Lösningsförslag till tentamen TMEL53 Digitalteknik M 2016-06-08

1a)

987/2	= 493	REST 1	LSB	
493/2	= 246	REST 1		
246/2	= 123	REST 0		
123/2	= 61	REST 1		
61/2	= 30	REST 1		
30/2	= 15	REST 0		
15/2	= 7	REST 1		
7/2	= 3	REST 1	↑	
3/2	= 1	REST 1		
1/2	= 0	REST 1	MSB	

$987_{10} = 1111011011_{2}$

b)

$\underbrace{1}_{1}\underbrace{111}_{7}\underbrace{011}_{3}\underbrace{011}_{3}$

$987_{10} = 1733_{8}$

c)

$\underbrace{1111}_{3}\underbrace{0110}_{D}\underbrace{11}_{B}$

$987_{10} = 3DB_{16}$

d)

$987_{10} = 1001\,10000111_{NBCD}$

e)

$0,8 \cdot 2 = 0,6 + 1$ MSB

$0,6 \cdot 2 = 0,2 + 1$ ↓

$0,2 \cdot 2 = 0,4 + 0$

$0,4 \cdot 2 = 0,8 + 0$

$0,8 \cdot 2 = 0,6 + 1$

. .
. .
. .

$0,8_{10} \approx 0.1100110011\ldots_{2}$

f)

DIVISION MED TVÅ → UTFÖR ETT HÖGERSKIFT

⇒ 00010100_{2} DIVIDERAT MED TVÅ = 00001010_{2}

g)

11101100

00010011 INVERTERA

\+ 1 ADDERA 1

00010100 = +20

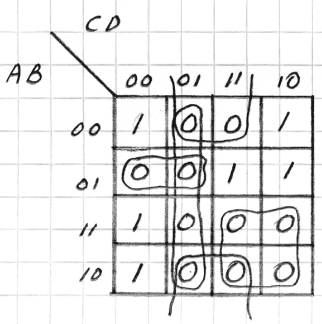
ALLTSÅ 11101100 = -20 ENLIGT TVÅ-KOMPLEMENTMETODEN

2a/ $$f = \overline{\bar{A}\left(\bar{B} + C\left(\bar{B} + CD\right)\right)} = \bar{\bar{A}} + \bar{B} + C\left(\bar{B} + CD\right) =$$

$$= A + \bar{\bar{B}} \cdot C\left(\bar{B} + CD\right) = A + B\left(C\bar{B} + CCD\right) =$$

$$= A + B\bar{B}C + BCCD = \underline{\underline{A + BCD}}$$

b/



$$f = \overline{\bar{A}B\bar{C} + \bar{C}D + \bar{B}D + AC} =$$

$$= \overline{\bar{A}B\bar{C}} \cdot \overline{\bar{C}D} \cdot \overline{\bar{B}D} \cdot \overline{AC} =$$

$$= \underline{\underline{\left(A + \bar{B} + C\right)\left(C + \bar{D}\right)\left(B + \bar{D}\right)\left(\bar{A} + \bar{C}\right)}}$$

c/ $$X\bar{Y} + Y\bar{Z} + XZ = X\bar{Y}\left(Z + \bar{Z}\right) + \left(X + \bar{X}\right)Y\bar{Z} + X\left(Y + \bar{Y}\right)Z =$$

$$= \underline{X\bar{Y}Z} + X\bar{Y}\bar{Z} + XY\bar{Z} + \bar{X}Y\bar{Z} + XYZ + \underline{X\bar{Y}Z} =$$

$$= \underline{\underline{\bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + X\bar{Y}Z + XY\bar{Z} + XYZ}} \quad \left(= P_2 + P_4 + P_5 + P_6 + P_7\right)$$

d/ $$f = CA \oplus \left(\bar{B} + A\right) = CA\overline{\left(\bar{B} + A\right)} + \overline{CA}\left(\bar{B} + A\right) =$$

