Lösningsförslag till tentamen TMEL53 Digitalteknik M 2016-08-23

1a)

$876/2$	= 438	REST	0	LSB
$438/2$	= 219	REST	0	
$219/2$	= 109	REST	1	
$109/2$	= 54	REST	1	
$54/2$	= 27	REST	0	
$27/2$	= 13	REST	1	
$13/2$	= 6	REST	1	
$6/2$	= 3	REST	0	
$3/2$	= 1	REST	1	
$1/2$	= 0	REST	1	MSB

↑ LÄSRIKTNING

$876_{10} = 1101101100_2$

b)

$$\underbrace{1}_{1}\underbrace{101}_{5}\underbrace{101}_{5}\underbrace{100}_{4}$$

$876_{10} = 1554_8$

c)

$$\underbrace{11}_{3}\underbrace{0110}_{6}\underbrace{1100}_{C}$$

$876_{10} = 36C_{16}$

d)

$876_{10} = 100001110110_{NBCD}$

e)

$0,7 \cdot 2$	$= 0,4$	$+ 1$	MSB
$0,4 \cdot 2$	$= 0,8$	$+ 0$	
$0,8 \cdot 2$	$= 0,6$	$+ 1$	
$0,6 \cdot 2$	$= 0,2$	$+ 1$	
$0,4 \cdot 2$	$= 0,8$	$+ 0$	
		$\vdots$	ETC.

↓ LÄSRIKTNING

$0,7_{10} \approx 0.10110\ldots$

f) MULTIPLIKATION MED TVÅ:

UTFÖR ETT VÄNSTERSKIFT

00010100 GÅNGER TVÅ = 00101000

g)

$$
\begin{array}{rl}
 & 11110110 \\
 & 00001001 \quad \text{INVERTERA} \\
+ & 1 \quad \text{ADDERA 1} \\
\hline
 & 00001010 \leftarrow +10
\end{array}
$$

ALLTSÅ $11110110_2 = -10_{10}$

ENLIGT TVÅKOMPLEMENTMETODEN

2a)

$$f = \overline{D + \bar{A}C + \bar{A}D + \left(\overline{\bar{B} + C + \bar{D}}\right)} =$$

$$= \overline{D + \bar{A}C + \bar{A}D + B\bar{C}D} =$$

$$= \overline{D\left(1 + \bar{A} + B\bar{C}\right) + \bar{A}C} =$$

$$= \overline{D + \bar{A}C} = \bar{D} \cdot \overline{\bar{A}C} =$$

$$= \bar{D}\left(A + \bar{C}\right) = A\bar{D} + \bar{C}\bar{D}$$

2b)

AB \ CD	00	01	11	10
00	0	1	1	1
01	0	1	1	1
11	0	1	1	0
10	0	1	1	0

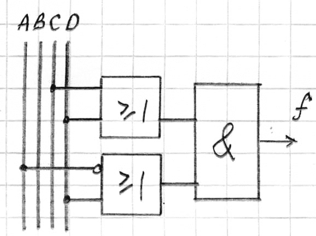
MINIMERAD PS-FORM

$\Rightarrow$ RINGA IN NOLLOR I KARNAUGHDIAGRAMMET

$$\bar{f} = \bar{C}\bar{D} + A\bar{D}$$

$$\Rightarrow f = \overline{\bar{C}\bar{D} + A\bar{D}} = \overline{\bar{C}\bar{D}} \cdot \overline{A\bar{D}} =$$

$$= \left(C + D\right) \cdot \left(\bar{A} + D\right)$$



2c)

$$X\bar{Y} + YZ + \bar{X}Z = X\bar{Y}\left(Z + \bar{Z}\right) + \left(X + \bar{X}\right)YZ + \bar{X}\left(Y + \bar{Y}\right)Z =$$

$$= X\bar{Y}Z + X\bar{Y}\bar{Z} + XYZ + \bar{X}YZ + \bar{X}YZ + \bar{X}\bar{Y}Z =$$

$$= X\bar{Y}Z + X\bar{Y}\bar{Z} + XYZ + \bar{X}YZ + \bar{X}\bar{Y}Z$$

2d)

$$f = A\bar{B} \oplus \left(C + A\right) = A\bar{B} \cdot \left(\overline{C + A}\right) + \overline{A\bar{B}} \cdot \left(C + A\right) =$$

$$= \underbrace{A\bar{B}\bar{C}\bar{A}}_{0} + \left(\bar{A} + B\right)\left(C + A\right) = \bar{A}C + \underbrace{\bar{A}A}_{0} + BC + AB =$$

$$= /\text{CONSENSUS}/ = \bar{A}C + AB$$

3a)

