SOLUTIONS. Exam Aug 10, 2006

TSTE80 Analog and Discrete-time Integrated Circuits.

Excercise 1.

a) A PMOS transistor is saturated when $V_{SD} > V_{eff} = V_{SG} - V_{tp}$.

Transistor **M1**: $V_{SD1} = V_{DD} - V_x = V_{SG1}$ i.e. $V_{SD1} > V_{SG1} - V_{tp1}$, so **M1** works in *saturation*.

Transistor **M2**: $V_{SD2} = V_x - V_{bias} = V_{SG2}$ i.e. $V_{SD2} > V_{SG2} - V_{tp2}$, so **M2** works in *saturation*.

A NMOS transistor is saturated when $V_{DS} > V_{eff} = V_{GS} - V_{tn}$.

Transistor **M3**: $V_{DS3} = V_{bias} - 0 = V_{GS3}$ i.e. $V_{DS3} > V_{GS3} - V_{tn3}$, so **M3** works in *saturation*.

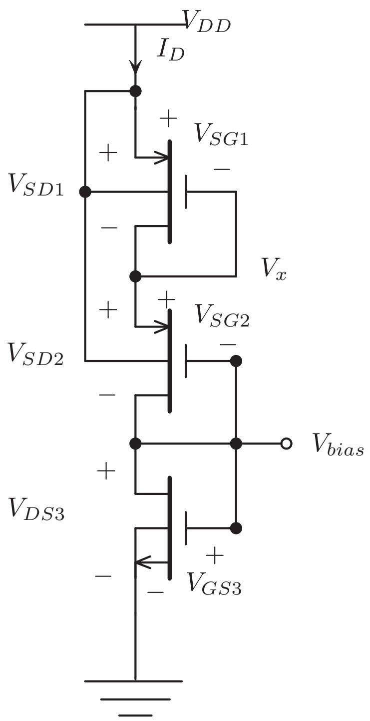

Figure 1: A bias circuit.

b) First note that $I_{D1} = I_{D2} = I_{D3} = I_D$

Transistor **M1**: $V_{SB1} = 0$ i.e. $V_{tp1} = V_{t0p}$.

Enclosed page of formulas gives:

$$\left(\frac{W}{L}\right)_1 = \frac{I_D}{\frac{1}{2}\mu_{0p}C_{oxp}V_{eff1}^2(1 + \lambda_p(V_{SD1} - V_{eff1}))} \quad (1)$$

$I_D = 5\ \mu$ A, $\mu_{0p}C_{oxp} = 58.5\ \mu$ A/V^2,

$V_{eff1} = V_{SG1} - V_{t0p} = V_{DD} - V_x - V_{t0p} = 3.3 - 0.6 - 0.62 = 2.08$ V ,
$\lambda_p = 0.05$.
Further $V_{SD1} = V_{SG1}$ gives $V_{SD1} - V_{eff1} = V_{t0p} = 0.62$ V.
Now equation (1) gives:

$$\left(\frac{W}{L}\right)_1 \approx 0.42 \tag{2}$$

Transistor **M2**: $V_{SB2} = V_{S2} - V_{B2} = V_x - V_{DD} = -1.25$ V.

Enclosed page of formulas gives:

$$V_{tp2} = V_{t0p} + \gamma(\sqrt{2\phi_F - V_{SB2}} - \sqrt{2\phi_F}) = 0.62 + 0.41(\sqrt{2.07} - \sqrt{0.82} = 0.8386 \text{ V} \tag{3}$$

$$\left(\frac{W}{L}\right)_2 = \frac{I_D}{\frac{1}{2}\mu_{0p}C_{oxp}V_{eff2}^2(1 + \lambda_p(V_{SD2} - V_{eff2}))} \tag{4}$$

