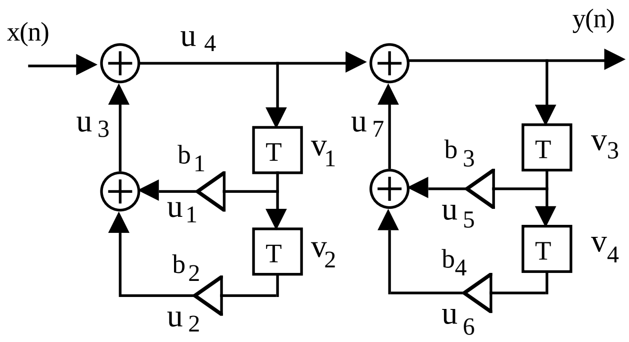
6.1 a) 1. Collapse nodes with transmittancies ±1.

2. Introduce node numbers according to the figure below.



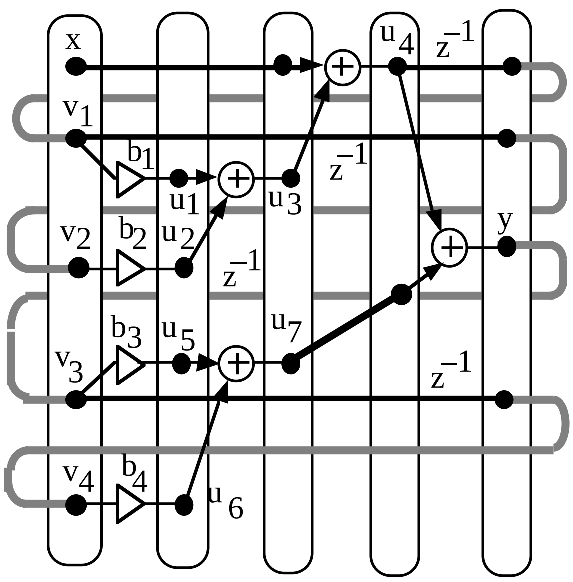
3. Remove all delay elements

4. Initial nodes : v_1, v_2, v_3, v_4, x
Executable operations: b_1, b_2, b_3, b_4
New nodes: u_1, u_2, u_5, u_6
Executable operations: +, +
New nodes: u_3, u_7
Executable operations: +
New nodes: u_4
Executable operations: +
New nodes: y

5. Complete the computation graph by adding the branches from the signal-flow graph. Note that the delay elment $v_3(n)$ does not correspond to physical memory.

b) The system of difference equations is

$u_1 := b_1\, v_1(n)$
$u_2 := b_2\, v_2(n)$
$u_5 := b_3\, v_3(n)$
$u_6 := b_4\, v_4(n)$
$u_3 := u_1 + u_2$
$u_7 := u_5 + u_6$
$u_4 := u_3 + x(n)$
$y(n) := u_4 + u_7$
$v_2(n+1) := v_1(n)$
$v_4(n+1) := v_3(n)$
$v_1(n+1) := u_4$
$v_3(n+1) := y(n)$