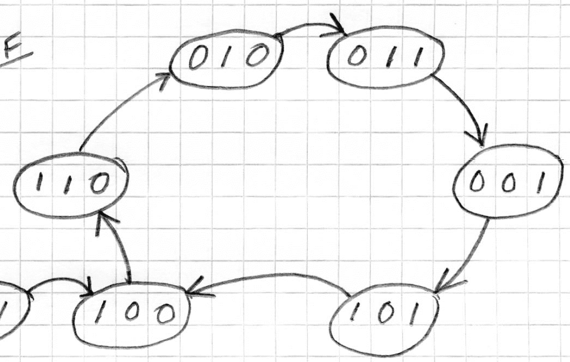
$J = \overline{Q_1 \oplus Q_2}$ $\qquad S = \overline{Q_3}$ $\qquad T = \overline{Q_1 \oplus Q_3}$

$K = Q_2 \cdot \overline{Q_3}$ $\qquad R = Q_3$

Q_1	Q_2	Q_3	J	K	S	R	T	Q_1^+	Q_2^+	Q_3^+
0	0	0	1	0	1	0	1	1	1	1
0	0	1	1	0	0	1	0	1	0	1
0	1	0	0	1	1	0	1	0	1	1
0	1	1	0	0	0	1	0	0	0	1
1	0	0	0	0	1	0	0	1	1	0
1	0	1	0	0	0	1	1	1	0	0
1	1	0	1	1	1	0	0	0	1	0
1	1	1	1	0	0	1	1	1	0	0

TILLSTÅNDSGRAF

$Q_1 Q_2 Q_3$



b)

Q_1	Q_2	Q_3	D_1	D_2	D_3	Q_1^+	Q_2^+	Q_3^+
0	0	0	1	1	1	1	1	1
0	0	1	1	0	1	1	0	1
0	1	0	0	1	1	0	1	1
0	1	1	0	0	1	0	0	1
1	0	0	1	1	0	1	1	0
1	0	1	1	0	0	1	0	0
1	1	0	0	1	0	0	1	0
1	1	1	1	0	0	1	0	0

$Q_1 \backslash Q_2Q_3$	00	01	11	10
0	1	1	0	0
1	1	1	1	0

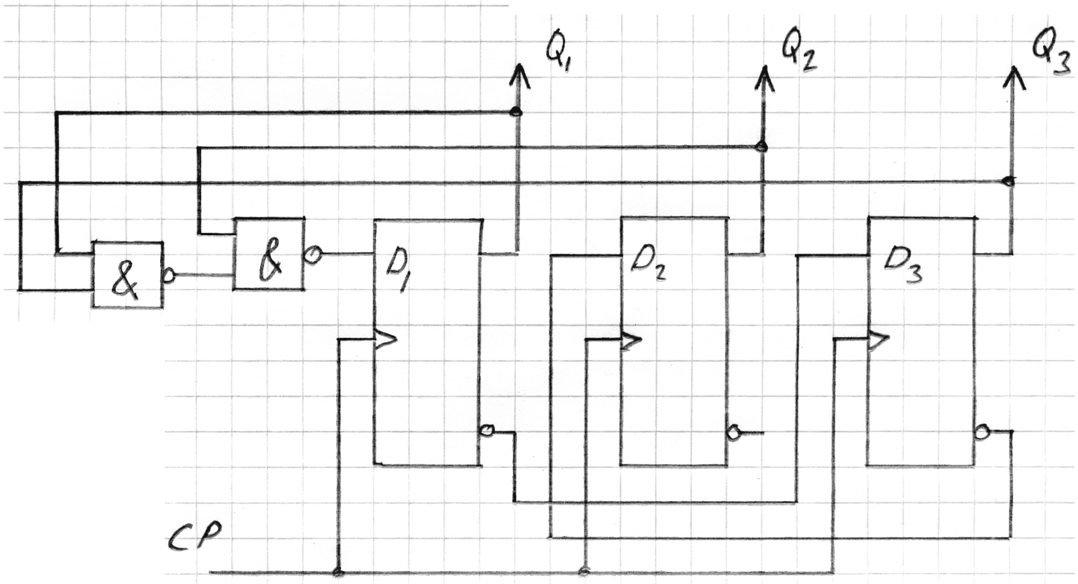
$$D_1 = \overline{Q_2} + Q_1 Q_3 = \overline{\overline{\overline{Q_2} + Q_1 Q_3}} =$$

$$= \overline{Q_2 \cdot \overline{Q_1 Q_3}}$$

I TABELLEN SER MAN DIREKT ATT :

