SOLUTIONS. Exam August 21, 2008
TSTE08 Analog and Discrete-time Integrated Circuits.

Exercise 1.

M1: Figure 1 gives: $V_{GS1} = V_{in} = 3$ V och $V_{DS1} = V_{out} = 0.03$ V

$V_{DS1} << V_{GS1} - V_{tn} = 2.5$ V, which means that transistor **M1** works in the *linear* region.

Enclosed formulas then give:

$$I_{D1} = \frac{\mu_{0n}C_{ox}}{2}\left(\frac{W}{L}\right)_1 \left(2(V_{GS1} - V_{Tn}) - V_{DS1}\right) V_{DS1} \qquad (1)$$

With $I_{D1} = I_{D2} = 20$ nA relation (1) gives:

$$20 \cdot 10^{-9} = 10 \cdot 10^{-9} \cdot \left(\frac{W}{10^{-6}}\right)_1 (2 \cdot 2.5 - 0.03)\, 0.03 \Rightarrow W_1 \approx 13.4\mu\text{m}$$

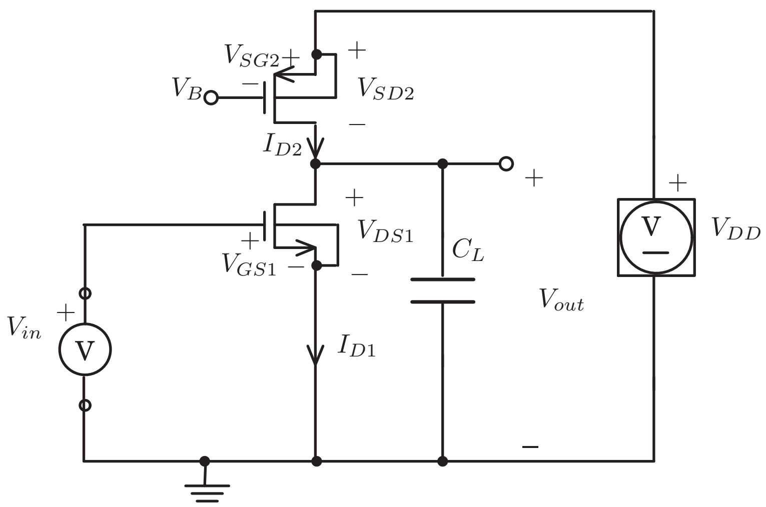

Figure 1: Inverter. Large signal analysis.

M2: From **Figure 1**: $V_{SG2} = V_{DD} - V_B = 2$ V and $V_{SD2} = V_{DD} - V_{out} = 2.97$ V

$V_{SD2} > V_{SG2} - V_{tp} = 1.4$ V, which means that transistor **M2** works in the *saturated* region.

Enclosed formulas than give:

$$I_{D2} = \frac{\mu_{0p}C_{ox}}{2}\left(\frac{W}{L}\right)_2 \left((V_{SG2} - V_{Tp})^2\right)\left(1 + \lambda_p(V_{SD2} - V_{eff2})\right) \qquad (2)$$

$I_{D1} = I_{D2} = 20$ nA inserted in (2):

$$20 \cdot 10^{-9} = 3 \cdot 10^{-9} \left(\frac{W}{10^{-6}}\right)_2 1.4^2(1 + 0.05 \cdot (2.97 - 1.4)) \Rightarrow W_2 \approx 3.19\mu\text{m}$$

Answer: $W_1 \approx 13.4\ \mu$m och $W_2 \approx 3.19\ \mu$m

Exercise 2.

a) **Figure 2 a)** shows a complete small signal equivalent. As the DC voltage source (V_{DD}) is ideal it will be replaced by a short circuit in the small signal equivalent circuit.