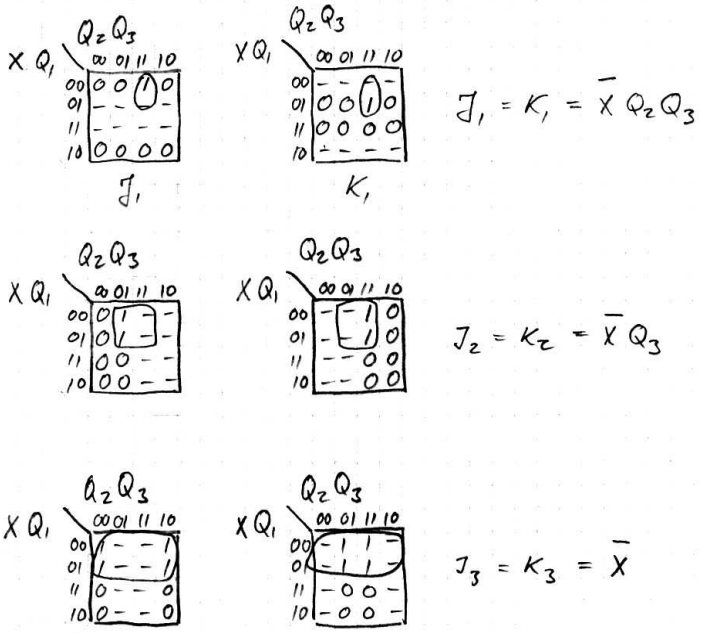
$$= CA\left(B\bar{A}\right) + \left(\bar{C} + \bar{A}\right)\left(\bar{B} + A\right) =$$

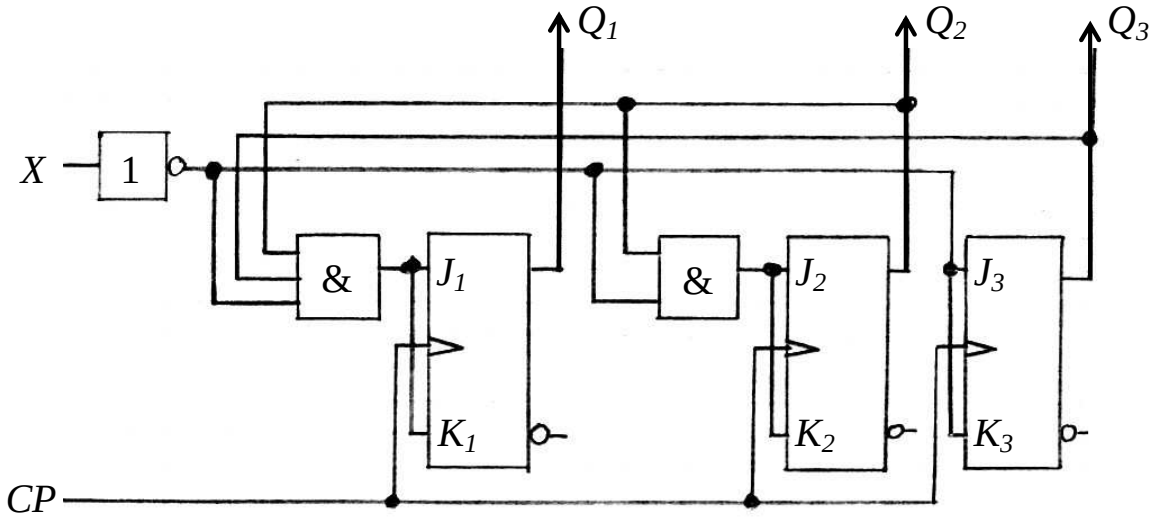
$$= 0 + \bar{B}\bar{C} + A\bar{C} + \bar{A}\bar{B} + \underbrace{A\bar{A}}_{0} = A\bar{C} + \bar{A}\bar{B}$$

↑ CONSENSUS

3.

