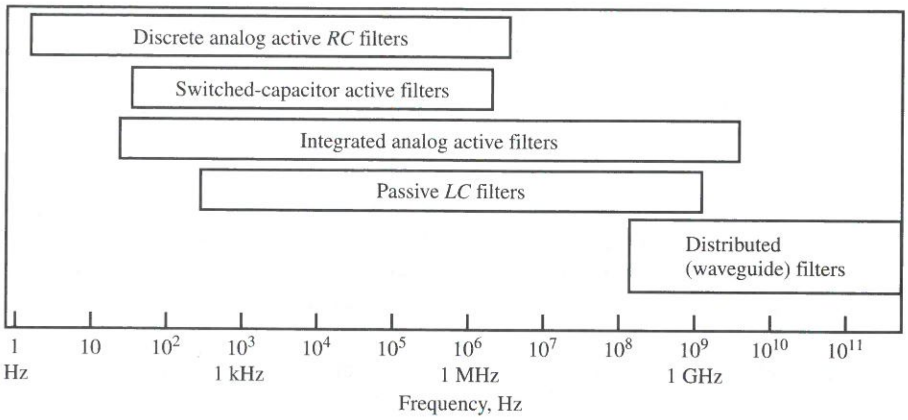

Figure 1.5 Choice of filter type as a function of the operating frequency range.

The meaning of several of these criteria will become clear as we progress in our discussion of active filters. Some guidelines for a possible choice of filter type can be obtained from Fig. 1.5 as a function of the desired frequency range of operation. The range of *LC* filters is limited at the low end by the bulk of the inductors and for high frequencies by parasitic and distributed effects. We see that compared to passive *LC* filters, discrete opamp-based active filters can realize filters for lower frequencies, but not for higher frequencies, whereas integrated analog filters, depending on the design and the type of devices used, can span the range from low audio frequencies to the gigahertz range. Switched-capacitor filters will be seen to be limited in their application range from about 10 Hz to about 1 MHz by impractical element sizes and by the bandwidth of the active devices. Microwave filters cover the highest frequency range by relying on distributed elements and waveguide designs. The indicated limitations of active filters depend, of course, on the active devices used: opamps or OTAs. The limits may be temporary and will change as technology advances and faster active devices become available.

If sensitivity to component variations and fabrication tolerances is important, passive *LC* filters often have an advantage. We will consider this aspect in Chapter 13. Although still used in large numbers, their design is not compatible with modern fully integrated systems. As our discussion progresses, we will learn that to address this difficulty many methods have been developed to simulate the performance of *LC* filters with active circuitry.

Finally, active filters require, of course, power supplies. The power supply voltages range anywhere from 1 V to 15 V, with typical designs at the time of this writing at or below 3.3 V. You will have learned in basic electronics courses that as the power supply voltage for biasing the active elements shrinks, so will the linear range over which the active devices can be used. Consequently, the usable linear signal level becomes smaller with reduced power supply voltages. Since active devices generate noise, which limits the smallest signals that can be processed, dynamic range becomes a serious concern for the designer. Dynamic range is defined as the difference between the largest undistorted signal and the noise level.

1.4 CIRCUIT ELEMENTS AND SCALING

It will become clear as our study progresses that filter design is primarily a frequency-domain matter and that we seldom make reference to time-domain quantities, such as rise time or overshoot. Design specifications or physical measurements are made in terms of frequency f in Hertz. However, it turns out to be much more convenient to use radian frequencies ω in rad/s rather than f. We will follow this practice and use ω as long as possible and only convert to f in the last step. Experience in design will show the advantages of this choice.

We will make extensive use of both magnitude and frequency scaling, and also of normalized element values as well as normalized values of frequency. This has several reasons. It avoids the need to use very small or very large component values, such as pF (10^{-12} F) capacitors and MΩ (10^6 Ω) resistors. It permits us to design filters whose critical specifications are on the frequency axis "in the neighborhood" of $\omega = 1$ rad/s. Further, it permits us to deal with only dimensionless specifications and components without having to be concerned with units, such as Hz, Ω, F, or H. Finally, the most important reason is that much of the work of filter designers is based on the use of design tables. In these tables so-called "prototype" lowpass transfer functions are assumed to have a passband along the *normalized* frequency ω in $0 \le \omega \le \omega_p = 1$ and a stopband in $1 < \omega_s < \omega < \infty$. See Fig. 1.3a. In addition, these prototype filters are designed with normalized dimensionless elements from which the real physical components are obtained by denormalization. The relationships between the physical elements R, L, and C and their normalized representations R_n, L_n, and C_n are

$$L_n = \frac{\omega_S}{R_S} L, \quad C_n = \omega_S R_S C, \quad R_n = \frac{1}{R_S} R \qquad \text{(1.13a, b, c)}$$

