-Solutions -
Continuous-time Filters

Description

This chapter contains the solutions for ATIK lessons on continuous-time filters. The relevant theory as well related/identical exercises can be found in [1], [2]. The exercises are intended to stress the advantages of active-filter circuits, methodology of translating a given filter specification into an opamp based circuit. The exercises also draw attention to the sensitiviy of filter structures to element mismatches. Along with each solution, the pertinent sections in the reference text books are indicated as an aid to the students.

Contents

History

Rev	Date	Comments	Issued by
PA1	2012-08-31	Created and released.	Prakash Harikumar

1 First-order filter

The related theory for this exercise is given in pp. 87-88 of [1]. Following are the advantages of active-filter implementation

- Active filters can provide gain which passive structures are unable to do

- Higher-order filters can be realized by direct-cascading of lower order filters without any need for buffering. For cascading of passive filters, buffering is essential to provide isolation

- In active filters, poles and zeros can be placed independent of each other

- There are no restrictions on the pole or zero locations

- Ideal inverting integrator and differentiator can be realized only with active elements.

2 Bilinear transfer function

The related theory and numerical examples for this exercise are provided in pp 84-90 of [1]. The bilinear transfer function is defined as

$$T(s) = K\frac{s + z_1}{s + p_1} \tag{1}$$

Following can be said about the circuit diagrams (a)-(d) provided in the question

- Circuit (a) and (d) realize the same bilinear transfer function given by (1). But circuit (d) is more preferable for implementing the given specification since DC biasing of the opamp is facilitated in circuit (d) by the feedback resistor which connects the input and output terminals.In both the circuits, the pole and zero can be set independently anywhere on the negative real axis.

- Circuits (b) and (c) realize second-order functions. The manner in which the capacitors are connected determines the order of the transfer function realized.

- Transfer function of circuit (c) is given by (2).It has two negative real poles and zeros at origin and infinity. Circuit (c) is a band-pass filter.

$$T(s) = -\frac{sC_1R_2}{(sC_1R_1 + 1)(sC_2R_2 + 1)} \tag{2}$$

- Transfer function of circuit (b) is given by (3). From (3), it is seen that the circuit has poles at $s = 0$ and $s = \infty$. The circuit provides infinite gain to the input at these two frequencies and hence is highly nonlinear and thus useless from a theoretical perspective. The result of (3) is based on an unrealistic device model and hence not valid.

$$T(s) = -\frac{(sC_1R_1 + 1)(sC_2R_2 + 1)}{sC_2R_1} \tag{3}$$

An active circuit realization of (1) is shown in Fig. 1. For the circuit in Fig. 1, the transfer function is given by

$$T(s) = -\frac{C_1}{C_2}\frac{s + G_1/C_1}{s + G_2/C_2} \tag{4}$$

from which we can identify $z_1 = G_1/C_1 = 1/C_1R_1$, $p_1 = G_2/C_2 = 1/C_2R_2$ and $K = C_1/C_2$. We are given high-frequency gain = $20\ dB$ and $z_1 = 1\ KHz$, $p_1 = 10\ KHz$. $K = 10^{(20/20)} = 10$. The bilinear transfer function for the given specification can be written as

$$T(s) = -10\frac{s + 2\pi \cdot 1000Hz}{s + 2\pi \cdot 10,000Hz} \tag{5}$$
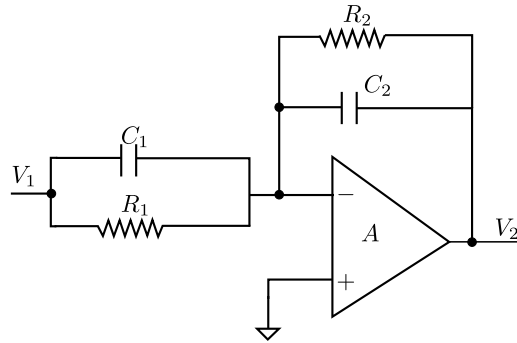
Figure 1: Active realization of bilinear transfer function