$X\ Q_1\ Q_2\ Q_3$	$J_1 K_1$	$J_2 K_2$	$J_3 K_3$	$Q_1^+\ Q_2^+\ Q_3^+$
0 0 0 0	0 -	0 -	1 -	0 0 1
0 0 0 1	0 -	1 -	- 1	0 0 1 0
0 0 1 0	0 -	- 0	1 -	0 1 0
0 0 1 1	1 -	- 1	- 1	0 1 0 0
0 1 0 0	- 0	0 -	1 -	0 1 0 1
0 1 0 1	- 0	1 -	- 1	0 1 1 0
0 1 1 0	- 0	- 0	1 -	0 1 1 1
0 1 1 1	- 1	- 1	- 1	1 0 0 0
1 0 0 0	0 -	0 -	0 -	0 0 0 0
1 0 0 1	0 -	0 -	- 0	0 0 0 1
1 0 1 0	0 -	- 0	0 -	0 0 1 0
1 0 1 1	0 -	- 0	- 0	0 0 1 1
1 1 0 0	- 0	0 -	0 -	0 1 0 0
1 1 0 1	- 0	0 -	- 0	1 0 0 1
1 1 1 0	- 0	- 0	0 -	1 1 0
1 1 1 1	- 0	- 0	- 0	1 1 1

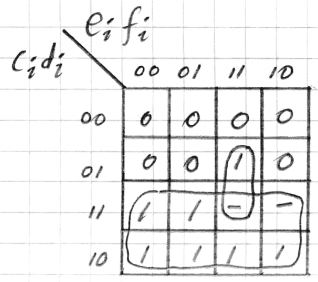




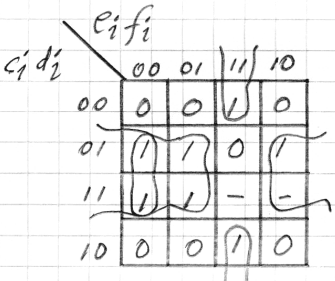
4a)

c_i	d_i	e_i	f_i	c_{i+1}	d_{i+1}	e_{i+1}
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	1	0
0	1	0	0	0	1	0
0	1	0	1	0	1	1
0	1	1	0	0	1	1
0	1	1	1	1	0	0
1	0	0	0	1	0	0
1	0	0	1	1	0	1
1	0	1	0	1	0	1
1	0	1	1	1	1	0
1	1	0	0	1	1	0
1	1	0	1	1	1	1
1	1	1	0	-	-	-
1	1	1	1	-	-	-

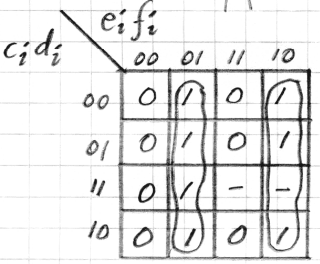
MAX 6 ETTOR KAN HA FÖREKOMMIT INNAN SISTA BLOCKET.