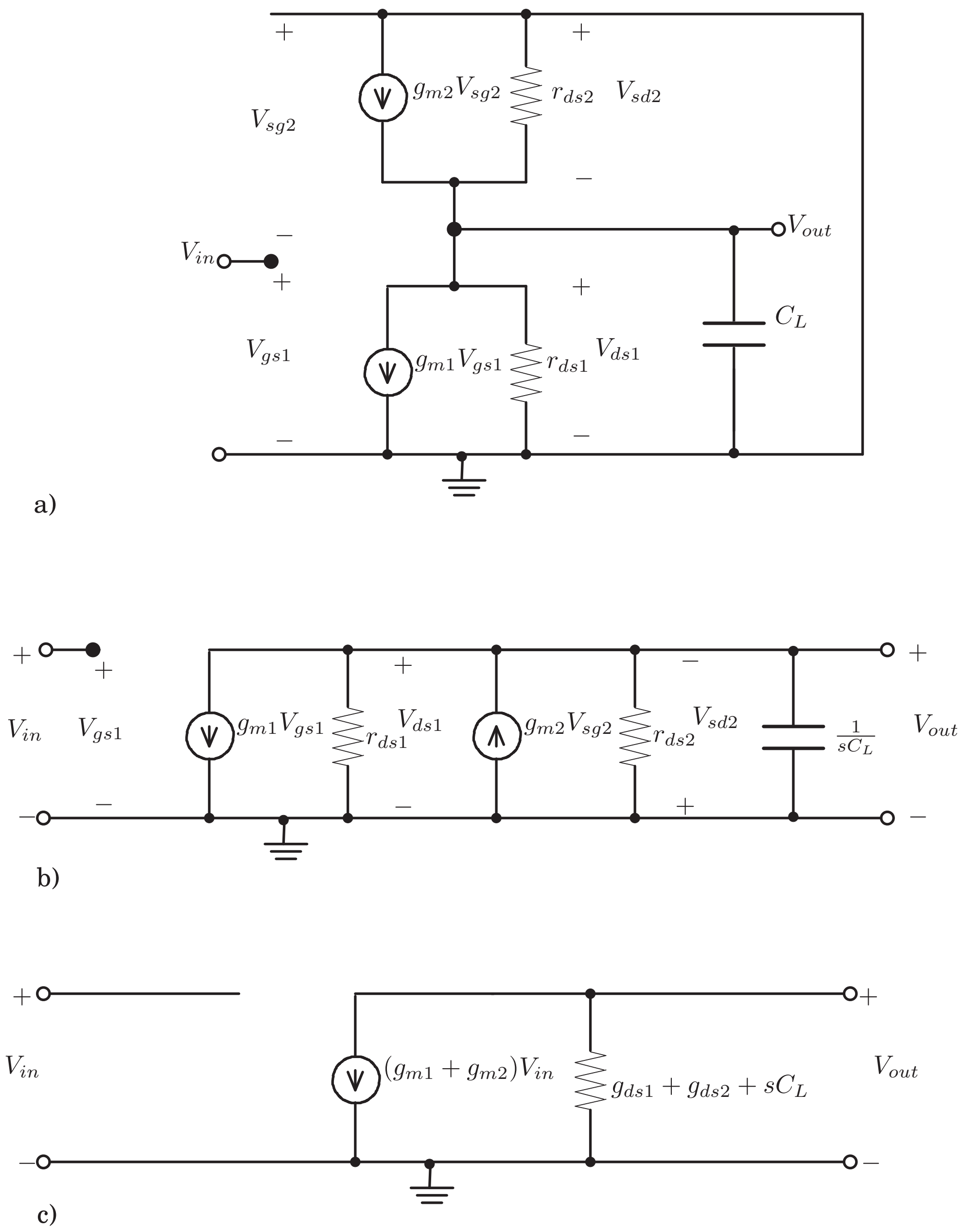

Figure 2: Complete small signal equivalent.

b) **Figure 2b)** is obtained by rewriting **figure 2a)**. Note that $V_{sg2} = -V_{gs1}$.

The equivalent circuit in **Figure 2c)** (the asked for equivalent) is obtained from **Figure 2b)** by:

1. Note that resistors $1/g_{ds1}$, $1/g_{ds2}$ and the capacitor $\frac{1}{sC_L}$ are parallel, then the total admittans obtains by adding the admittanses g_{ds1}, g_{ds2} and sC_L.
2. Observe that $V_{sg2} = -V_{gs1} = -V_{in}$ which yields that the current source $g_{m2}V_{sg2}$ in **Figure 2b)** can be changed to a current source $g_{m2}V_{in}$ with opposite direction. This current source is parallel to the current source $g_{m1}V_{gs1}$. Thus they can be added to one current source $(g_{m1} + g_{m2})V_{in}$.