It is common practice to apply normalization the the element values and frequencies. The benefits and method of normalization is found in pp. 11 of [1]. If the frequency parameter is scaled by $\omega_s = 2\pi \cdot 1000$ rad/s, the result is

$$T(s) = -10\frac{s_n + 1}{s_n + 10} \tag{6}$$

where $s_n = s/\omega_s$. Then we have normalized values given by $1/R_{1n}C_{1n} = 1$, $1/R_{2n}C_{2n} = 10$, $C_{1n}/C_{2n} = 10$. Let $C_1 = 50 \; nF$ and recalling that the frequency is normalized by $\omega_s = 2\pi \cdot 1000 \; rad/s$, we have

$$R_1 = \frac{1}{2\pi \cdot 1000 \cdot 1 \cdot 50 \; nF} = 318 \; \Omega \tag{7}$$

$$C_2 = C_1/10 = 5 \; nF \tag{8}$$

$$R_2 = \frac{1}{2\pi \cdot 1000 \cdot 10 \cdot 5 \; nF} = 318 \; \Omega \tag{9}$$

We see that at the origin (DC), $T(0) = -R_2/R_1 = -1$ and at high frequencies $T(\infty) = -C_1/C_2 = -10$. The circuit in Fig. 1 is an inverting, high-pass filter.

3 Higher-order filters starting point

3.1 Effect of cascading

A cascade of passive RC-sections is shown in Fig. 2. Considering the effect of loading, we have

$$\frac{V_3}{V_1} = T_1(s)T_2(s)\frac{Z_{i2}(s)}{Z_{i2}(s) + Z_{o2}(s)}, \tag{10}$$

where

$$T_1(s) = T_2(s) = \frac{1}{1 + sRC} \tag{11}$$

$$Z_{i2}(s) = R + \frac{1}{sC} \tag{12}$$

$$Z_{o1}(s) = \frac{1}{sC + \frac{1}{R}} \tag{13}$$

Thus the transfer function of the passive RC-cascade is given by

$$T(s) = \frac{V_3}{V_1} = \left(\frac{1}{1 + sRC}\right)^2 \frac{R + \frac{1}{sC}}{\left(R + \frac{1}{sC}\right) + \left(\frac{R}{1+sCR}\right)} = \frac{1}{(sRC)^2 + 3sCR + 1} \tag{14}$$

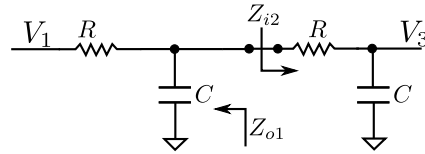| No	Rev	Date		Repo	Page
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Figure 2: Cascade of passive RC-sections

Ideally i.e. for $Z_{i2} = \infty$ and $Z_{o1} = 0$, the transfer function of the cascade is

$$T_1(s) = T_2(s) = \left(\frac{1}{1 + sRC}\right)^2 = \frac{1}{(sRC)^2 + 2sCR + 1} \tag{15}$$

From (15) and (14) we can conclude that the transfer function is significantly altered when loading effect is considered. Thus passive filter sections are not suitable for cascading in order to realize higher-order filters unless buffers (opamps) are utilized to isolate the sections. Active filter sections have the advantage that they can be cascaded directly without the need for buffering.

3.2 Choice of poles

For the effect of loading to be negligible we need to ensure that $Z_{i2} \gg Z_{o1}$. Since we have two poles given by $R_1 C_1$ and $R_2 C_2$,

$$R_2 + \frac{1}{sC_2} \gg \frac{1}{sC_1 + \frac{1}{R_1}} \tag{16}$$

4 Biquads

This exercise is same as Example 3.11 given in pp 107-109 of [1]. From the given break frequencies and the 20 dB/decade, we have $p_1 = 100 \ rad/s$, $p_2 = 10^5 \ rad/s$, $z_1 = 10^3 \ rad/s$, $z2 = 10^4 \ rad/s$. The transfer function for the second-order filter can be written as

$$T(s) = K\frac{(s + z_1)(s + z_2)}{(s + p_1)(s + p_2)} = \frac{(s + 10^3)(s + 10^4)}{(s + 10^2)(s + 10^5)} \tag{17}$$

where K, the DC gain is 1, since $T(0) = 0 \ dB$. $T(s)$ can be written as a product of bilinear functions as

$$T(s) = T_1(s)T_2(s) = \frac{(s + 10^3)}{(s + 10^2)} \cdot \frac{(s + 10^4)}{(s + 10^5)} \tag{18}$$