$I_D = 5\ \mu$ A, $\mu_{0p}C_{oxp} = 58.5\ \mu$ A/V^2,
$V_{eff2} = V_{SG2} - V_{tp2} = V_x - V_{bias} - V_{tp2} = 2.05 - 0.6 - 0.8386 = 0.6114$ V ,
$\lambda_p = 0.05$.
Further $V_{SD2} = V_{SG2}$ gives $V_{SD2} - V_{eff2} = V_{tp2} = 0.8386$ V.
Now equation (4) gives:

$$\left(\frac{W}{L}\right)_2 \approx 0.44 \tag{5}$$

Transistor **M3**: $V_{BS3} = 0$ i.e $V_{tn3} = V_{t0n} = 0.47$.

$$\left(\frac{W}{L}\right)_3 = \frac{I_D}{\frac{1}{2}\mu_{0n}C_{oxn}V_{eff3}^2(1 + \lambda_n(V_{DS3} - V_{eff3}))} \tag{6}$$

$I_D = 5\ \mu$ A, $\mu_{0n}C_{oxn} = 180\ \mu$ A/V^2,
$V_{eff3} = V_{GS3} - V_{t0n} = V_{bias} - 0 - V_{tp2} = 0.6 - 0.47 = 0.13$ V ,
$\lambda_n = 0.03$.
Further $V_{DS3} = V_{GS3}$ gives $V_{DS3} - V_{eff3} = V_{t0n} = 0.47$ V.
Now equation (6) gives:

$$\left(\frac{W}{L}\right)_3 \approx 3.24 \tag{7}$$

Answer: $\left(\frac{W}{L}\right)_1 \approx 0.42$, $\left(\frac{W}{L}\right)_2 \approx 0.44$ and $\left(\frac{W}{L}\right)_3 \approx 3.24$

Exercise 2.

Figure 2 gives the complete small signal equivalent circuit (SSEC):
Notice that:

- $V_{gs1} = V_{g1} - V_{s1} = V_{in}$
- $V_{gs2} = V_{g2} - V_{s2} = V_{in}$
- $V_{gs3} = 0 - V_{s3} = -V_x$
- $V_{gs4} = 0 - V_{s4} = -V_y$

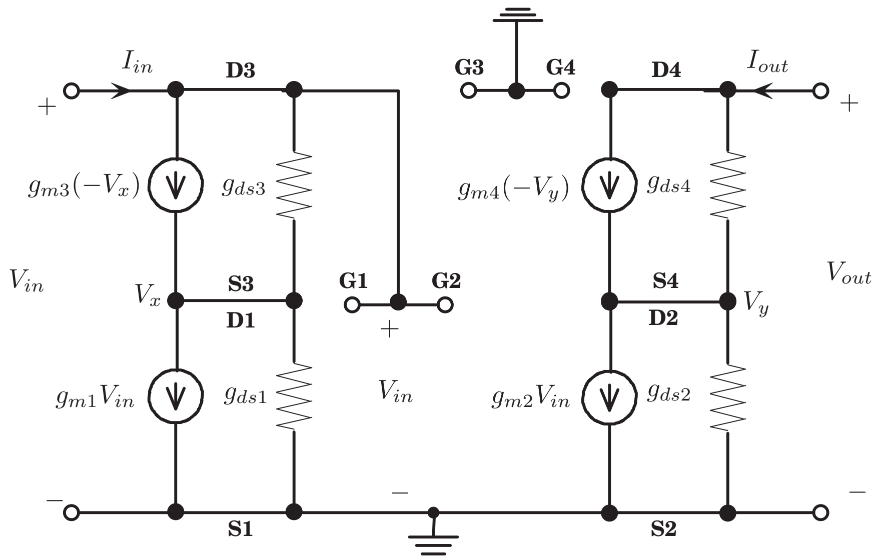

Figure 2: Small signal equivalent circuit.