In Eq. (1.13), R_S is an arbitrary scaling resistor (in Ohms) that normalizes the impedance level and ω_S is the radian frequency (in rad/s) that scales and normalizes the frequency axis, such that $\omega/\omega_S = 1$, usually, at the passband corner. These expressions, as well as their inverses,

$$L = \frac{R_S}{\omega_S} L_n, \quad C = \frac{1}{\omega_S R_S} C_n, \quad \text{and} \quad R = R_S R_n \qquad \text{(1.14a, b, c)}$$

are easy to remember by noting that R, L, and C have units of Ω, H, and F whereas R_n, L_n, and C_n are dimensionless numbers. Thus, the scaling factors ω_S and R_S do not only change the numerical values of the elements or frequency parameters, they can be seen to remove or restore the units depending on the direction of the transformation. As an example, assume that a prototype filter was designed, and design tables indicate that $R_n = 1$, $L_n = 3.239$, and $C_n = 1.455$ are the required normalized components. If the impedance level is selected as $R_S = 1{,}200\ \Omega$ and the frequency was normalized by $\omega_S = 10.8$ Mrad/s $= 10{,}800{,}000$ rad/s, we compute the real inductor value from Eq. (1.14a) as

$$L = \frac{1{,}200\ \Omega}{10.8 \times 10^6\ \text{MHz}} 3.239 = 360\ \mu\text{H}$$

Similarly, we find for the other components $R = 1.2$ kΩ and $C = 112$ pF. We still point out that *all* components with physical units (Ω, S, H, F, s, Hz) are scaled, but dimensionless parameters, such as gain, are not. Thus, in the above example where $R_S = 1{,}200\ \Omega$ was chosen as the resistor to scale the impedance level, a transconductance of value $g_m = 245\ \mu$S is normalized to

$$g_{m,n} = R_S g_m = 1200\ \Omega \times 245\ \mu\text{S} = 0.294$$

but an amplifier gain of value $K = 45$ dB keeps its value K in the normalized circuit.

TABLE 1.2 Typical Component Values in Discrete and Integrated Realizations

	Discrete	Integrated
Tolerances	1–20%	10–40% absolute 0.1–1% for ratios
Resistors		
Preferred range	1–100 kΩ	Process dependent: values with 10% to 30% tolerances in the range of 50 Ω–1 kΩ
Lower limit	0.05–1 kΩ	
Upper limit	100–500 kΩ	
Capacitors		
Readily realizable	5 pF–1 μF	0.5–5 pF
Practical	0.5 pF–10 μF	0.2–10 pF
Marginally practical	0.2 pF–500 μF	0.1–50 pF
Inductors		
Readily realizable	1 μH–10 mH	Real inductors with large losses of the order of 10 nH or less
Practical	0.1 μH–50 mH	
Marginally practical	100 nH–1 H	

It will become clear in the chapters to follow that ordinarily there is no unique solution to the design problem. One of the decisions that the designer has to make is that of element size. Making appropriate choices will become easier with experience, and selecting a suitable impedance normalization factor, R_S, will help. Table 1.2 serves in guiding the selection. Whether an element value is conveniently realizable depends on the chosen technology; here we distinguish between discrete filter designs and filters to be implemented on integrated circuits. Note that in integrated circuits, *ratios* of *like* components can be very accurate with careful layout and processing, but untuned absolute values of components can have very large tolerances. This is the reason why the parameters of integrated-circuit opamps are not predictable with any accuracy or reliability and why in the design of active filters it is very important to make the filter independent of the opamp parameters. Practical sizes of resistors and capacitors are limited by the available silicon area on an IC chip. Integrated inductors are very small and at the same time very lossy. Simulated inductors can be larger and less lossy, but they add noise; it is not difficult to implement a simulated inductor in the range of many mH or even H.

The design of active filters generally requires accurate components. Typically, resistors with 1 or 5% tolerances are used in discrete circuits, more rarely in less critical applications 10 or 20% resistors will suffice. On the other hand, capacitors with 10 or even 20% tolerances are more readily available and are preferred to save cost. As a rule, suitable capacitor values are preselected, such as 0.1 or 0.01 μF, because fewer standard capacitor values are available for the filter designer to choose from. It makes little sense to compute a capacitor for a specified filter to three or more digits and then find out that no company manufactures that capacitor. The resistors needed for the filter are determined from these predetermined values and a specified frequency. For example, frequency is set by an RC product as $f_0 = 1/(RC)$; then, for $f_0 = 12$ kHz and choosing $C = 0.01\mu$F we find $R = 1/(2\pi f_0 C) = 1.326$ kΩ. The next closest 1% resistor can be chosen for the design. If the resulting tolerances of the RC product are too large, the resistor must be trimmed. We should note that the fact that components with at best 1% tolerances are available to the filter designer does *not* mean that the computations leading to the element values can be carried out to only two or three significant digits. The numerical mathematics in filter design is as a rule very ill conditioned, especially for high-order filters, so many digits should be retained in the calculations to achieve valid results. The problem is that many intermediate results involve small differences of two relatively large numbers. For instance, suppose a step in the algebra calls for the difference of