For realizing the $T_1(s)$ and $T_2(s)$, we can use the inverting opamp circuit shown in Fig. 3. To realize the biquad, we need to cascade two such structures. For the first section,

$$z_1 = \frac{1}{R_{11}C_{11}} = 10^3 \ rad/s \tag{19}$$

$$p_1 = \frac{1}{R_{21}C_{21}} = 10^2 \ rad/s \tag{20}$$

For the second section,

$$z_2 = \frac{1}{R_{12}C_{12}} = 10^4 \ rad/s \tag{21}$$

$$p_2 = \frac{1}{R_{22}C_{22}} = 10^5 \ rad/s \tag{22}$$

Let all capacitors be $0.01 \ \mu F$. Then using (19)-(22), we can find the resistor values to be $R_{11} = 100 \ k\Omega$, $R_{21} = 1 \ M\Omega$, $R_{12} = 10 \ k\Omega$, $R_{22} = 1 \ k\Omega$. The resulting circuit is shown in Fig. 4.

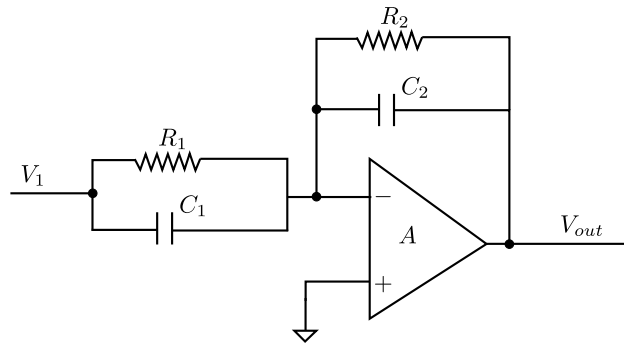| No	Rev	Date		Repo	Page
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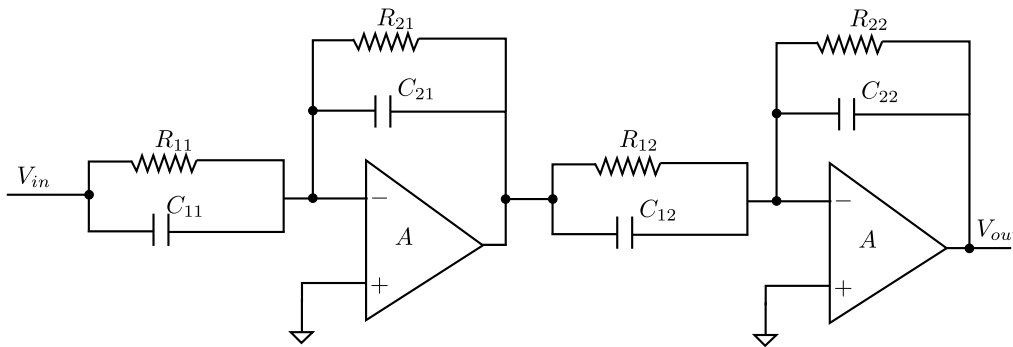INSTITUTE OF TECHNOLOGY

Figure 3: Inverting opamp circuit

Figure 4: Biquad circuit realization

5 Tow-Thomas biquad

The theory and solution for this exercise are provided in pp 129-134 of [1].