<u>Determine r_{in}</u>
KCL gives:

$$I_{in} = g_{m3}(-V_x) + (V_{in} - V_x)g_{ds3} \quad (8)$$
$$I_{in} = V_{in}g_{m1} + V_x g_{ds1} \quad (9)$$

Equations (8) and (9) give the input resistance:

$$\underline{\underline{r_{in} = \frac{V_{in}}{I_{in}} = \frac{g_{ds1} + g_{ds3} + g_{m1}}{g_{m1}(g_{ds3} + g_{m3}) + g_{ds1}g_{ds3}}}} \quad (10)$$

<u>Determine r_{out}</u> (OBS! Put V_{in} to zero when calculating r_{out})
KCL gives:

$$I_{out} = g_{m4}(-V_y) + (V_{out} - V_y)g_{ds4} \quad (11)$$
$$I_{out} = V_{in}g_{m2} + V_y g_{ds2} \quad (12)$$

Putting $V_{in} = 0$ gives the output resistans:

$$\underline{\underline{r_{out} = \frac{V_{out}}{I_{out}} = \frac{g_{ds2} + g_{ds4} + g_{m4}}{g_{ds2}g_{ds4}}}} \quad (13)$$

Exercise 3.

a) This exercise is solved using the charge redistribution analysis. First, the reference direction of the charge is chosen. Next, the charge of the capacitors are computed for time t, $t+\tau$, and $t+2\tau$.

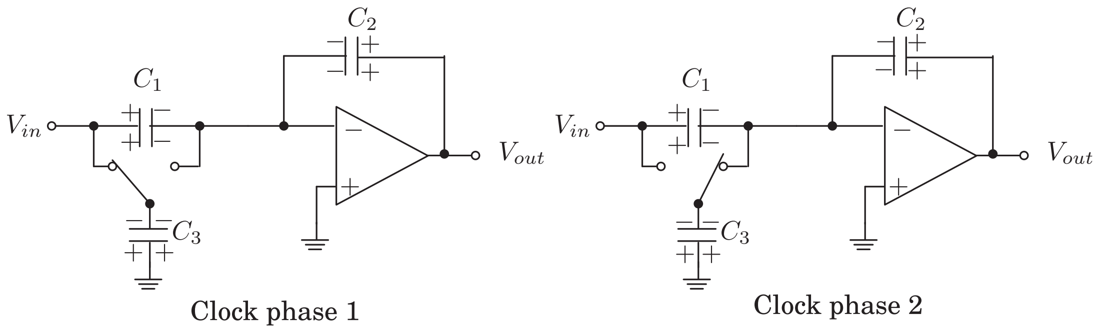

Figure 3: A switched-capacitor circuit in clock phase 1 and clock phase 2.

For time t:

$$\begin{aligned} q_1(t) &= (V_{in}(t) - 0)C_1 \\ q_2(t) &= (V_{out}(t) - 0)C_2 \\ q_3(t) &= (0 - V_{in}(t))C_3 \end{aligned} \tag{14}$$

For time $t+\tau$:

$$\begin{aligned} q_1(t+\tau) &= (V_{in}(t+\tau) - 0)C_1 \\ q_2(t+\tau) &= (V_{out}(t+\tau) - 0)C_2 \\ q_3(t+\tau) &= (0 - 0)C_3 \end{aligned} \tag{15}$$

For time $t+2\tau$:

$$\begin{aligned} q_1(t+2\tau) &= (V_{in}(t+2\tau) - 0)C_1 \\ q_2(t+2\tau) &= (V_{out}(t+2\tau) - 0)C_2 \\ q_3(t+2\tau) &= (0 - V_{in}(t+2\tau))C_3 \end{aligned} \tag{16}$$

Equations for the charge conservation:

1)The capacitors can not discharge through the opamp, so the total charge on the capacitors att the end of clock cycle 1 is equal to the total charge on the capacitors during the whole clock cycle 2:

$$-q_1(t) - q_2(t) - q_3(t) = -q_1(t+\tau) - q_2(t+\tau) - q_3(t+\tau) \tag{17}$$

2) Since no charge can be given by opamp the total charge on C_1 and C_2 must be the same at $t+2\tau$ as at $t+\tau$:

$$-q_2(t+\tau) - q_3(t+\tau) = -q_2(t+2\tau) - q_3(t+2\tau) \tag{18}$$

Equations (14) and (15) together with equation(17) give:

$$V_{in}(t)C_1 + V_{out}(t)C_2 - V_{in}(t))C_3 = V_{in}(t+\tau)C_1 + V_{out}(t+\tau)C_2 \tag{19}$$