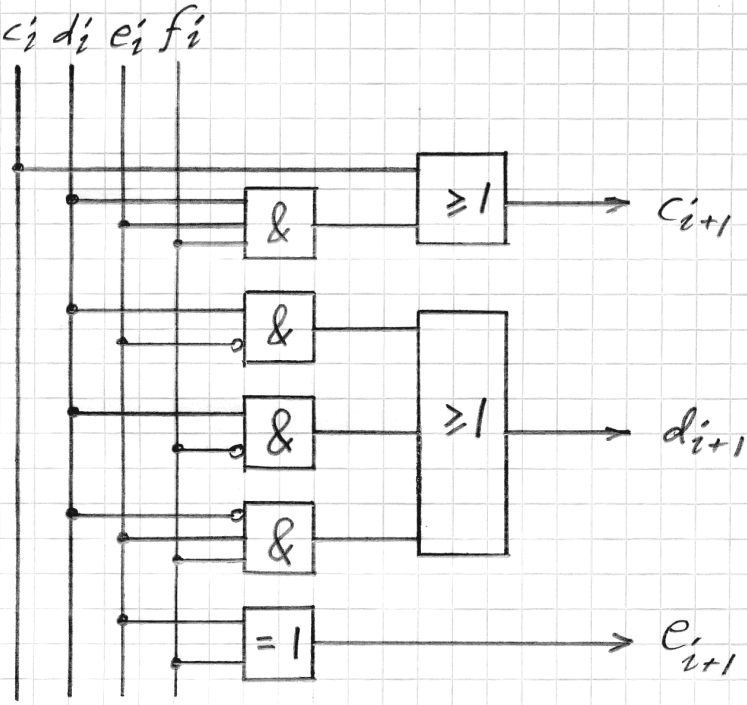
$$c_{i+1} = c_i + d_i e_i f_i$$



$$d_{i+1} = d_i \bar{e}_i + d_i \bar{f}_i + \bar{d}_i e_i f_i$$



$$e_{i+1} = \bar{e}_i f_i + e_i \bar{f}_i = e_i \oplus f_i$$



b/

$c_i d_i$ \ $e_i f_i$	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	1	1	-	-
10	1	1	1	1

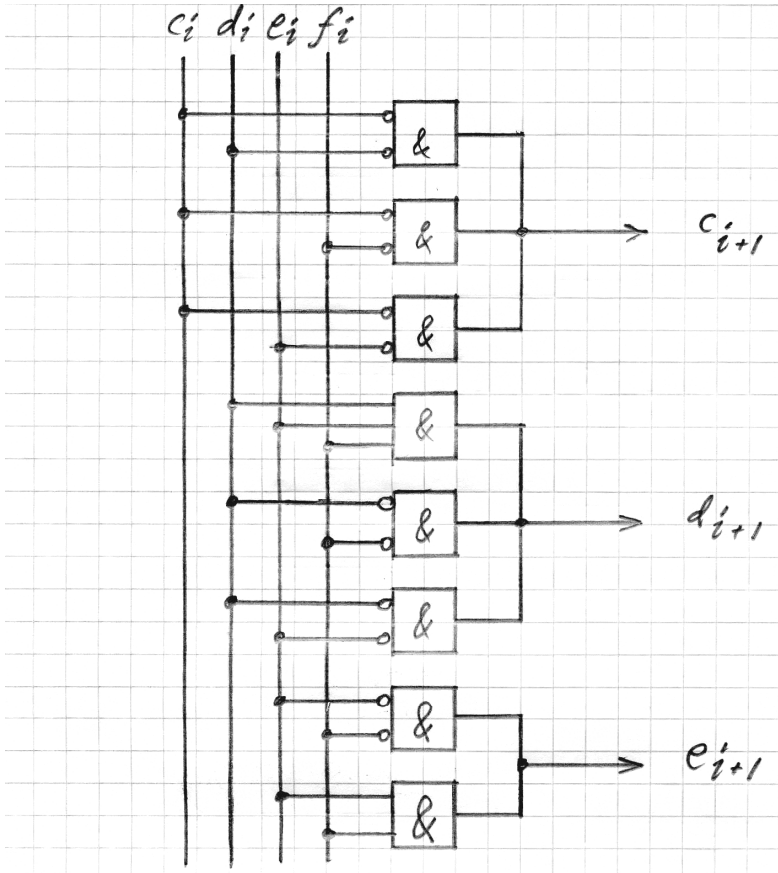
$$c_{i+1} = \overline{\bar{c}_i \bar{d}_i + \bar{c}_i \bar{f}_i + \bar{c}_i \bar{e}_i} = \overline{\bar{c}_i \bar{d}_i} \cdot \overline{\bar{c}_i \bar{f}_i} \cdot \overline{\bar{c}_i \bar{e}_i}$$