5.1 Implement the Tow-Thomas filter circuit

We are provided with the block diagram to realize the transfer function, which should be used to implement the ciruit with real components. We know that it is easy to realize an ideal, inverting integrator but not an ideal non-inverting one. Since negative feedback is needed in all loops to ensure stability, it is not possible to use two inverting integrators directly. In order to solve this difficulty, the block diagram provided in the question is modified as shown in Fig. 5, where a non-inverting integrator is realized as a cascade of an inverting integrator and an inverter. These are shown in Fig. 6 and Fig. 7 respectively. In addition to these circuits, we also need a summer which can merge with the integrator as shown in Fig. 8. The module T_1 shown in Fig. 8 realizes
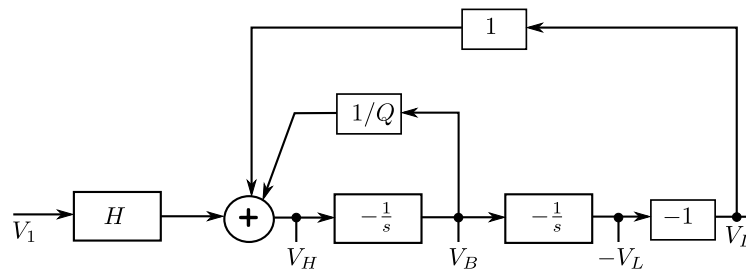

Figure 5: Modified block digram using two inverting integrators and an inverter

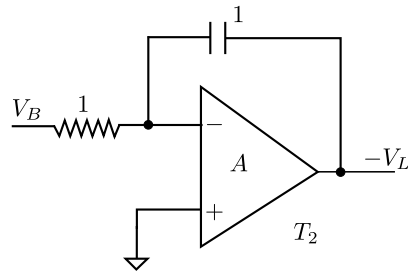| No	Rev	Date		Repo	Page
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Figure 6: Inverting integrator
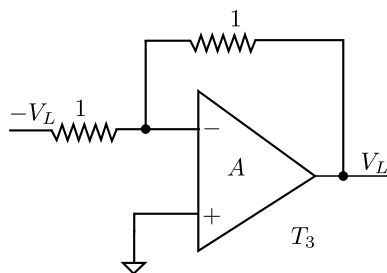
Figure 7: Inverter