Equations (15) and (16) together with equation(18) give:

$$V_{in}(t+\tau)C_1 + V_{out}(t+\tau)C_2 = V_{in}(t+2\tau)C_1 + V_{out}(t+2\tau)C_2 \tag{20}$$

Equations (19) and (20) yields:

$$V_{in}(t)C_1 + V_{out}(t)C_2 - V_{in}(t))C_3 = V_{in}(t+2\tau)C_1 + V_{out}(t+2\tau)C_2 \tag{21}$$

Setting $2\tau = T$ gives the differens equation:

$$C_1V_{in}(t) + C_2V_{out}(t) - C_3V_{in}(t) = C_1V_{in}(t+T) + C_2V_{out}(t+T) \tag{22}$$

Z-transforming (22) gives:

$$C_1V_{in}(z) + C_2V_{out}(z) - C_3V_{in}(z) = C_1zV_{in}(z) + C_2zV_{out}(z) \tag{23}$$

Rewriting (23):

$$V_{out}(z) = \frac{C_1 - C_3 - zC_1}{-C_2 + zC_2} \cdot V_{in}$$

Using that $C_3 = 2C_1$ finally gives:

$$\underline{\underline{V_{out}(z) = -\frac{C_1(z+1)}{C_2(z-1)} \cdot V_{in}}} \tag{24}$$

b) Switches, capacitors, and the operational amplifier introduce parasitic capacitors into the circuit as is shown in Figure 4.

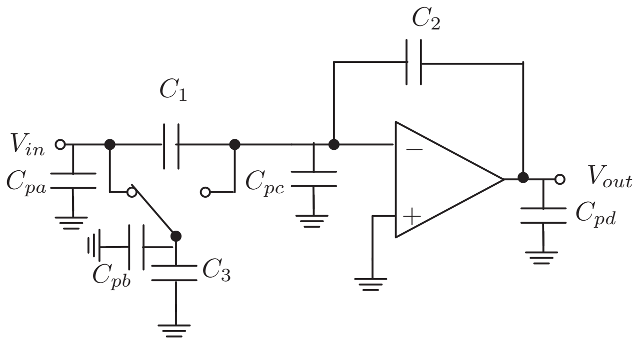

Figure 4: SC-circuit with parasitics in clock phase 1.

- C_{pa} is connected between the ideal input voltage source and ground where the input source can source/sink as much charge as is required. Hence, this parasitics do not change the transfer function.
- C_{pb} is in parallel with C_3 and thereby it **will change** the transfer function.
- C_{pc} is connected between ground and virtual ground thereby not change the transfer function.
- C_{pd} is always connected to the ideal output of the operational amplifier and ground and thereby will not be a part of the transfer function.

Hence, the circuit **is sensitive** to capacitive parasitics when the transfer function is of concern.

Exercise 4.

a) Using KCL in node **A** and node **B** respectively in **Figure 5** gives:

$$\begin{aligned}
\textbf{A}: \quad & g_{mI}V_{in} + (g_I + sC_I)V_x + sC_c(V_x - V_{out}) &= 0 \qquad (25)\\
\textbf{B}: \quad & g_{mII}V_x + (g_{II} + sC_{II})V_{out} + sC_c(V_{out} - V_x) &= 0 \qquad (26)
\end{aligned}$$

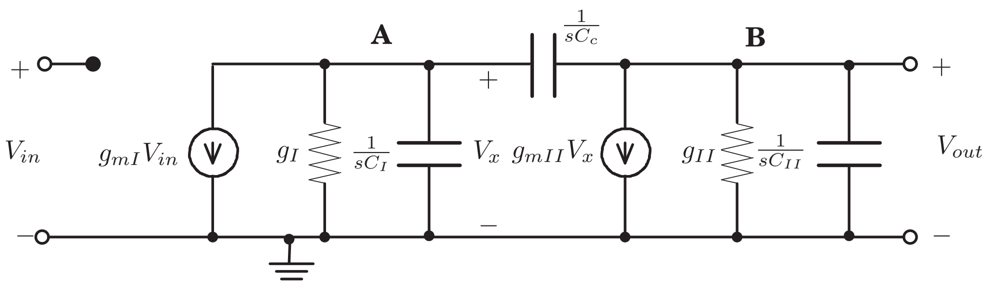

Figure 5: A small-signal model of a two-stage OTA.