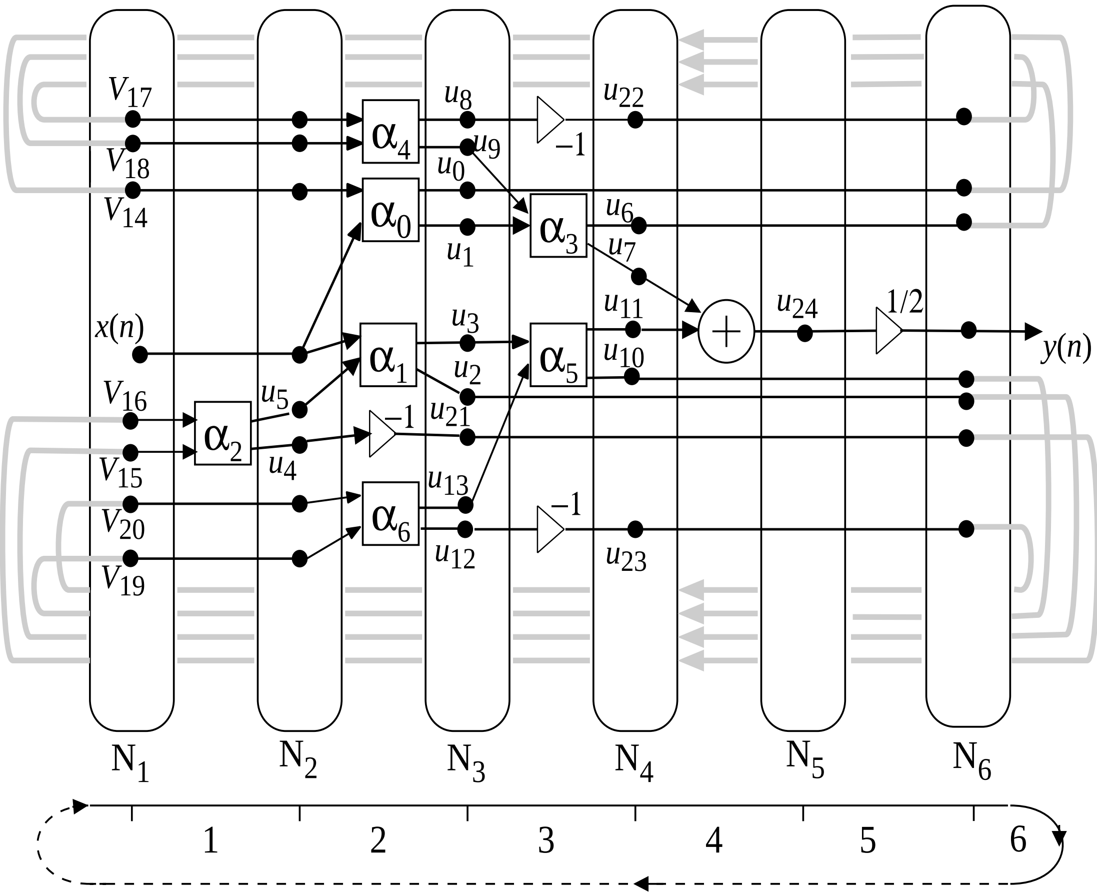


An intermediate node (u_i) can be eliminated if it appears only in one place on the right hand side. Thus, we can, for example, eliminate u_1. We get

$u_4 := b_1\, v_1(n) + b_2\, v_2(n) + x(n)$
$y(n) := u_4 + b_3\, v_3(n) + b_4\, v_4(n)$
$v_2(n+1) := v_1(n)$
$v_4(n+1) := v_3(n)$
$v_1(n+1) := u_4$
$v_3(n+1) := y(n)$

c)

6.2 The critical path in the filter is two cascade adaptors. Derive the SFG in precedence form with adaptors as basic operators, i.e., processing elements.



Finally, we obtain the difference equations in computational order for the filter.

N_1	$V_{15}(n) = u_2(n{-}1)$
	$V_{16}(n) = u_{21}(n{-}1)$
N_1 -> N_2	
adaptor	$d_2(\ n) = V_{15}(n) - V_{16}(n)$
α_2	$f_2(n) = \alpha_2 \cdot d_2(n)$
N_2	$u_4(n) = V_{15}(n) + f_2(n)$
	$u_5(n) = V_{16}(n) + f_2(n)$
	$V_{14}(n) = u_0(n{-}1)$
	$V_{17}(n) = u_6(n{-}1)$
	$V_{18}(n) = u_{22}(n{-}1)$
	$V_{19}(n) = u_{10}(n{-}1)$
	$V_{20}(n) = u_{23}(n{-}1)$
N_2 -> N_3	
adaptor	$d_0(\ n) = x(n) - V_{14}(n)$
α_0	$f_0(n) = \alpha_0 \cdot d_0(n)$
N_2 -> N_3	
adaptor	$d_1(\ n) = x(n) - u_5(n)$
α_0	$f_1(n) = \alpha_1 \cdot d_1(n)$
N_2 -> N_3	
adaptor	$d_4(\ n) = V_{17}(n) - V_{18}(n)$
α_0	$f_4(n) = \alpha_4 \cdot d_4(n)$

$N_2 \rightarrow N_3$ adaptor α_0

$d_6(n) = V_{19}(n) - V_{20}(n)$
$f_4(n) = \alpha_6 \cdot d_6(n)$

N_3

$u_0(n) = x(n) + f_0(n)$
$u_1(n) = V_{14}(n) + f_0(n)$
$u_2(n) = x(n-1) + f_1(n)$
$u_3(n) = u_5(n-1) + f_1(n)$
$u_8(n) = V_{17}(n-1) + f_4(n)$
$u_9(n) = V_{18}(n-1) + f_4(n)$
$u_{12}(n) = V_{19}(n-1) + f_6(n)$
$u_{13}(n) = V_{20}(n-1) + f_6(n)$
$u_{21}(n) = -1 \cdot V_4$

