4.28 a) Let $x(n)$ denote the input signal. We form a new input signal according to

$$x_i(m) = x(n) \text{ for } m = 2n \text{ and } = 0 \text{ otherwise.}$$

The interpolator is described by the difference equation

$$y(m) = \frac{1}{2}[x_i(m) + x_i(m-2)] + x_i(m-1)$$

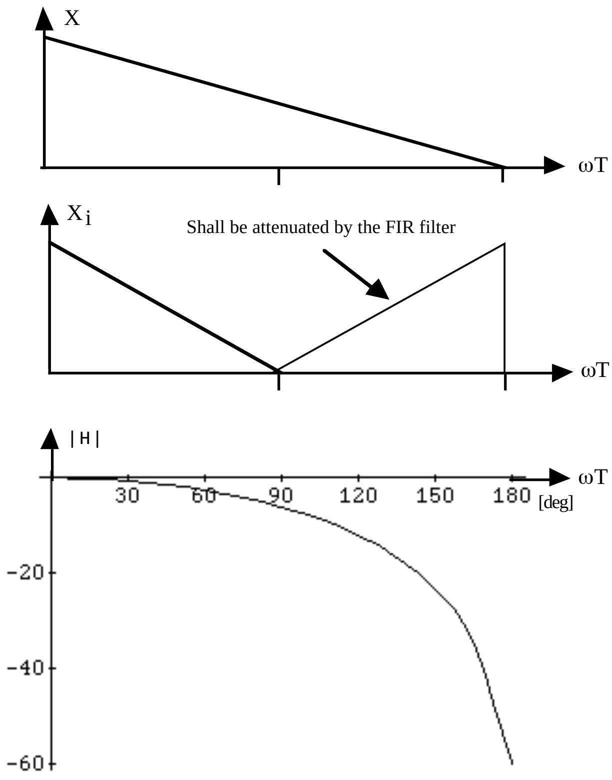
Only the first factor above contributes with an interpolated value for m = even since $x_i(m-1) = 0$. In the next sample interval m = odd. Hence only the second term contributes to the output, $x_i(m-1)$, $(= x(n))$, while both $x_i(m) = 0$ and $x_i(m-2) = 0$.

b) $H(z) = \frac{1}{2}[1 + z^{-2}] + z^{-1} = \frac{1}{2z^2}[z^2 + 2z + 1]$

Selecting a unity gain for the filter we get $H(z) = \frac{z^2 + 2z + 1}{4z^2}$

c) A double pole for $z = 0$ and a double zero for $z = -1$

d) The magnitude of the Fourier transform is shown below for the original input signal, X, the new input signal, X_i and the magnitude function for the FIR filter.



e) Ideally the magnitude function shall be = 1 for $0 \le \omega T \le \frac{\pi}{2}$ and = 0 for $\frac{\pi}{2} < \omega T \le \pi$. The phase function shall be linear. Obviously, this is a poor FIR filter since the attenuation of the unwanted image is very poor.

f) $h(m) = \frac{1}{4}, \frac{1}{2}, \frac{1}{4}, 0, 0, 0, \ldots$

The filter is an FIR filter with length three. The filter order is 2.