Equation (26) gives:

$$V_x = -\frac{g_{II} + sC_{II} + sC_c}{g_{mII} - sC_c} \qquad (27)$$

Equation (27) inserted in (25) gives the transfer function:

$$H_(s) = \frac{V_{out}}{V_{in}} = \frac{g_{mI}(g_{mII} - sC_c)}{g_I g_{II} + s((C_{II} + C_c)g_I + (C_I + C_c)g_{II} + C_c g_{mII}) + s^2(C_I C_{II} + C_c(C_I + C_{II}))} \qquad (28)$$

b) Assuming that $g_{mII} >> g_I$, $g_{mII} >> g_{II}$, $g_{mII} >> g_I$, $C_c >> C_I$, $C_{II} >> C_I$ and that C_{II} and C_c is of the same order, equation (28) gives:

$$H_(s) = \frac{V_{out}}{V_{in}} = \frac{g_{mI}(g_{mII} - sC_c)}{g_I g_{II} + sC_c g_{mII} + s^2 C_c C_{II}} \qquad (29)$$

$s = 0$ in equation (29) gives the DC gain:

$$A_0 = \frac{g_{mI} g_{mII}}{g_I g_{II}} \qquad (30)$$

The nominator i equation (29) gives one zero z_1 at

$$z_1 = \frac{g_{mII}}{C_c} \qquad (31)$$

To determine the poles look at the denominator $N(s)$ of equation (29):

$$N(s) = g_I g_{II} + sC_c g_{mII} + s^2 C_c C_{II} = C_c C_{II}(\frac{g_I g_{II}}{C_c C_{II}} + s\frac{g_{mII}}{C_{II}} + s^2) \qquad (32)$$

With the poles p_1 and p_2 the denominator can be written as:

$$C_c C_{II}(s - p_1)(s - p_2) = C_c C_{II}(p_1 p_2 - (p_1 + p_2)s + s^2) \qquad (33)$$

As $|p_2| >> |p_1|$ equation (33) can be approximated as:

$$C_c C_{II}(p_1 p_2 - (p_1 + p_2)s + s^2) \approx C_c C_{II}(p_1 p_2 - p_2 s + s^2) \tag{34}$$

Identification between equations (32) and (34):

$$p_2 = -\frac{g_{mII}}{C_{II}} \tag{35}$$

$$p_1 p_2 = \frac{g_I g_{II}}{C_c C_{II}} \Rightarrow p_1 = -\frac{g_I g_{II}}{g_{mII} C_c} \tag{36}$$

As $|p_2| >> |p_1|$ the unity-gain frequency ω_u is approximately given by $\omega_u \approx A_0|p_1|$ i.e.

$$\omega_u \approx A_0|p_1| = \frac{g_{mI} g_{mII}}{g_I g_{II}} \cdot \frac{g_I g_{II}}{g_{mII} C_c} = \frac{g_{mI}}{C_c} \tag{37}$$

Answer:

$$A_0 = \frac{g_{mI} g_{mII}}{g_I g_{II}},\ z_1 = \frac{g_{mII}}{C_c},\ p_1 = -\frac{g_I g_{II}}{g_{mII} C_c},\ p_2 = -\frac{g_{mII}}{C_{II}},\ \omega_u \approx \frac{g_{mI}}{C_c} \tag{38}$$

Exercise 5.

a) **Figure 6a** gives a small-signal equivalent circuit, including noie-sources.