$N_3 \rightarrow N_4$ adaptor α_3

$d_3(n) = u_1(n) - u_9(n)$
$f_3(n) = \alpha_3 \cdot d_3(n)$

$N_3 \rightarrow N_5$ adaptor α_5

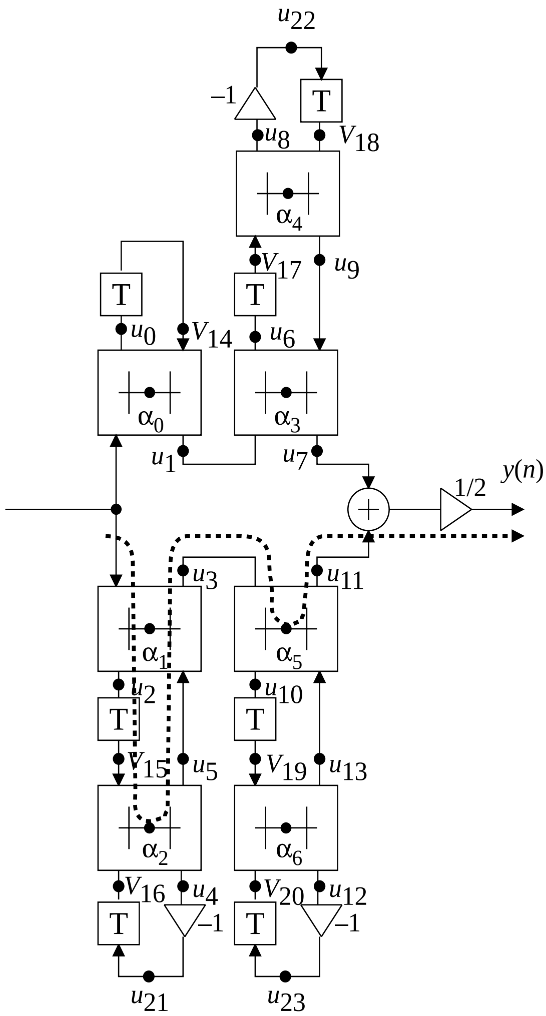
$d_5(n) = u_3(n) - u_{13}(n)$
$f_1(n) = \alpha_5 \cdot d_5(n)$

N_4

$u_6(n) = u_1(n) + f_3(n)$
$u_7(n) = u_9(n) + f_3(n)$
$u_{10}(n) = u_3(n-1) + f_5(n)$
$u_{11}(n) = u_{13}(n-1) + f_5(n)$
$u_{22}(n) = -1 \cdot u_8$
$u_{23}(n) = -1 \cdot u_{12}$

N_5

$y_0(n) = u_7(n) - u_{11}(n)$
$y(n) = (1/2) \cdot y_0(n)$



6.4 Using z-transform, the transfer function is

$$H(z) = Z\{h(n)\} = 1 + 2^{-1} + \dots + z^{-M} = \frac{1 - z^{-(M+1)}}{1 - z^{-1}}$$

which gives $Y(z) = H(z)X(z) = \dfrac{1 - z^{-(M+1)}}{1 - z^{-1}} X(z)$

or

$$Y(z) = z^{-1}Y(z) + X(z) - z^{-(M+1)}X(z)$$

Apply the inverse z-transform, the corresponding difference equation is

$$y(n) = y(n-1) + x(n) - x(n-M-1)$$

The high level language realization is left to the reader. The imporatant sequences inside the loop are

{read in input $x(n)$}
{read in old input $x(n-M-1)$ and $y(n-1)$}
{compute $y(n)$}
{overwrite the $x(n-M-1)$ with $x(n)$}
{update $y(n-1)$ with $y(n)$}

6.6 a) The sets of nodes are

N_1: {1, 3, 5, 7, 8}, N_2: {2, 6}, and N_3: {4}

The precedence graph for the multiplications has to sets. In the first set belongs

N_1: $\{a_1, a_2, a_3, a_4, b_1, b_2, b_3\}$

and in the second set

N_2: $\{c_1, c_2\}$.

