

3.9 A system is causal if and only if

$$x_1(n) = x_2(n) \text{ for } n \leq n_0 \text{ fi } y_1(n) = y_2(n) \text{ for } n \leq n_0$$

Now, an LSI system is described by the convolution

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

We must have $y_1(n_0) = y_2(n_0) \Rightarrow$

$$y_1(n_0) - y_2(n_0) = \sum_{k=-\infty}^{\infty} [x_1(k) - x_2(k)] h(n_0-k) = \sum_{k=n_0}^{\infty} [x_1(k) - x_2(k)] h(n_0-k)$$

since $x_1(k) = x_2(k)$ for $k \leq n_0$.

Thus, $h(n_0-k) = 0$ for $k \leq n_0$ since all terms in the convolution must be zero, i.e., $h(n) = 0$ for $n < 0$. On the other hand, if $h(n) = 0$ for $n < 0$, we have

$$y(n_0) = \sum_{k=-\infty}^{\infty} x(k) h(n_0-k) = \sum_{k=-\infty}^{n_0} x(k) h(n_0-k)$$

$\Rightarrow y(n)$ is independent of $x(k)$ for $k > n_0$, i.e., independent of future input samples.