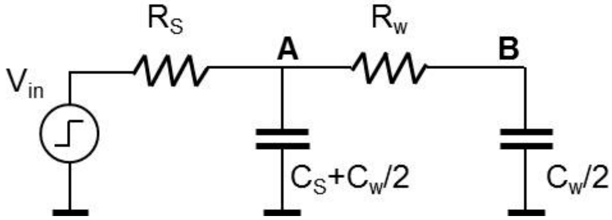
TSTE86 Homework 4: Solution

a) Circuit model with a lumped wire model (π)



Wire resistance and capacitance calculated with respect to wire length L are

$R_W = \frac{L}{W} r_W = 150L$ mΩ/μm

$C_W = c_a WL + 2c_f L = 95L$ aF/μm

b) Inherent propagation delay is $t_{P,inh} = 0.69 R_S C_S$

Propagation delay from the input to the end of the wire $t_P = 0.69\left(R_S(C_S + C_W) + R_W C_W / 2\right)$

hence we have $2R_S C_S = R_S(C_S + C_W) + R_W C_W / 2$,

which can be simplified to $R_S C_S = R_S C_W + R_W C_W / 2$

Next using the expressions for R_W and C_W we have

$R_S C_S = R_S(c_a W + 2c_f)L + \frac{r_W}{2W}(c_a W + 2c_f)L^2$

which we rewrite as $7.125 \times 10^{-6} L^2 + 0.689L - 43.5 = 0$ (where L is expressed in μm)

Hence, $L \cong \frac{43.5}{0.689} \cong 63$ μm

c) $t_p = 0.69 \cdot 2R_S C_S = 60$ ps

while the propagation of the wire $t_{pW} = t_p - 0.69R_S(C_S + C_W) = 0.69R_W C_W / 2$

so $t_{pW} = 0.69R_W C_W / 2 = 28.4$ fs, that is $t_{pW} \ll t_p$ and $R_W \ll R_S$, but $C_W = C_S$

d) Propagation delay of the loaded driver is $t_{Pd} = 0.69R_S(C_S + C_W)$

while of the wire $t_{pW} = 0.69R_W C_W / 2$

hence $R_S(C_S + C_W) = R_W C_W / 2$, which can be rewritten as $7.125 \cdot 10^{-6} L^2 - 0.689L - 43.5 = 0$

Hence $L \cong \frac{0.689 + 0.69}{14.25 \cdot 10^{-6}} \cong 0.097 \cdot 10^6$ μm = 97 mm

The circuit propagation delay is $t_p = 0.69R_W C_W = 0.69 \cdot 150 \cdot 10^{-3} \cdot 95 \cdot 10^{-18} \cdot L^2$ (with L in μm)

$t_p = 92.5 \cdot 10^{-9}$ s = 92.5 ns while C_W = 8.9 pF and R_W = 14.5 kΩ, hence $C_S \ll C_W$ and $R_W = 2R_S$