$c_i d_i$ \ $e_i f_i$	00	01	11	10
00	0	0	1	0
01	1	1	0	1
11	1	1	-	-
10	0	0	1	0

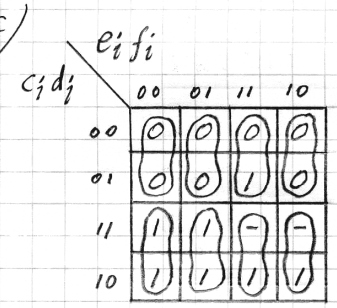
$$d_{i+1} = \overline{d_i e_i f_i + \bar{d}_i \bar{f}_i + \bar{d}_i \bar{e}_i} = \overline{d_i e_i f_i} \cdot \overline{\bar{d}_i \bar{f}_i} \cdot \overline{\bar{d}_i \bar{e}_i}$$

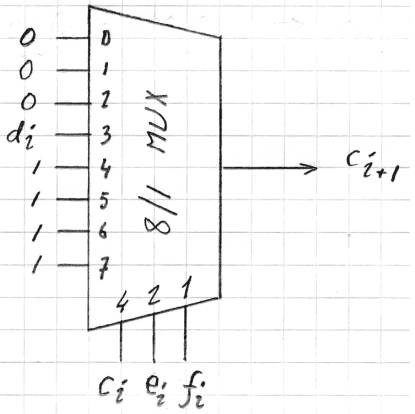
$c_i d_i$ \ $e_i f_i$	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	0	1	-	-
10	0	1	0	1

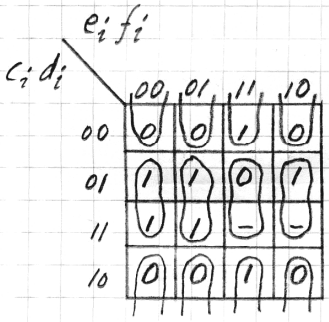
$$e_i = \bar{e}_i \bar{f}_i + e_i f_i = \overline{\bar{e}_i \bar{f}_i} \cdot \overline{e_i f_i}$$

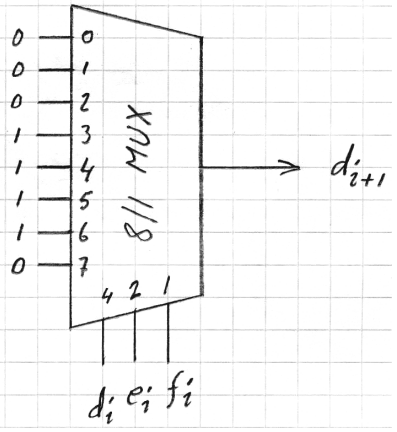

c_i d_i e_i f_i
&
c_{i+1}
d_{i+1}
e_{i+1}

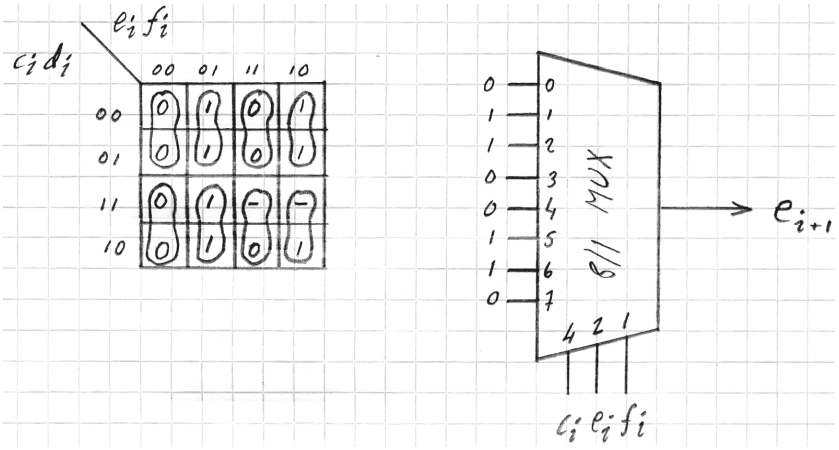
c)