The multiplier c_1 shall precede a_2 and b_1 while c_2 shall precede $a_3, a_4, b_2,$ and b_3.

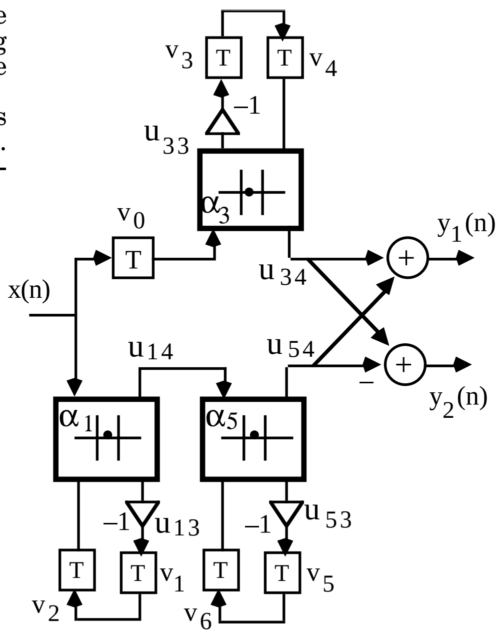
b) The set of difference equations in computable order is

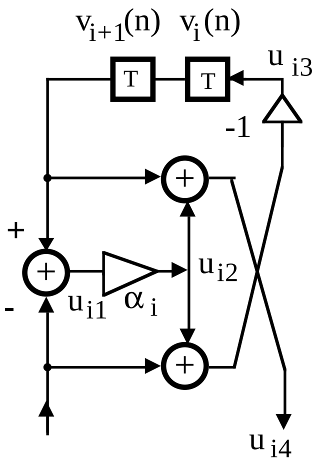
$$\begin{aligned}
&u_2 := a_2\, v_1(n) + b_1\, v_3(n)\\
&u_6 := a_3\, v_1(n) + a_4\, v_5(n) + b_2\, v_7(n) + b_3\, v_8(n)\\
&y(\mathrm{n}) = v_4(n) := a_1\, v_1(n) + c_1\, u_2 + c_2\, u_6\\
&v_8(\mathrm{n}) := v_7(n-1)\\
&v_7(\mathrm{n}) := u_6\\
&v_3(\mathrm{n}) := u_2\\
&v_5(\mathrm{n}) := v_1(n) \;\text{ where } v_1(\mathrm{n}) = x(n).
\end{aligned}$$

c)

6.7 The figure below shows the internal structure of an allpass section and the node numbering where i = 1, 3, and 5. The basic operations are additions and multiplications.

An analysis of the precedence relations yields the difference equations shown in Table. Seven time slots are required for the arithmetic operations.





Bireciprocal WDF allpass section.

Nodes	Equations
N_2	$u_{11} := -x(n) + v_2(n)$ $u_{31} := -v_0(n) + v_4(n)$
N_3	$u_{12} := \alpha_1\, u_{11}$ $u_{32} := \alpha_3\, u_{31}$
N_4	$u_{13} := -(u_{12} + x(n))$ $u_{14} := u_{12} + v_2(n)$ $u_{33} := -(u_{32} + v_0(n))$ $u_{34} := u_{32} + v_4(n)$