$D_2 = \overline{Q_3}$ OCH $D_3 = \overline{Q_1}$

KOPPLINGSSCHEMA :



4.

c_i	d_i	e_i	f_i	c_{i+1}	d_{i+1}
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	0	0	1
1	0	0	0	0	1
1	0	1	1	0	1
1	1	0	–	–	–
1	1	1	–	–	–

c_i \ $d_i e_i$	00	01	11	10
0	0	1	0	1
1	0	1	–	–

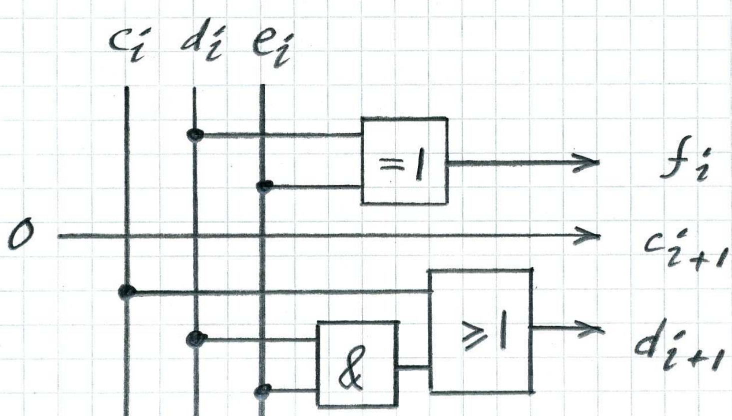
$$f_i = \bar{d_i} e_i + d_i \bar{e_i} = d_i \oplus e_i$$

c_i \ $d_i e_i$	00	01	11	10
0	0	0	0	0
1	0	0	–	–

$$c_{i+1} = 0$$

c_i \ $d_i e_i$	00	01	11	10
0	0	0	1	0
1	1	1	–	–

$$d_{i+1} = c_i + d_i e_i$$



5a)

A	B	C	a_i	b_i	c_i	C_{i+1}	S_i
0	0	0	0	0	0	0	0
0	0	1	1	0	1	1	0
0	1	0	1	1	0	1	0
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	1	0	1
1	1	0	0	1	0	0	1
1	1	1	1	1	1	1	1

b)

A \ BC	00	01	11	10
0	0	1	1	1
1	0	0	1	0

$AB = 00 \Rightarrow C_{i+1} = C$

$AB = 01 \Rightarrow C_{i+1} = 1$

$AB = 10 \Rightarrow C_{i+1} = 0$

$AB = 11 \Rightarrow C_{i+1} = C$

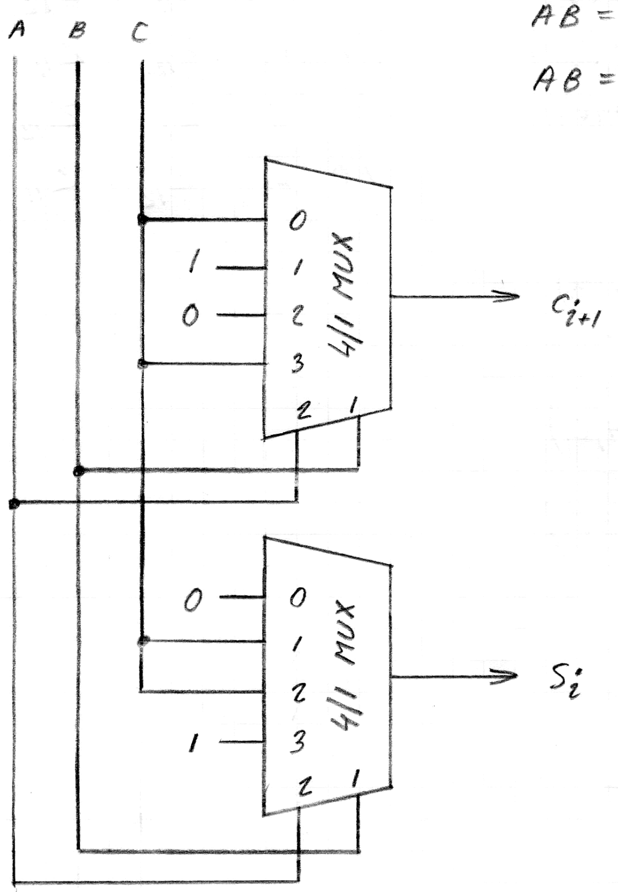
A \ BC	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$AB = 00 \Rightarrow S_i = 0$