$e_i f_i$
$c_i d_i$
00 01 11 10
00 0 0 0 0
01 0 0 1 0
11 1 1 - -
10 1 1 1 1


0 0 0 d_i 1 1 1 1
0 1 2 3 4 5 6 7
MUX 8/1
4 2 1
c_i e_i f_i
c_{i+1}


$e_i f_i$
$c_i d_i$
00 01 11 10
00 0 0 1 0
01 1 1 0 1
11 1 1 - -
10 0 0 1 0

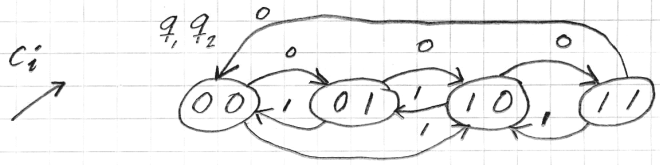

0 0 0 1 1 1 1 0
0 1 2 3 4 5 6 7
MUX 8/1
4 2 1
d_i e_i f_i
d_{i+1}



5 a/

c_i	q_1	q_2	c_{i+1}	D	T	q_1^+	q_2^+
0	0	0	0	0	1	0	1
0	0	1	0	1	1	1	0
0	1	0	0	1	1	1	1
0	1	1	1	0	1	0	0
1	0	0	0	1	0	1	0
1	0	1	1	0	1	0	0
1	1	0	1	0	1	0	1
1	1	1	1	1	1	1	0

$D = s_i \qquad T = c_{i+1} + \overline{c_i}$



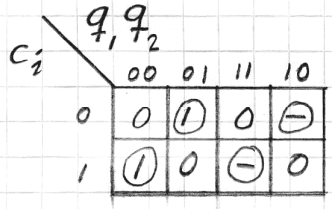
b/

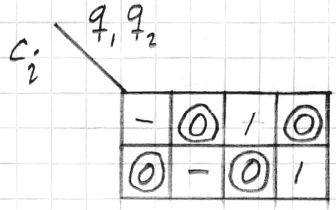
0	1	
1	0	
1	1	
0	0	
1	0	
0	0	
0	1	
1	0	

q_1^+ q_2^+

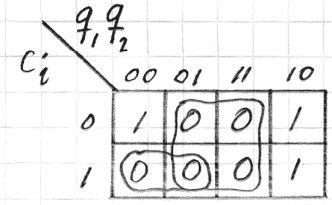
c)

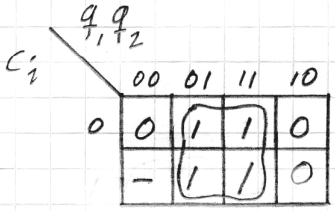
c_i	q_1	q_2	S_1	R_1	S_2	R_2	q_1^+	q_2^+
0	0	0	0	–	1	0	0	1
0	0	1	1	0	0	1	1	0
0	1	0	–	0	1	0	1	1
0	1	1	0	1	0	1	0	0
1	0	0	1	0	0	–	1	0
1	0	1	0	–	0	1	0	0
1	1	0	0	1	1	0	0	1
1	1	1	–	0	0	1	1	0



