3.2 Apply the definition of z-transform to the sequences using $Z\{u(n)\} = \dfrac{z}{z-1}$

(a) $x_1(nT)$. The period is 6, and the z-transform is

$$Z\{x_1(nT)\} = \sum_{n=-\infty}^{\infty} x_1(nT)z^{-n}$$

$$= \sum_{n=0}^{\infty} (0z^0 - az^{--1} + 0z^{-2} + az^{-3} + 0z^{-4} - az^{-5})z^{-6n}$$

$$= \sum_{n=0}^{\infty} (-az^{-1} + az^{-3} - az^{-5})z^{-6n} = (-az^{-1} + az^{-3} - az^{-5}) \sum_{n=0}^{\infty} z^{-6n}$$

$$= \frac{-a(z^5 - z^3 + z)}{z^6 - 1}$$

(b) $x_2(nT)$. The period is 7, and the z-transform is

$$Z\{x_2(nT)\} = \sum_{n=-\infty}^{\infty} x_2(nT)z^{-n}$$

$$= \sum_{n=0}^{\infty} (0az^0 + az^{-1} - az^{-2} - az^{-3} + az^{-4} + az^{-5} - az^{-6})z^{-7n}$$

$$= (az^0 + az^{-1} - az^{-2} - az^{-3} + az^{-4} + az^{-5} - az^{-6}) \sum_{n=0}^{\infty} z^{-7n}$$

$$= a(1 + z^{-1} - z^{-2} - z^{-3} + z^{-4} + z^{-5} - z^{-6})\frac{z^7}{z^7 - 1}$$

$$= \frac{a(z^7 + z^6 - z^5 - z^4 + z^3 + z^2 - z^1)}{z^7 - 1}$$

(c) $x_3(nT)$. The period is 7, and the z-transform is

$$Z\{x_3(nT)\} = \sum_{n=-\infty}^{\infty} x_3(nT)z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^0 + 0z^{--1} - az^{-2} - 0z^{-3} + az^{-4} + 0z^{-5} - az^{-6})z^{-7n}$$

$$= (az^0 - az^{-2} + az^{-4} - az^{-6}) \sum_{n=0}^{\infty} z^{-7n} = \frac{a(z^7 - z^5 + z^3 - z^1)}{z^7 - 1}$$

(d) $x_4(nT)$. The period is 7, and the z-transform is

$$Z\{x_4(nT)\} = \sum_{n=-\infty}^{\infty} x_4(nT)z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^0 + 0z^{-1} - az^{-2} - az^{-3} + 0z^{-4} + az^{-5} - 0z^{-6})z^{-7n}$$

$$= (az^0 - az^{-2} - az^{-3} - az^{-5}) \sum_{n=0}^{\infty} z^{-7n} = \frac{a(z^7 - z^5 - z^4 + z^2)}{z^7 - 1}$$

(e) $x_5(nT)$. The period is 7, and the z-transform is

$$Z\{x_5(nT)\} = \sum_{n=-\infty}^{\infty} x_5(nT)z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^0 + 2az^{-1} + az^{-2} - az^{-3} - 2az^{-4} - az^{-5} + 0z^{-6})z^{-7n}$$

$$= a(1 + 2z^{-1} + z^{-2} - z^{-3} - 2z^{-4} - z^{-5} + z^{-6}) \sum_{n=0}^{\infty} z^{-7n}$$

$$= \frac{a(z^7 + 2z^6 + z^5 - z^4 - 2z^3 - z^2 + z^1)}{z^7 - 1}$$