$AB = 01 \Rightarrow S_i = C$

$AB = 10 \Rightarrow S_i = C$

$AB = 11 \Rightarrow S_i = 1$



c)

A \ BC	00	01	11	10
0	0	1	1	1
1	0	0	1	0

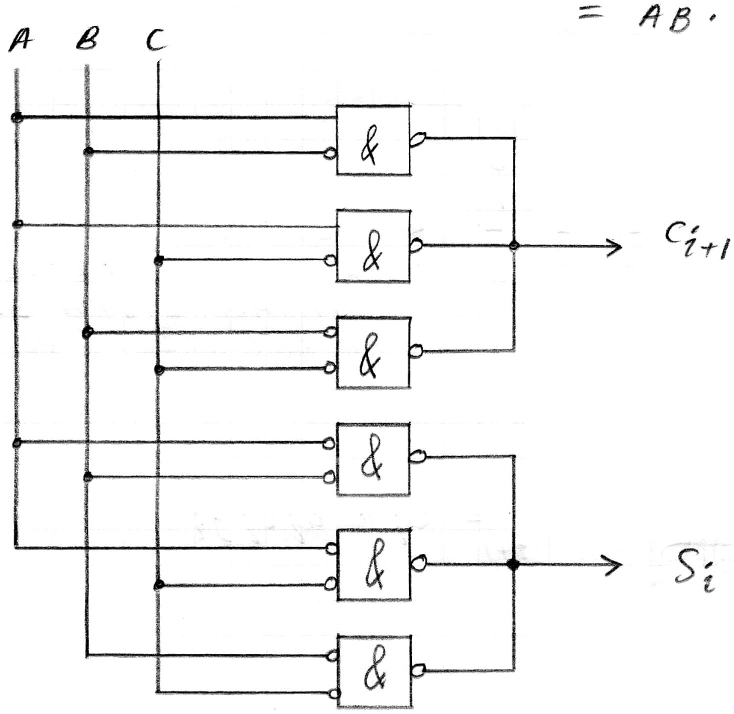
$$C_{i+1} = \overline{A\bar{B} + A\bar{C} + \bar{B}\bar{C}} =$$

$$= \overline{A\bar{B}} \cdot \overline{A\bar{C}} \cdot \overline{\bar{B}\bar{C}}$$

A \ BC	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$S_i = \overline{\bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C}} =$$

$$= \overline{\bar{A}\bar{B}} \cdot \overline{\bar{A}\bar{C}} \cdot \overline{\bar{B}\bar{C}}$$



6a)

	X = 0	X = 1
Q	$Q^+(u)$	$Q^+(u)$
A	A (1)	A (1)
B	C (1)	F (0)
C	A (1)	B (1)
D	D (1)	D (1)
E	D (1)	F (1)
F	C (1)	F (0)

1-EKUIVALENS : GÖR EN FÖRSTA UPPDELNING I GRUPPER MED AVSEENDE PÅ UTSIGNALER

$$\Sigma_{11} = (B, F)$$

$$\Sigma_{12} = (A, C, D, E)$$

	X = 0	X = 1
Q	$Q^+(u)$	$Q^+(u)$
A	Σ_{12}	Σ_{12}
B	Σ_{12}	Σ_{11}
C	Σ_{12}	Σ_{11}
D	Σ_{12}	Σ_{12}
E	Σ_{12}	Σ_{11}
F	Σ_{12}	Σ_{11}