Figure 2c yields:

$$V_{out} = -\frac{(g_{m1} + g_{m2})V_{in}}{g_{ds1} + g_{ds2} + sC_L} \tag{3}$$

$$(3) \Rightarrow \frac{V_{out}}{V_{in}} = -\frac{g_{m1} + g_{m2}}{g_{ds1} + g_{ds2} + sC_L}$$

c) $s = j\omega$ yields the transfer function $H(\omega)$:

$$H(\omega) = -\frac{g_{m1} + g_{m2}}{g_{ds1} + g_{ds2} + j\omega C_L} \Rightarrow \mid H(\omega) \mid = \frac{g_{m1} + g_{m2}}{\left((g_{ds1} + g_{ds2})^2 + (\omega C_L)^2\right)^{1/2}} \tag{4}$$

Unity-gain frequency is that angular frequency ω_u when $\mid H(\omega) \mid = 1$.

$$\mid H(\omega) \mid = 1 \overset{(4)}{\Rightarrow} (g_{ds1}+g_{ds2})^2+(\omega C_L)^2 = (g_{m1}+g_{m2})^2 \Rightarrow \omega_u = \frac{\left((g_{m1} + g_{m2})^2 - (g_{ds1} + g_{ds2})^2\right)^{1/2}}{C_L}$$

Answer: $\omega_u = \frac{\left((g_{m1} + g_{m2})^2 - (g_{ds1} + g_{ds2})^2\right)^{1/2}}{C_L} \approx \frac{g_{m1} + g_{m2}}{C_L}$

Exercise 3.

a)

Phase I:

- $V_{INP} = +E, V_{INN} = 0 \Rightarrow$ M1 conducts and M3 blocks.
- M3 blocks $\Rightarrow I_4 = 0$
  M1 conducts $\Rightarrow I_1 = I_2 = I_0$
- M2 and M4 constitute a current mirror and as M2 and M4 are identical $I_3 = I_1 = I_0$
- M3 blocks $(I_4 = 0) \Rightarrow I_5 = I_3 = I_0$

Phase II:

- $V_{INP} = 0, V_{INN} = +E \Rightarrow$ M1 blocks and M3 conducts.
- M1 blocks $\Rightarrow I_6 = I_7 = 0$
- M2 and M4 constitute a current mirror $\Rightarrow I_8 = I_6 = 0$
- M3 conducts $\Rightarrow I_9 = I_0$
- $I_8 = 0$ and $I_9 = I_0 \Rightarrow I_{10} = -I_9 = -I_0$

b)

$$\text{Definition: Slew-Rate (SR) =max}\frac{dv_{out}(t)}{dt}$$

For capacitor C_L we have that:

$$i_{CL}(t) = C_L\frac{dv_{CL}(t)}{dt} = C_L\frac{dv_{out}(t)}{dt} \Rightarrow \frac{dv_{out}}{dt} = \frac{i_{CL}(t)}{C_L}$$

Thus the maximum value of $\frac{dv_{out}(t)}{dt}$ obtains when $i_{CL}(t)$ has it maximum value, which according to a) is I_0.

Answer: Slew-Rate $= \frac{I_0}{C_L}$

Exercise 4.

Figure 3 a) shows a small signal equivalent and **Figure 3 b)** a redrawn version, where we have used the fact that $V_{gs2} = 0$ (giving $g_{m2}V_{gs2} = 0$) and that g_{ds1}, g_{ds2} and $\frac{1}{sC_L}$ are parallel.

