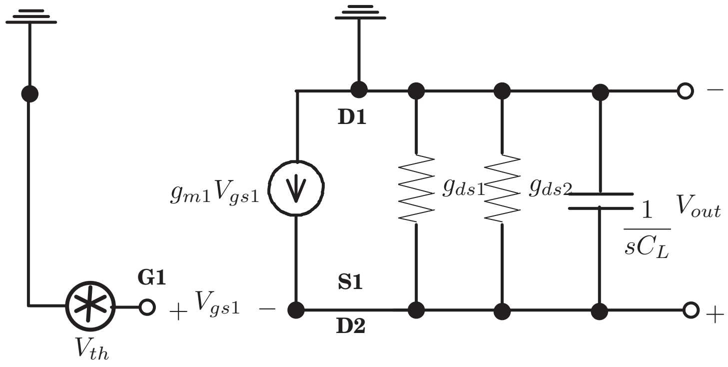

Figure 4: Small signal equivalent circuit.

and

$$P_{out,noise} = \int_0^\infty R_{out}(f)df = \int_0^\infty \frac{1}{1+(\frac{2\pi f C_L}{g_{m1}})^2} \cdot \frac{8kT}{3} \cdot \frac{1}{g_{m1}} df \tag{11}$$

(11) gives

$$P_{out,noise} = \frac{8kT}{3} \cdot \frac{1}{g_{m1}} \cdot \frac{g_{m1}}{2\pi C_L} \left[\arctan \frac{2\pi f C_L}{g_{m1}}\right]_0^\infty = \frac{8kT}{6\pi C_L} \cdot \frac{\pi}{2} = \frac{2KT}{3C_L} \tag{12}$$

Answer:

$$R_{out}(f) = \frac{1}{1+(\frac{2\pi f C_L}{g_{m1}})^2} \cdot \frac{8kT}{3} \cdot \frac{1}{g_{m1}} \tag{13}$$

$$P_{out,noise} = \frac{2KT}{3C_L} \tag{14}$$

Exercise 5

Least Significant Bit (LSB) = Q=$\frac{2}{2^{10}} = 1.96$ mV

I.e. the smallest input signal that can be compared is $V_{IH} - V_{IL} = 1.96$ mV

$(V_{OH} - V_{OL}) = A_v(V_{IH} - V_{IL}) \Rightarrow A_v \geq \frac{V_{OH} - V_{OL}}{(V_{IH} - V_{IL})_{min}} = \frac{3}{1.96 \cdot 10^{-3}} = 1536$ or 63.7 dB

Answer: $A_v \geq 63.7$ dB