

TSTE86 Homework 1: Solution

For this circuit, we have $V_{GS} = V_{in}$ and $V_{DS} = V_{out} = V_{DD} - RI_D$. Further, $V_{SB} = 0 \Rightarrow V_T = V_{T0}$, and $\lambda = 0 \Rightarrow$ current can be written

$$V_{in} \leq V_{T0} \Rightarrow I_D = 0 \rightarrow \langle \text{cutoff} \rangle$$

$$V_{in} > V_{T0} \Rightarrow I_D = k' \frac{W}{L} V_{min} \left(V_{in} - V_{T0} - \frac{V_{min}}{2} \right),$$

$$\text{where } V_{min} = \min(V_{in} - V_{T0}, V_{DD} - RI_D, V_{DSAT}) \rightarrow \langle \text{saturated, resistive, velocity sat.} \rangle$$

1. $V_{in} = 0 \text{ V}$

$$V_{in} \leq V_{T0} \Rightarrow I_D = 0 \Rightarrow V_{out} = V_{DD}$$

$$\boxed{\text{MOSFET is cutoff, } I_D = 0, \text{ and } V_{out} = 2.5 \text{ V}}$$

2. $V_{in} = 0.5 \text{ V}$

$$V_{in} > V_{T0} \Rightarrow \text{guess MOSFET is saturated} \Rightarrow V_{min} = V_{in} - V_{T0} = 0.07 \text{ V}$$

$$I_D = k' \frac{W}{L} \frac{(V_{in} - V_{T0})^2}{2} = 0.85 \mu\text{A}$$

$$V_{out} = V_{DD} - RI_D = 2.49 \text{ V}$$

$$\text{Check mode: } V_{min} = \min(0.07, 2.49, 0.63) \text{ V} \rightarrow \text{saturated is correct}$$

$$\boxed{\text{MOSFET is saturated, } I_D = 0.85 \mu\text{A, and } V_{out} = 2.49 \text{ V}}$$

3. $V_{in} = 1.0 \text{ V}$

$$V_{in} > V_{T0} \Rightarrow \text{guess MOSFET is saturated} \Rightarrow V_{min} = V_{in} - V_{T0} = 0.57 \text{ V}$$

$$I_D = k' \frac{W}{L} \frac{(V_{in} - V_{T0})^2}{2} = 56 \mu\text{A}$$

$$V_{out} = V_{DD} - RI_D = 1.88 \text{ V}$$

$$\text{Check mode: } V_{min} = \min(0.57, 1.88, 0.63) \text{ V} \rightarrow \text{saturated is correct}$$

$$\boxed{\text{MOSFET is saturated, } I_D = 56 \mu\text{A, and } V_{out} = 1.88 \text{ V}}$$

4. $V_{in} = 1.5 \text{ V}$

$$V_{in} > V_{T0} \Rightarrow \text{guess MOSFET is saturated} \Rightarrow V_{min} = V_{in} - V_{T0} = 1.07 \text{ V}$$

$$I_D = k' \frac{W}{L} \frac{(V_{in} - V_{T0})^2}{2} = 198 \mu\text{A}$$

$$\text{Check mode: } V_{min} = \min(1.07, 0.32, 0.63) \text{ V} \rightarrow \text{saturated is incorrect}$$

$$\text{Reguess MOSFET is velocity saturated} \Rightarrow V_{min} = V_{DSAT} = 0.63 \text{ V}$$

$$I_D = k' \frac{W}{L} V_{DSAT} \left(V_{in} - V_{T0} - \frac{V_{DSAT}}{2} \right) = 164 \mu\text{A}$$

$$V_{out} = V_{DD} - RI_D = 0.69 \text{ V}$$

Check mode: $V_{\min} = \min(1.07, 0.69, 0.63) \text{ V} \rightarrow$ velocity saturated is correct

MOSFET is velocity saturated, $I_D = 164 \mu\text{A}$, and $V_{\text{out}} = 0.69 \text{ V}$

5. $V_{\text{in}} = 2.0 \text{ V}$

$V_{\text{in}} > V_{T0} \Rightarrow$ guess MOSFET is velocity saturated $\Rightarrow V_{\min} = V_{\text{DSAT}} = 0.63 \text{ V}$

$$I_D = k' \frac{W}{L} V_{\text{DSAT}} \left(V_{\text{in}} - V_{T0} - \frac{V_{\text{DSAT}}}{2} \right) = 273 \mu\text{A}$$

$V_{\text{out}} = V_{\text{DD}} - RI_D = -0.50 \text{ V} \rightarrow$ unrealistic voltage

Reguess MOSFET is resistive $\Rightarrow V_{\min} = V_{\text{out}} \Rightarrow I_D = k' \frac{W}{L} V_{\text{out}} \left(V_{\text{in}} - V_{T0} - \frac{V_{\text{out}}}{2} \right)$

$$V_{\text{out}} = V_{\text{DD}} - RI_D = V_{\text{DD}} - Rk' \frac{W}{L} V_{\text{out}} \left(V_{\text{in}} - V_{T0} - \frac{V_{\text{out}}}{2} \right) \Rightarrow$$

$$Rk' \frac{W}{L} \frac{1}{2} V_{\text{out}}^2 - \left[1 + Rk' \frac{W}{L} (V_{\text{in}} - V_{T0}) \right] V_{\text{out}} + V_{\text{DD}} = 0 \Leftrightarrow$$

$$2.415V_{\text{out}}^2 - 6.1681V_{\text{out}} + 2.5 = 0 \Rightarrow V_{\text{out}} = \begin{cases} 0.40 \text{ V} \\ (3.26 \text{ V} > V_{\text{DD}}) \end{cases} \Rightarrow I_D = 191 \mu\text{A}$$

Check mode: $V_{\min} = \min(1.57, 0.40, 0.63) \text{ V} \rightarrow$ resistive is correct

MOSFET is resistive, $I_D = 191 \mu\text{A}$, and $V_{\text{out}} = 0.40 \text{ V}$

6. $V_{\text{in}} = 2.5 \text{ V}$

$V_{\text{in}} > V_{T0} \Rightarrow$ guess MOSFET is resistive $\Rightarrow V_{\min} = V_{\text{out}} \Rightarrow$

$$Rk' \frac{W}{L} \frac{1}{2} V_{\text{out}}^2 - \left[1 + Rk' \frac{W}{L} (V_{\text{in}} - V_{T0}) \right] V_{\text{out}} + V_{\text{DD}} = 0 \Leftrightarrow$$

$$2.415V_{\text{out}}^2 - 10.9981V_{\text{out}} + 2.5 = 0 \Rightarrow V_{\text{out}} = \begin{cases} 0.30 \text{ V} \\ (4.37 \text{ V} > V_{\text{DD}}) \end{cases} \Rightarrow I_D = 200 \mu\text{A}$$

Check mode: $V_{\min} = \min(2.07, 0.30, 0.63) \text{ V} \rightarrow$ resistive is correct

MOSFET is resistive and, $I_D = 200 \mu\text{A}$, and $V_{\text{out}} = 0.30 \text{ V}$

Plot $V_{\text{out}}(V_{\text{in}})$