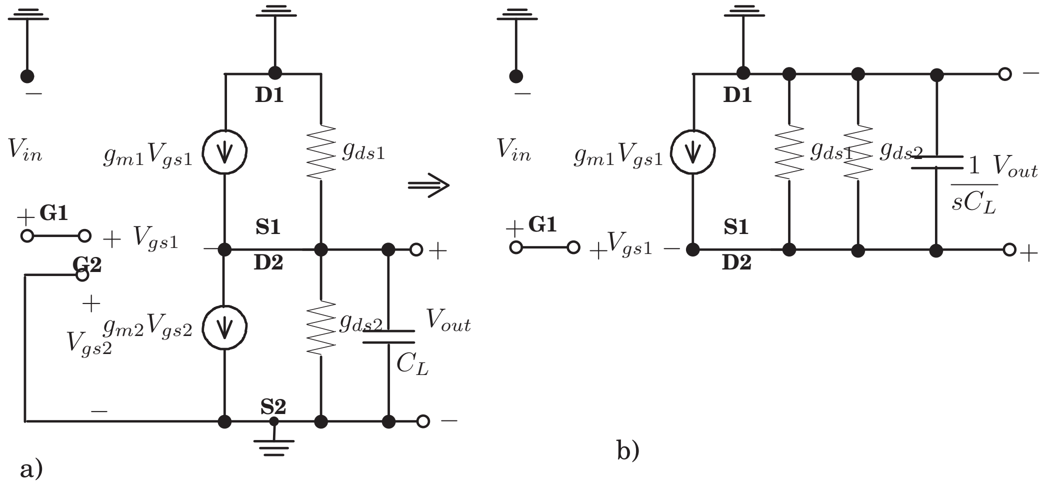

Figure 3: Small signal equivalent circuit.

Determine V_{out}/V_{in}**:**
Figure 3 b) gives:

$$V_{gs1} = V_{in} - V_{out} \tag{5}$$

and

$$V_{out} = g_{m1}V_{gs1} \cdot \frac{1}{g_{ds1} + g_{ds2} + sC_L} \tag{6}$$

(5) inserted in (6) gives:

$$V_{out} = g_{m1}(V_{in} - V_{out}) \cdot \frac{1}{g_{ds1} + g_{ds2} + sC_L} \tag{7}$$

(7) gives the transfer function:

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{\frac{g_{m1}}{g_{ds1}+g_{ds2}+sC_L}}{1 + \frac{g_{m1}}{g_{ds1}+g_{ds2}+sC_L}} = \frac{g_{m1}}{g_{m1} + g_{ds1} + g_{ds2} + sC_L} \tag{8}$$

As $g_m >> g_{ds}$ following approximation of $H(s)$ obtains:

$$H(s) \approx \frac{g_{m1}}{g_{m1} + sC_L} = \frac{1}{1 + \frac{sC_L}{g_{m1}}} \tag{9}$$

Determine R_{out}**:**
Set the input signal $V_{in} = 0$ and introduce the noisy voltage source V_{Th} with spectral density $R_{Th}(f) = \frac{8kT}{3} \cdot \frac{1}{g_{m1}}$ (from enclosed formulas) between **G1** and ground. See **Figure 4**. Note that V_{Th} will have the same position as V_{in} in **Figure 3 b)**, i.e. between **G1** and ground. Which means that eqn. (9) also gives the relation between V_{Th} and V_{out}.

The relation $R_{out}(f) = |H(f)|^2 R_{in}(f)$ gives (introduce $s = j2\pi f$ in $H(s)$):

$$R_{out}(f) = \left(\frac{1}{\sqrt{1 + (\frac{2\pi f C_L}{g_{m1}})^2}}\right)^2 R_{in}(f) = \frac{1}{1 + (\frac{2\pi f C_L}{g_{m1}})^2} \cdot \frac{8kT}{3} \cdot \frac{1}{g_{m1}} \tag{10}$$

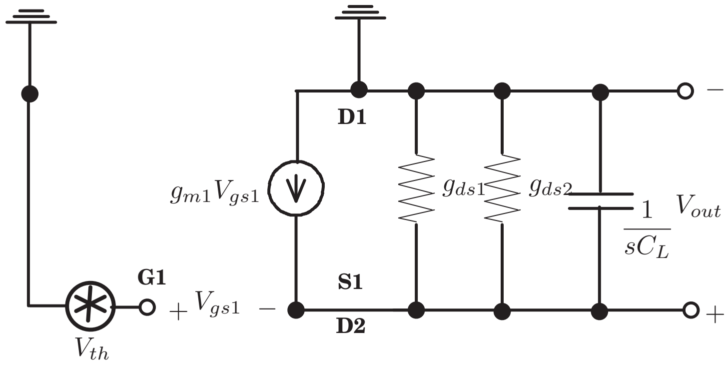

Figure 4: Small signal equivalent circuit.

and

$$P_{out,noise} = \int_0^\infty R_{out}(f)df = \int_0^\infty \frac{1}{1+(\frac{2\pi f C_L}{g_{m1}})^2} \cdot \frac{8kT}{3} \cdot \frac{1}{g_{m1}} df \tag{11}$$