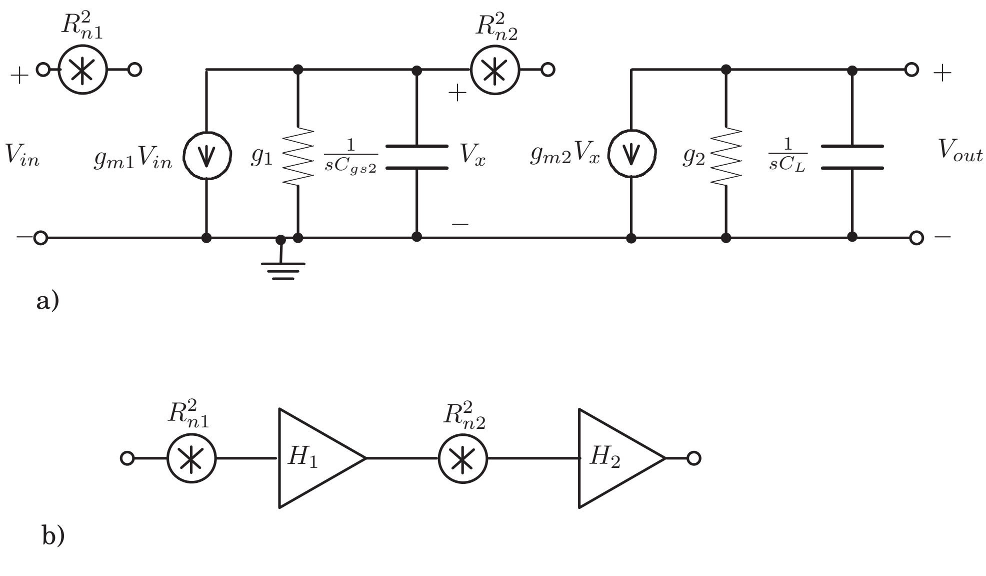

Figure 6: a) A small-signal equivalent. b) Equivalent circuit for determing output noise spectral density

As the noise sources are uncorrelated the output noise spectral density can be computed as **Figure 5b)** discribes, i.e. by the following formula

$$R_{out}(\omega) = |H_1(\omega)|^2 |H_2(\omega)|^2 R_{n1}(\omega) + |H_2(\omega)|^2 R_{n2}(\omega) \tag{39}$$

where $H_1(\omega)$ is the transfer function for the first stage and $H_2(\omega)$ the transfer function for the second stage.

From **Figure 5a)** $H_1(s) = V_x(s)/V_{in}(s)$ and $V_x(s) = -g_{m1}V_{in}(s) \cdot \frac{1}{g_{ds1}+sC_{gs2}}$ which yiels

$$H_1(s) = -\frac{g_{m1}}{g_{ds1} + sC_{gs2}} \Rightarrow H_1(\omega) = -\frac{g_{m1}}{g_{ds1} + j\omega C_{gs2}} \tag{40}$$

In the same way $H_2(s)$ is calculated to:

$$H_2(s) = -\frac{g_{m2}}{g_{ds2} + sC_L} \Rightarrow H_2(\omega) = -\frac{g_{m2}}{g_{ds2} + j\omega C_L} \tag{41}$$

Equations (39)-(41) gives following spectral density of the output noise (here we also utilize that $g_{m1} = g_{m2} = g_m$ and $g_{ds1} = g_{ds2} = g_{ds}$):

$$R_{out}(\omega) = R_{n1}(\omega)\frac{g_m^2}{g_{ds}^2 + \omega^2 C_{gs2}^2} \cdot \frac{g_m^2}{g_{ds}^2 + \omega^2 C_L^2} + R_{n2}(\omega)\frac{g_m^2}{g_{ds}^2 + \omega^2 C_L^2} \tag{42}$$

Using that $R_{n1}(\omega) = R_{n2}(\omega) = \frac{8kT}{3}\frac{1}{g_m}$ (from enclosed page of formulas) equation (42) gives:

$$R_{out}(\omega) = \frac{8kT}{3} \cdot \frac{1}{g_m} \cdot \frac{g_m^2}{g_{ds}^2 + \omega^2 C_L^2}\left(\frac{g_m^2}{g_{ds}^2 + \omega^2 C_{gs2}^2} + 1\right) \tag{43}$$

Which gives the answer:

$$\underline{\underline{R_{out}(\omega) = \frac{8kT}{3} \cdot \frac{g_m}{g_{ds}^2 + \omega^2 C_L^2}\left(\frac{g_m^2}{g_{ds}^2 + \omega^2 C_{gs2}^2} + 1\right)}} \tag{44}$$

b) Equation (43) gives the amplitude spectrum of the circuit:

$$|H(\omega)| = \left(\frac{g_m^2}{g_{ds}^2 + \omega^2 C_L^2}\left(\frac{g_m^2}{g_{ds}^2 + \omega^2 C_{gs2}^2} + 1\right)\right)^{\frac{1}{2}} \tag{45}$$