$$1.324495 - 1.323122 = 0.001373$$

Being mislead by the available 1% components, a designer may choose to carry only three digits, $1.32 - 1.32 = 0.00$, clearly a meaningless result. Even computing to four digits, $1.324 - 1.323 = 0.001$, leaves only one digit, which has a 38% error. Let us emphasize, therefore, that computations in filter design must be carried out with 7 to 10 or, for high-order filters, even more digits. We shall throughout this text carry out all computations to the required accuracy, but keeping practice in mind, use element values to only two or three digits. If circuit performance calls for higher accuracy tuning will be assumed in our designs.

PROBLEMS

1.1 The input voltage of a filter is $v_i(t) = \sqrt{2}\cos(\omega t + 2.68)$ and its output voltage is $v_2(t) = \sqrt{2} \cdot 5.34 \cos(\omega t + 4.87)$. At the applied frequency ω, determine the gain in dB and the phase shift in degrees implemented by the filter.

1.2 At the frequency $f = 12$ kHz, a filter is designed to attenuate the input signal by 78 dB. Find the amplitude of the output signal if the 12-kHz input has an amplitude of 1 V.

1.3 A wide-band input signal of amplitude 100 mV is applied to the filter. In the stopband, the remaining signal components at the filter's output must be no larger than 45 μV. Determine the required stopband attenuation α of the filter in dB.

1.4 If an amplifier has 35 dB gain at $f_c = 100$ MHz and shifts the phase by $-42°$, determine the output signal delivered if the input is $v_{in}(t) = 2.4\cos(\omega_c t + 45°)$.

1.5 Identify the filter type (lowpass, bandpass, etc.) described by the following attenuation specifications and calculate the widths of the transition band(s).

(a) $\alpha_{max} = 0.01$ dB in $f \le 3.4$ kHz; $\alpha_{min} = 45$ dB in 9.6 kHz $\le f \le \infty$

(b) $\alpha_{max} = 0.01$ dB in 12.5kHz $\le f \le 24$ kHz; $\alpha_{min} = 45$ dB in $f \le 7$ kHz and $f \ge 40$ kHz

(c) $\alpha_{min} = 85$ dB in 12.5 kHz $\le f \le 24$ kHz; $\alpha_{max} = 1$ dB in $f \le 7$ kHz and $f \ge 40$ kHz

(d) $\alpha_{min} = 60$ dB in $f \le 24$ kHz; $\alpha_{max} = 0.5$ dB in $f \ge 40$ kHz

(e) $\alpha_{max} = 0.1$ dB in $f \le 360$ kHz; $\alpha_{min} = 80$ dB in 600 kHz $\le f$

(f) $\alpha_{max} = 3$ dB in 1 MHz $\le f \le 2.4$ MHz; $\alpha_{min} = 75$ dB in $f \le 730$ kHz and $\alpha_{min} = 48$ dB in $f \ge 7.8$ MHz

1.6 The transfer function of a filter is specified to equal

$$T(s) = \frac{2\left(s^2 + 9.32\right)}{s^4 + 1.322s^3 + 0.976s^2 + 0.750s + 1}$$

The frequency is normalized by $f_0 = 18$ kHz. Determine the gain in dB at dc. Calculate the rate of attenuation increase in dB per decade at high frequencies. At which frequencies is the attenuation infinite?

1.7 According to a design table, the normalized components of a passive LC filter are $L_1 = 1.2547$, $L_2 = 0.9873$, $L_3 = 0.8765$, $C_1 = 2.5632$, $C_2 = 1.5764$, and $R_S = R_L = 1$. The impedance level is normalized by $R_0 = 300\ \Omega$ and the normalizing frequency is $f_0 = 10.8$ MHz. Find the values of the denormalized components.

1.8 The normalized components of an active filter were computed to be $R_1 = 1.243$, $R_2 = R_3 = 1.677$, $R_4 = 6.888$, and $C_1 = C_2 = 0.765$; the amplifier gain is required to be $K = 1.93$. The normalizing frequency is $f_0 = 360$ kHz. Choose the impedance level such that the filter can be built with $C = 0.05$ nF capacitors and determine the remaining elements of the circuit, including the final value of amplifier gain.

1.9 Calculate the rate of attenuation increase in dB/octave and in dB/decade as f approaches zero and infinity in the function