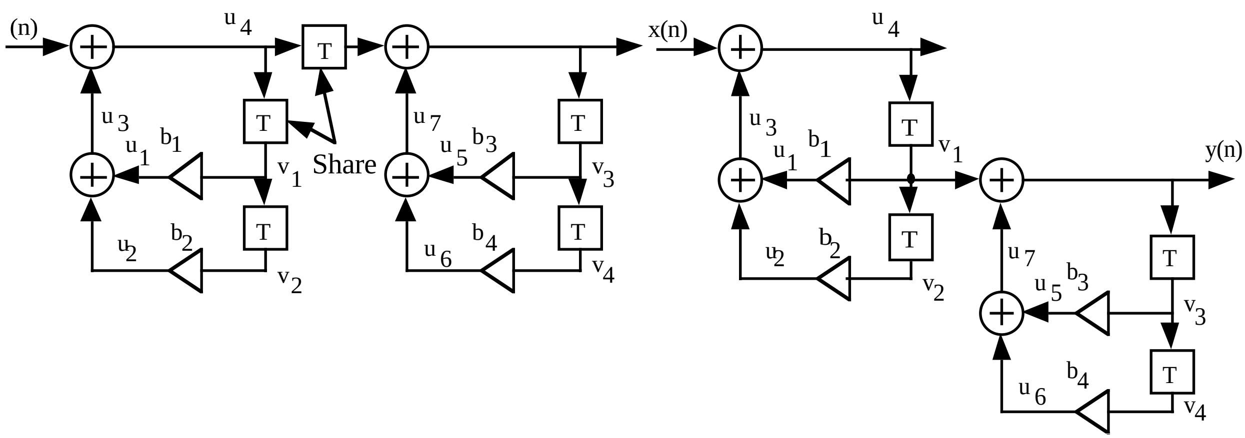
N_5	$u_{51} := -u_{14} + v_6(n)$
N_6	$u_{52} := \alpha_5 u_{51}$
N_7	$u_{53} := -(u_{52} + u_{14})$ $u_{54} := u_{52} + v_6(n)$
N_8	$y_1(n) := u_{34} + u_{54}$ $y_2(n) := u_{34} - u_{54}$
N_1	$v_2(n+1) := v_1(n)$ $v_1(n+1) := u_{13}$ $v_4(n+1) := v_3(n)$ $v_3(n+1) := u_{33}$ $v_6(n+1) := v_5(n)$ $v_5(n+1) := u_{53}$ $v_0(n+1) := x(n)$

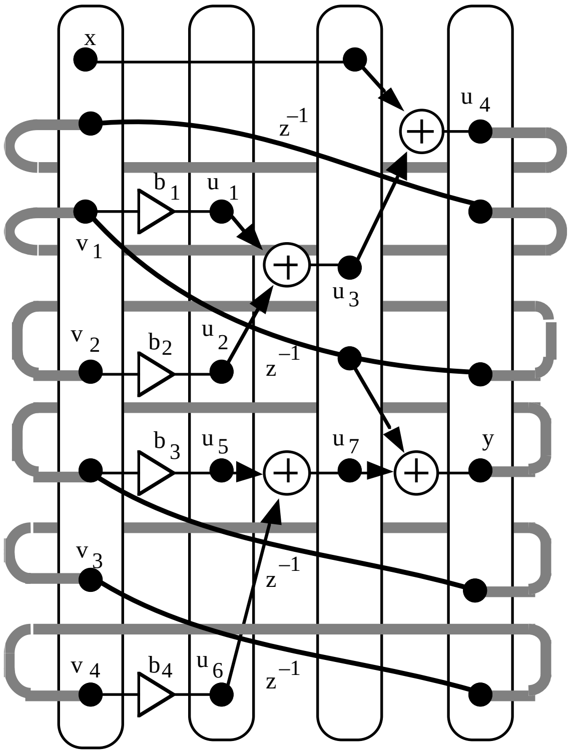
Difference equations in computable order.

6.10 The structure has a delay-free loop. Hence, the algorithm is not sequentially computable and it does not exist a computational order for the difference equations.

6.11 a) Propagate a dealy element between the two sections as shown below and make the subsequent simplification.

The precedence form is shown below. The required number of time slots have been reduced by one.





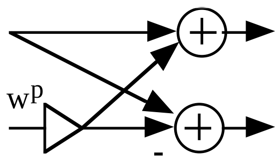
b)

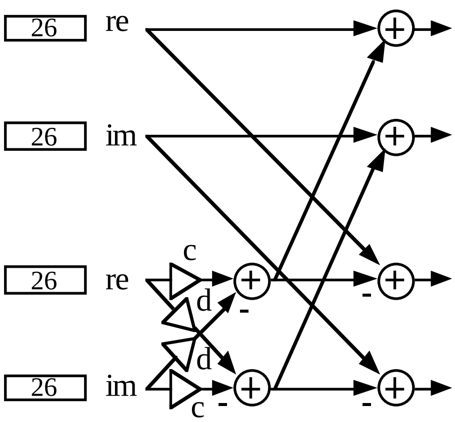
c)

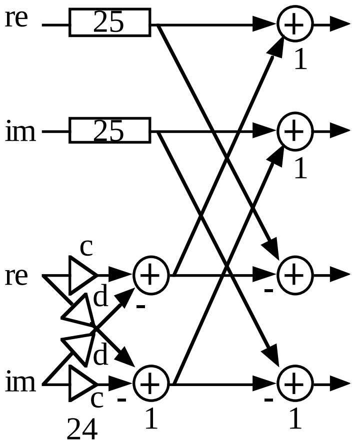
6.12 The Cooley-Tukey's butterfly is shown to the right. A complex multiplication can be written in terms of real multiplications

$$(a + j\,b)(c + j\,d) = (ac - bd) + j(ad + bc)$$

Propagation of the 26 shimming delays gives the result shown below. 50 D-elements are required.