Figure 8: Integrator-summer circuit

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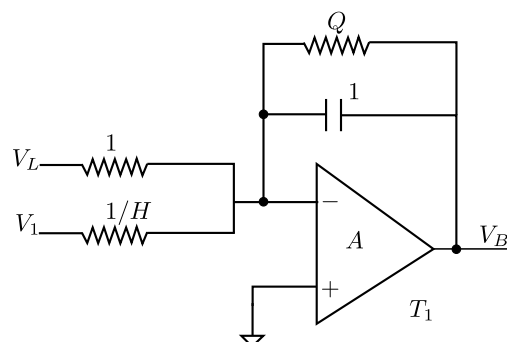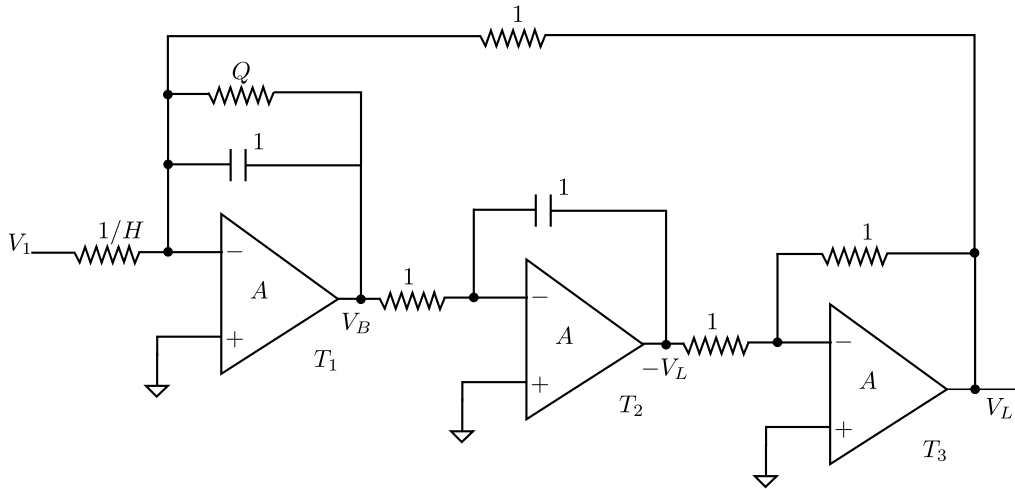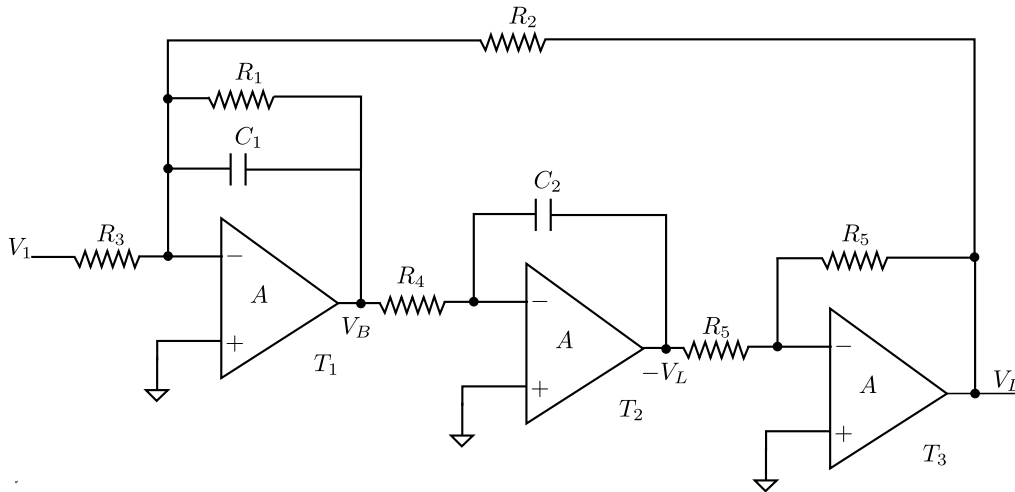
Figure 9: Tow-Thomas biquad with normalized elements

Figure 10: Tow-Thomas biquad

$$V_B = -\frac{1}{s + \frac{1}{Q}}(V_L + HV_1) \tag{23}$$

From Fig. 5 we have $-V_L = -\frac{1}{s}V_B$ and $V_L = (-1)(-V_L)$. The full circuit with normalized elements is shown in Fig. 9. With the circuit elements labeled. the Tow-Thomas biquad is shown in Fig. 10.

5.2 Derive low-pass and band-pass transfer functions

Analysis of Fig. 10, provides the following relations,

$$V_B = -\frac{1}{sC_1 + 1/R_1}\left(\frac{V_L}{R_2} + \frac{V_1}{R_3}\right) \tag{24}$$

$$-V_L = -\frac{1}{sC_2}\frac{V_B}{R_4} \tag{25}$$

$$V_L = -\frac{R_5}{R_5}(-V_L) \tag{26}$$

From (24)-(26), we have,

$$V_L = (-1) \cdot -\frac{1}{sC_2}\frac{V_B}{R_4} = \frac{1}{sC_2R_4} \cdot -\frac{1}{sC_1 + 1/R_1}\left(\frac{V_L}{R_2} + \frac{V_1}{R_3}\right) \tag{27}$$

Re-arranging the terms in (27) we get the low-pass transfer function $T_L(s)$,

$$T_L(s) = \frac{V_L}{V_1} = -\frac{R_1R_2/R_3}{s^2R_1C_1R_2C_2R_4 + sR_2C_2R_4 + R_1} \tag{28}$$

Dividing numerator and denominator of (28) by $R_1C_1R_2C_2R_4$, we get the final form of the low-pass transfer function as

$$T_L(s) = -\frac{1/(R_3R_4C_1C_2)}{s^2 + s/(R_1C_1) + 1/(R_2R_4C_1C_2)} \tag{29}$$