Eqn.(45) shows that we have a lowpass-filter with maximum at $\omega = 0$ and that

$$|H(0)| = \left(\frac{g_m^2}{g_{ds}^2}\left(\frac{g_m^2}{g_{ds}^2} + 1\right)\right)^{\frac{1}{2}} \tag{46}$$

The 3 dB cut-off frequency ω_0 determins of the relation $|H(\omega_0)| = \frac{1}{\sqrt{2}} \cdot |H(\omega)|_{max}$ i.e. $|H(\omega_0)| = \frac{1}{\sqrt{2}} \cdot |H(0)|$ in our example. For convenience in this example we can rewrite this condition as:

$$|H(\omega_0)|^2 = \frac{1}{2} \cdot |H(0)|^2 \tag{47}$$

Eqns. (45)-(47) give:

$$\frac{g_m^2}{g_{ds}^2 + \omega_0^2 C_L^2}\left(\frac{g_m^2}{g_{ds}^2 + \omega_0^2 C_{gs2}^2} + 1\right) = \frac{1}{2} \cdot \frac{g_m^2}{g_{ds}^2}\left(\frac{g_m^2}{g_{ds}^2} + 1\right) \tag{48}$$

Rewriting eqn. (48) gives:

$$\frac{1}{1 + \left(\frac{\omega_0 C_L}{g_{ds}}\right)^2}\left(\frac{g_m^2}{g_{ds}^2\left(1 + \left(\frac{\omega_0 C_{gs2}}{g_{ds}}\right)^2\right)} + 1\right) = \frac{1}{2} \cdot \left(\frac{g_m^2}{g_{ds}^2} + 1\right) \tag{49}$$

As $C_L >> C_{gs2}$ we put C_{gs2} to zero which gives the approximation:

$$\frac{1}{1+\left(\frac{\omega_0 C_L}{g_{ds}}\right)^2} \approx \frac{1}{2} \tag{50}$$

Eqn. (5) gives the 3 dB cut-off frequency:

$$\omega_0 \approx \frac{g_{ds}}{C_L} \Rightarrow f_{3dB} \approx \frac{g_{ds}}{2\pi C_L} \tag{51}$$

The noise-bandwidth concept gives that:

$$P_{out,noise} \approx R_{out} \cdot |H(\omega_0)|^2 \cdot \frac{\pi}{2} \cdot f_{3dB} = \frac{8kT}{3}\frac{1}{g_m} \cdot \frac{g_m^2}{g_{ds}^2}\left(\frac{g_m^2}{g_{ds}^2}+1\right) \cdot \frac{\pi}{2} \cdot \frac{g_{ds}}{2\pi C_L} \tag{52}$$

Which gives the answer:

$$\underline{\underline{P_{out,noise} \approx \frac{2kT}{3} \cdot \frac{g_m}{g_{ds}}\left(\frac{g_m^2}{g_{ds}^2}+1\right) \cdot \frac{1}{C_L}}} \tag{53}$$

c) As $g_m \sim \sqrt{I_D}$ and $g_{ds} \sim I_D$ and $I_D = I_{bias}$ in this example we have:

$$P_{out,noise} \sim \frac{1}{\sqrt{I_{bias}}}\left(\frac{1}{I_{bias}}+1\right) \tag{54}$$

Obviously $P_{out,noise}$ will decrese if I_{bias} is increased.
The DC gain is

$$|H(0)| = \left(\frac{g_m^2}{g_{ds}^2}\left(\frac{g_m^2}{g_{ds}^2}+1\right)\right)^{\frac{1}{2}}$$

I.e.

$$|H(0)|^2 \sim \frac{1}{I_{bias}}\left(\frac{1}{I_{bias}}+1\right) \tag{55}$$

Which shows that also the DC gain will decrease if I_{bias} is increased.