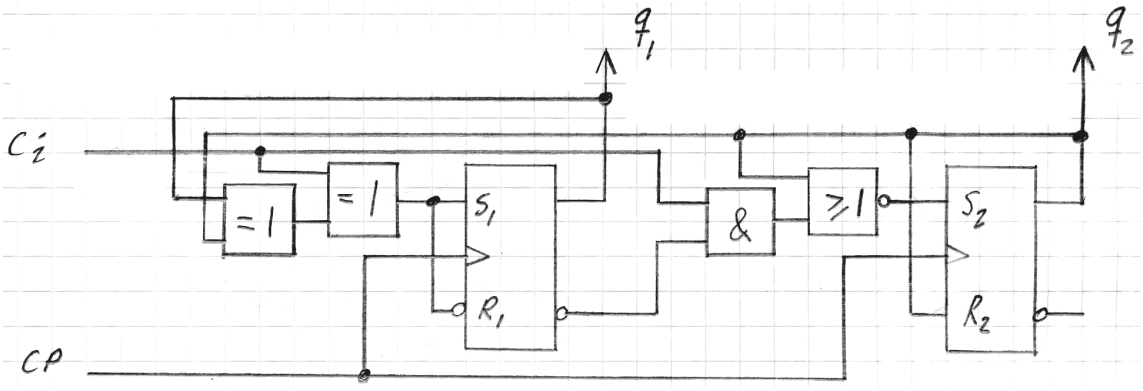


$$S_1 = c_i \bar{q}_1 \bar{q}_2 + c_i q_1 q_2 + \bar{c}_i \bar{q}_1 q_2 + \bar{c}_i q_1 \bar{q}_2 = c_i \oplus q_1 \oplus q_2 \qquad R_1 = \bar{S}_1$$





$$S_2 = \overline{c_i \bar{q}_1 + q_2} \qquad R_2 = q_2$$



6.

q	$X=0$	$X=1$
A	A(1)	A(1)
B	C(1)	F(0)
C	A(1)	B(1)
D	D(1)	D(1)
E	D(1)	F(1)
F	C(1)	F(0)
G	D(0)	A(0)
H	A(0)	D(0)
	$q^+(u)$	

1-EKVIVALENS

$\Sigma_{11}(G, H)$

$\Sigma_{12}(B, F)$

$\Sigma_{13}(A, C, D, E)$

q	$X=0$	$X=1$
A	Σ_{13}	Σ_{13}
B	Σ_{13}	Σ_{12}
C	Σ_{13}	Σ_{12}
D	Σ_{13}	Σ_{13}
E	Σ_{13}	Σ_{12}
F	Σ_{13}	Σ_{12}
G	Σ_{13}	Σ_{13}
H	Σ_{13}	Σ_{13}

2-EKVIVALENS

$\Sigma_{21}(A, D)$

$\Sigma_{22}(G, H)$

$\Sigma_{23}(B, F)$

$\Sigma_{24}(C, E)$

q	$X=0$	$X=1$
A	Σ_{21}	Σ_{21}
B	Σ_{24}	Σ_{23}
C	Σ_{21}	Σ_{23}
D	Σ_{21}	Σ_{21}
E	Σ_{21}	Σ_{23}
F	Σ_{24}	Σ_{23}
G	Σ_{21}	Σ_{21}
H	Σ_{21}	Σ_{21}

3-EKVIVALENS

$\Sigma_{31}(A, D)$

$\Sigma_{32}(B, F)$

$\Sigma_{33}(C, E)$

$\Sigma_{34}(G, H)$

3-EKVIVALENS = 2-EKVIVALENS OVS
TILLSTÅNDEN D, H, F OCH E ÄR "ÖVERFLÖDIGA".

TILLSTÅNDSTABELLEN KAN MINSKAS NER

q	x=0	x=1
A	A(1)	A(1)
B	C(1)	B ~~A~~(0)
C	A(1)	B(1)
G	A ~~A~~(0)	A(0)

FÖRSLAG TILL NY KODNING

A	0 0
B	0 1
C	1 0
G	1 1

x	q_1	q_2	D_1	D_2	q_1^+	q_2^+	u
0	0	0	0	0	0	0	1
0	0	1	1	0	1	0	1
0	1	0	0	0	0	0	1
0	1	1	0	0	0	0	0
1	0	0	0	0	0	0	1
1	0	1	0	1	0	1	0
1	1	0	0	1	0	1	1
1	1	1	0	0	0	0	0

x \ q_1q_2	00	01	11	10
0	0	(1)	0	0
1	0	0	0	0

x \ q_1q_2	00	01	11	10
0	0	0	0	0
1	0	(1)	0	(1)

$$D_1 = \bar{x}\bar{q}_1 q_2 = \overline{x + q_1 + \bar{q}_2} \qquad D_2 = x\bar{q}_1 q_2 + x q_1 \bar{q}_2 = x(q_1 \oplus q_2)$$