(11) gives

$$P_{out,noise} = \frac{8kT}{3} \cdot \frac{1}{g_{m1}} \cdot \frac{g_{m1}}{2\pi C_L} \left[\arctan \frac{2\pi f C_L}{g_{m1}}\right]_0^\infty = \frac{8kT}{6\pi C_L} \cdot \frac{\pi}{2} = \frac{2KT}{3C_L} \tag{12}$$

Answer:

$$R_{out}(f) = \frac{1}{1+(\frac{2\pi f C_L}{g_{m1}})^2} \cdot \frac{8kT}{3} \cdot \frac{1}{g_{m1}} \tag{13}$$

$$P_{out,noise} = \frac{2KT}{3C_L} \tag{14}$$

Exercise 5

a) Circuits for the two phases is shown in **Figure 5**.

Charge analysis:

1) At $t-\tau$:

$$q_1(t-\tau) = C_1 v_1(t-\tau)$$
$$q_2(t-\tau) = C_2 v_2(t-\tau)$$

2) At t:

$$q_1(t) = 0$$
$$q_2(t) = C_2 V_2(t)$$

Charge conservation:

$$q_1(t-\tau) + q_2(t-\tau) = q_1(t) + q_2(t) \overset{q_1(t)=0}{\Rightarrow} C_1 v_1(t-\tau) + C_2 v_2(t-\tau) = C_2 v_2(t) \tag{15}$$

3) At $t+\tau$:

$$q_1(t+\tau) = C_1 v_1(t+\tau)$$
$$q_2(t+\tau) = C_2 v_2(t+\tau)$$

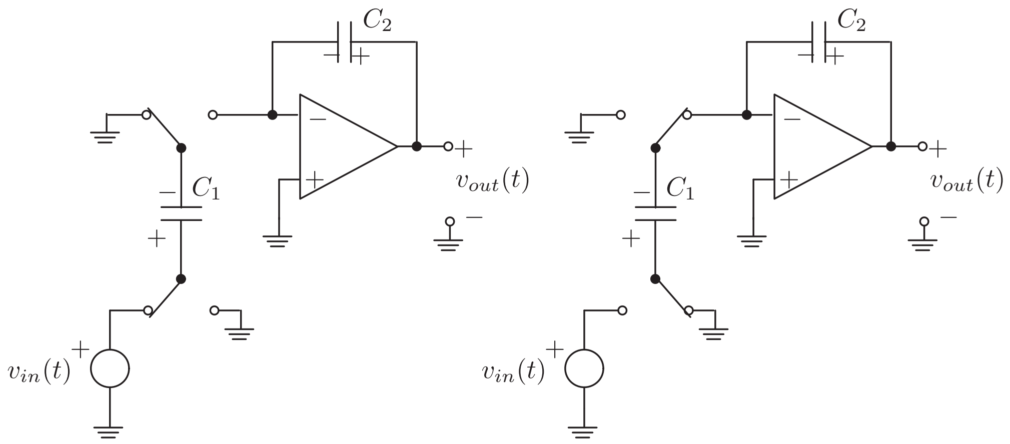

Figure 5: SC-circuit.

C_2 keeps its charge from t till $t+\tau$:

$$q_2(t+\tau) = q_2(t) \Rightarrow v_2(t+\tau) = v_2(t) \tag{16}$$

(16) in (15) gives:

$$C_1 v_1(t-\tau) + C_2 v_2(t-\tau) = C_2 v_2(t+\tau)$$

$kT = t+\tau$ and $T = 2\tau$ yields:

$$C_1 v_1(kT-T) + C_2 v_2(kT-T) = C_2 v_2(kT) \Rightarrow v_2(kT) = v_2(kT-T) + \frac{C_1}{C_2} \cdot v_1(kT-T) \tag{17}$$

Identifying (17) and given differens equation gives:

$$\underline{\underline{a = \frac{C_1}{C_2} \text{ and } b = 1}}$$

b) The charge analysis will be the same as in a), just replace $v_- = 0$ with $v_- = -v_2/A$.
Charge analysis:

1) At $t-\tau$:

$$q_1(t-\tau) = C_1 V_1(t-\tau)$$
$$q_2(t-\tau) = C_2 V_2(t-\tau)(1+\tfrac{1}{A})$$

2) At t:

$$q_1(t) = 0$$
$$q_2(t) = C_2 v_2(t)(1+\tfrac{1}{A})$$

Charge conservation:

$$q_1(t-\tau)+q_2(t-\tau) = q_1(t)+q_2(t) \Rightarrow C_1 v_1(t-\tau)+C_2(1+\frac{1}{A})v_2(t-\tau) = (\frac{C_1}{A}+C_2(1+\frac{1}{A}))v_2(t) \tag{18}$$

3) At $t+\tau$:

$$q_1(t+\tau) = C_1 v_1(t+\tau)$$
$$q_2(t+\tau) = C_2 v_2(t+\tau)(1+\tfrac{1}{A})$$

C_2 keeps its charge from t to $t+\tau$:

$$q_2(t+\tau) = q_2(t) \Rightarrow v_2(t+\tau) = v_2(t) \tag{19}$$

(19) introduced in (18) yields:

$$C_1 v_1(t-\tau) + C_2(1+\frac{1}{A})v_2(t-\tau) = (\frac{C_1}{A} + C_2(1+\frac{1}{A}))v_2(t+\tau)$$

$kT = t+\tau$ and $T = 2\tau$ yield:

$$\begin{aligned} &C_1 v_1(kT-T) + C_2(1+\tfrac{1}{A})v_2(kT-T) = (\tfrac{C_1}{A} + C_2(1+\tfrac{1}{A}))v_2(kT) \\ &\Rightarrow v_2(kT) = \frac{C_2(1+\frac{1}{A})}{\frac{C_1}{A}+C_2(1+\frac{1}{A})} \cdot v_2(kT-T) + \frac{C_1}{\frac{C_1}{A}+C_2(1+\frac{1}{A})} \cdot v_1(kT-T) \end{aligned} \tag{20}$$

Identifying (20) and given difference equation gives:

$$\underline{\underline{a}} = \frac{C_1}{\frac{C_1}{A} + C_2(1+\frac{1}{A})} = \underline{\underline{\frac{C_1}{C_2(1+\frac{1}{A}\cdot(1+\frac{C_1}{C_2}))}}}$$

$$\underline{\underline{b}} = \frac{C_2(1+\frac{1}{A})}{\frac{C_1}{A} + C_2(1+\frac{1}{A})} = \underline{\underline{\frac{1}{1+\frac{C_1}{C_2(1+A)}}}}$$

c) Capacitive parasitics in SC-circuits arise in nodes connected to either capacitors, amplifiers or/and switches. This means that every node has an associated parasitic. **Figure 6** shows those parasitics that possbly can affect the transfer function.

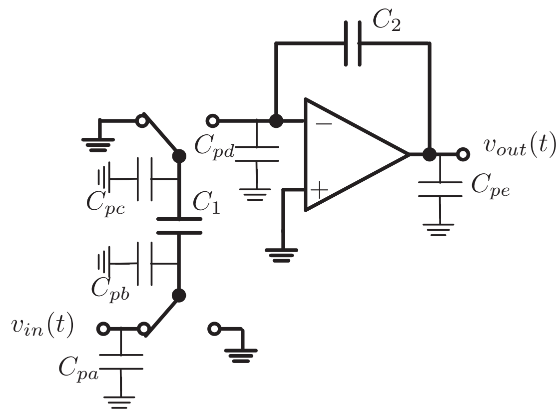

Figure 6: SC-circuit with parasitics.

- C_{pa} doesn't affect the transfer function as it is connected between the input souce and ground. The source can vive/take as much charge as needed.
- C_{pb} is charging during phase 1 from the input source and discharge to ground in next phase, reesulting in no affect on transfer function.
- C_{pc} is connected between ground and ground in phase 1 and between ground and virtual ground in phase 2. The transfer function will not be affected.
- C_{pd} is connected between ground and virtual ground in both phases which leaves the transfer function uneffected.

- C_{pe} is connected between ground and output node, which can give/take as much charge as needed. Thus no affect on the transfer function.

All other parasitics are short circuited to ground, and will not affect the transfer function.

Thus the transfer function is quite insensitive for capacitive parasitics. Which was to be proven.