To derive the band-pass transfer function we divide numerator and denominator of (25) by V_1,

$$-\frac{V_L}{V_1} = -\frac{1}{sC_2}\frac{V_B}{R_4V_1} = -\frac{1}{sC_2R_4}\frac{V_B}{V_1} \tag{30}$$

which gives

$$-T_L(s) = -\frac{1}{sC_2R_4}T_B(s) \tag{31}$$

The band-pass transfer function can then found using (31) and (29) as

$$T_B(s) = (-sC_2R_4) \cdot -T_L(s) = -\frac{(R_1/R_3) \cdot s/(R_1C_1)}{s^2 + s/(R_1C_1) + 1/(R_2R_4C_1C_2)} \tag{32}$$

5.3   Derive ω_0^2, Q, H for the low-pass case

In order to derive expressions for ω^2, Q, H we compare (29) with the standard low-pass transfer function given by (33)

$$T_L(s) = \frac{\pm H\omega_0^2}{s^2 + (\omega_0/Q)s + \omega_0^2} \tag{33}$$

We have to determine six circuit elements $(R_1 - R_4, C_1, C_2)$ but have only three parameters (ω_0, H, Q). Hence we arbitrarily select three of the components and then examine the consequences on the remaining three. Since frequency scaling is used, $\omega_0 = 1$ and we intend to use magnitude scaling. Choose $C_1 = C_2 = 1$, $R_4 = 1$. Comparing (29) and (33) we have $\omega_0^2 = 1/R_2$, $Q = R_1/\sqrt{R_2}$ and $H = R_2/R_3$. Since $\omega_0 = 1$, we find that $R_2 = 1, R_1 = Q$, $R_3 = 1/H$.

5.4   Circuit tuning

An important property of this biquad circuit is that it can be orthogonally tuned, i.e.

- R_2 can be adjusted to a specified value of ω_0

- R_1 can then be adjusted to give specified Q value without changing ω_0

- Finally R_3 can be adjusted to give desired value of H (circuit gain) without affecting either ω_0 or Q.

6   Sensitivity analysis

The description of sensitivity is provided in Chapter 12 of [1]. For this exercise, the sensitivty analysis of the inverting amplifier is provided in Example 12.1 (pp. 457-458) of [1] while that of the passive low-pass RLC circuit is provided in Example 12.2 (pp. 460-461) of [1].
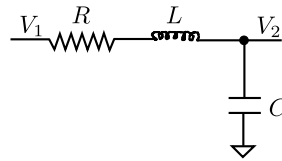
Figure 11: Passive RLC circuit

6.1 Passive RLC low-pass circuit

The passive RLC low-pass circuit is shown in Fig. 11 The transfer function of the circuit in Fig. 11 is

$$T(s) = \frac{V_2}{V_1} = \frac{1/sC}{R + sL + 1/sC} = \frac{1}{s^2 LC + sCR + 1} \tag{34}$$

Comparing (34) with the standard low-pass transfer function given in (33) results in

$$\omega_0 = \frac{1}{\sqrt{LC}} = L^{-\frac{1}{2}} C^{-\frac{1}{2}} \tag{35}$$

$$Q = R^{-1} L^{\frac{1}{2}} C^{-\frac{1}{2}} \tag{36}$$

We have

$$Y = kx^a \Rightarrow S_x^Y = a \tag{37}$$

Utilizing (35), (36) and (37), we can find the sensitivity of ω_0 and Q to R, L and C. We get $S_L^{\omega_0} = -1/2$, $S_C^{\omega_0} = -1/2$ and $S_R^{\omega_0} = 0$. Similarly $S_L^Q = 1/2$, $S_C^Q = -1/2$ and $S_R^Q = -1$. Thus it is clear that

- A $+1\%$ change in L results in a -0.5% change in ω_0 and a $+0.5\%$ change in Q

