5.9 The input $x(n)$ is assumed to be scaled. Hence, the inputs to the multipliers with coefficients a is already scaled. Now, scale the next critical node, i.e., the inputs to the multipliers. In this case, the output node.

a) L_2-norm scaling. Determine the impulse response, by first determining the transfer function.

$$H(z) = \frac{a(z-1)}{z-b} = \frac{a}{1-b\,z^{-1}} - \frac{a\,z^{-1}}{1-b\,z^{-1}} = a\sum_{n=0}^{\infty}(b\,z^{-1})^n - \frac{a}{b}\sum_{n=1}^{\infty}(b\,z^{-1})^n$$

or

$$h(n) = \begin{cases} 0 & \text{for } n < 0 \\ a & \text{for } n = 0 \\ a(1-1/b)\,b^n & \text{for } n > 0 \end{cases}$$

We get:

$$S = \sum_{n=-\infty}^{\infty} h(n)^2 = \frac{2\,a^2}{1+b} \quad \text{which shall be} = 1 \;\Rightarrow a = \sqrt{\frac{1+b}{2}}$$

b) L_∞-norm scaling. Generally, it is difficult to find the maximal value of the magnitude function by analytical methods. However, in this simple case, (highpass filter) we have max for $z = -1$. Hence,

$$|H(e^{j\omega T})| = \frac{2\,a}{1+b} \quad \text{should be set to } 1 \Rightarrow a = \frac{1+b}{2}$$