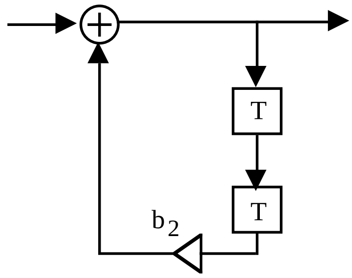


6.15 a) The critical loop in cascade form with direct form I or II first- and second-order sections is

$T_{min} = T_{mult} + T_{add}$

If the poles are located on the imaginary axis (corresponding to the bireciprocal WDF).

$T_{min} = (T_{mult} + T_{add})/2$



b) The critical loop in parallel form is the same as for the cascade form.

c) The critical loop in the lattice wave digital filter is between two adaptors

$T_{min} = 2(T_{mult} + 2\,T_{add})$

In bireciprocal lattice wave digital filters $T_{min} = T_{mult} + 2\,T_{add}$

d) The critical loop is also through two adaptors if the delay elements are places alternatingly in the upper and lower branches. Hence, direct form I and II are generally somewhat faster structures than lattice filters.

6.18 The magnitude function is not effected, but the group delay is increased with a factor T for every level of pipeline that is introduced.

6.23 a)

b) The original transfer function is $H(z) = 1/(1 - b\,z^{-1})$. Add two poles and zeros at $z = b\,e^{\pm j2\pi/3}$. We get

$H(z) = (1 + b\,z^{-1} + b^{\,2} z^{-2})/(1 - b^{\,3} z^{-3}) = (z^2 + b\,z + b^{\,2})/(z^3 - b^{\,3})$

6.24 a)

b) The original transfer function is $H(z) = 1/(1 - b_1\,z^{-1} - b_2\,z^{-2})$

There are two complex conjugate poles at $z = r\,e^{\pm j\theta}$. We add two poles and zeros at $z = r\,e^{\pm j\theta + j2\pi/3}$ and at $z = r\,e^{\pm j\theta - j2\pi/3}$. We get

$H(z) = (1 + b_1\,z^{-1} + (b_1^{\,2} + b_2)z^{-2} - b_1 b_2\,z^{-3} + b_2^{\,2}\,z^{-4})/(1 - (b_1^{\,3} + b_1 b_2)z^{-3} - b_2^{\,3}\,z^{-6}) =$

$= (z^6 + b_1\,z^5 + (b_1^{\,2} + b_2)z^4 - b_1 b_2\,z^3 + b_2^{\,2}\,z^2)/(z^6 - (b_1^{\,3} + b_1 b_2)z^3 - b_2^{\,3})$

6.29 a) The maximal sample frequency is determined by the critical loop.

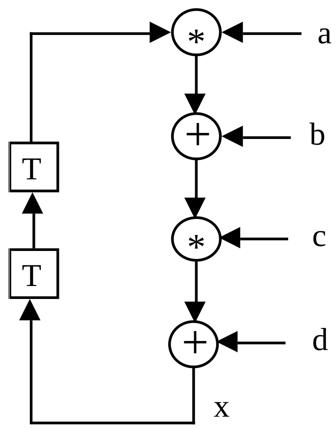
$T_{sample\ min} = (2\,t_{mult} + 2\,t_{add})/2 = (2 \cdot 2 + 2 \cdot 1)/2 = 3$

b) We have $x = d + c(b + a\,z^{-2}\,x)$

This expression can be rewritten by using the distributive rule

$x = d + c\,b + c\,a\,z^{-2}\,x$

This does, however, not change the critical loop, as shown in the figure. If we now use the associative rule, the expression can be rewritten



$$x = ((c\ a)\ z^{-2}\ x + (d + c\ \mathrm{b}))$$

The cirtical loop has now only two operations. We have

$$T_{sample\ min} = (t_{mult} + t_{add})/2 = (2 + 2\ )/2 = 2$$

Note that the number of operations has changed and the length of the other computational paths that are not part of the critical loop has also changed.