TILLSTÅND C OCH E MÅSTE SÄRAS FRÅN A OCH D

2-EKVIVALENS: NY GRUPPINDELNING

$$\Sigma_{21} = (B, F)$$

$$\Sigma_{22} = (A, D)$$

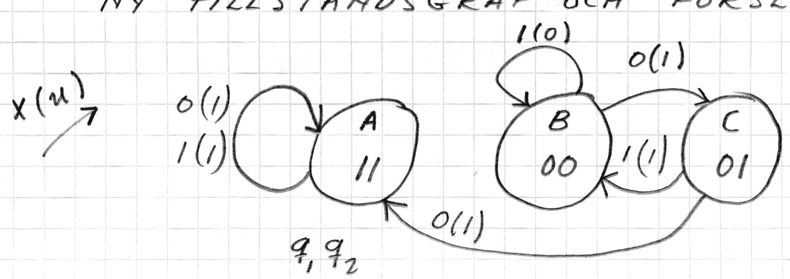
$$\Sigma_{23} = (C, E)$$

	$X = 0$	$X = 1$
Q	$Q^+(u)$	$Q^+(u)$
A	Σ_{22}	Σ_{22}
B	Σ_{23}	Σ_{21}
C	Σ_{22}	Σ_{21}
D	Σ_{22}	Σ_{22}
E	Σ_{22}	Σ_{21}
F	Σ_{23}	Σ_{21}

NU KAN INGEN NY GRUPPINDELNING GÖRAS.
TILLSTÅNDEN D, E OCH F ÄR ÖVERFLÖDIGA.

	$X = 0$	$X = 1$
Q	$Q^+(u)$	$Q^+(u)$
A	$A(1)$	$A(1)$
B	$C(1)$	B ~~A~~ (0)
C	$A(1)$	$B(1)$

NY TILLSTÅNDSGRAF OCH FÖRSLAG TILL KODNING:



FÖRE CP					EFTER CP		
x	q_1	q_2	D_1	D_2	q_1^+	q_2^+	u
0	0	0	0	1	0	1	1
0	0	1	1	1	1	1	1
0	1	0	–	–	–	–	–
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	0	0	1
1	1	0	–	–	–	–	–
1	1	1	1	1	1	1	1

$x \backslash q_1 q_2$	00	01	11	10
0	0	1	1	–
1	0	0	1	–

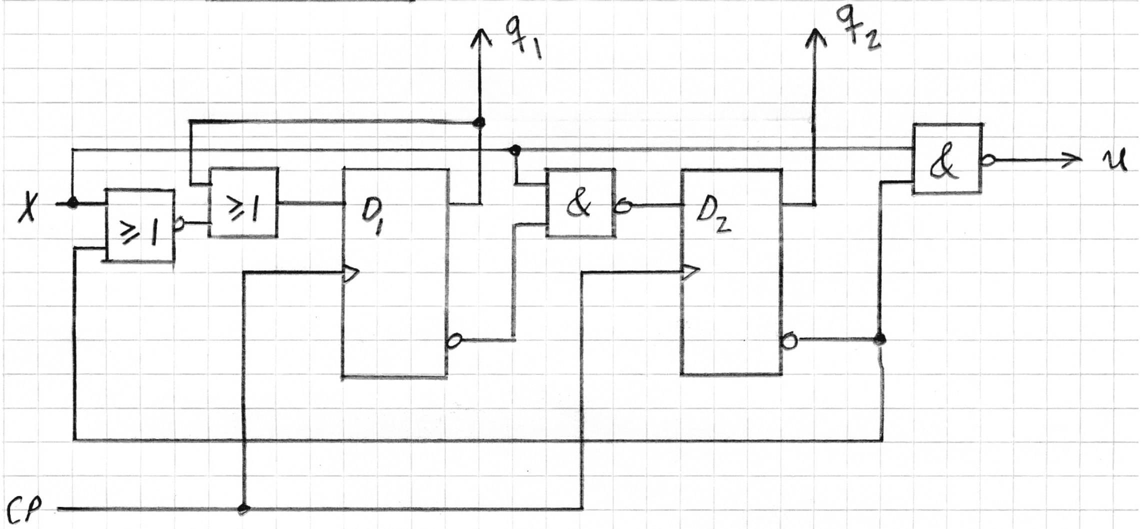
$$D_1 = q_1 + \bar{x} q_2 = q_1 + \left(\overline{x + \bar{q}_2}\right)$$

$x \backslash q_1 q_2$	00	01	11	10
0	1	1	1	–
1	0	0	1	–

$$D_2 = \bar{x} + q_1 = \overline{x \bar{q}_1}$$

$x \backslash q_1 q_2$	00	01	11	10
0	1	1	1	–
1	0	1	1	–

$$u = \bar{x} + q_2 = \overline{x \bar{q}_2}$$



b)