- A $+1\%$ change in C results in a -0.5% change in ω_0 and a -0.5% change in Q

- A $+1\%$ change in R results in no change in ω_0 and a -0.1% change in Q

These sensitivities are considered low. The circuit is an example of a LC ladder filter. LC ladder filters tend to have very low sensitivities. The total changes are computed as

$$\frac{d\omega_0}{\omega_0} = S_L^{\omega_0} \frac{dL}{L} + S_C^{\omega_0} \frac{dC}{C} + S_R^{\omega_0} \frac{dR}{R} = -\frac{1}{2} \left(\frac{dL}{L} + \frac{dC}{C} \right) \tag{38}$$

$$\frac{dQ}{Q} = S_L^Q \frac{dL}{L} + S_C^Q \frac{dC}{C} + S_R^Q \frac{dR}{R} = \frac{1}{2} \left(\frac{dL}{L} - \frac{dC}{C} \right) - \frac{dR}{R} \tag{39}$$

We have $\omega_0 = 1/\sqrt{LC}$ and $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$. If both L and C increase, their effects on ω_0 add because ω_0 depends on the product of the two elements. But the errors in L, C tend to cancel in Q which depends on a ratio of the two components. An increase in R, i.e. larger losses results in a lower Q, but does not affect ω_0.

6.2 Inverting amplifier

The schematic of an inverting amplifier is shown in Fig. 12. Summing the currents at node a and remembering that the opamp input current i_- is zero results in the equation,

$$\frac{V_1 - V_a}{R_1} + \frac{V_2 - V_a}{R_2} = 0 \tag{40}$$

Opamp imposes the condition that $V_x = 0 - V_a = -V_a = V_2/A$. Here A denotes the finite open-loop DC gain of the opamp. Thus $-V_a = V_2/A$. Substituting for V_a in (40) we have,

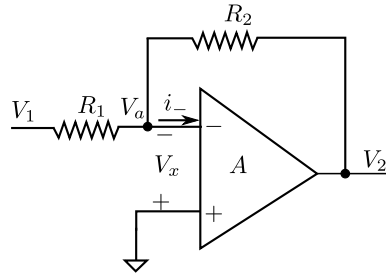$$\frac{V_1 + V_2/A}{R_1} + \frac{V_2 + V_2/A}{R_2} = 0 \tag{41}$$

Figure 12: Inverting amplifier

From (41), we can derive the transfer function of the inverting amplifier as

$$T = \frac{V_2}{V_1} = \frac{-R_2}{R_1} \frac{1}{(1+R_2/R_1)/A} = -\frac{G_0}{1+(1+G_0)/A} \tag{42}$$

where $G_0 = R_2/R_1$ is the amplifier's low-frequency gain in closed-loop. The sensitivity of T with respect to R_1 is

$$S_{R_1}^T = \frac{\partial T/T}{\partial R_1/R_1} = \frac{R_1}{T} \frac{\partial T}{\partial R_1} \tag{43}$$

Substituting for T from (42) in (43) gives

$$S_{R_1}^T = \frac{R_1}{T} \frac{\partial}{\partial R_1} \left(\frac{-R_2}{R_1 + (R_1 + R_2)/A} \right) = \frac{-R_1}{T} \frac{-R_2(1+1/A)}{(R_1 + (R_1+R_2)/A)^2} \tag{44}$$

If we evaluate (44) at $A = \infty$ (ideal opamp), we get

$$S_{R_1 (A=\infty)}^T = \frac{-R_1}{-G_0} \cdot \frac{-R_2}{R_1^2} = -1 \tag{45}$$

Similarly, the sensitivity of T with respect to R_2 is given by

$$S_{R_2}^T = \frac{R_2}{T} \frac{\partial T}{\partial R_2} \tag{46}$$

Substituting for T from (42) in (46) and performing the differentiation gives

$$S_{R_2}^T = -\frac{(R_1 + (R_1 + R_2)/A - \frac{R_2}{A}}{\left(R_1 + (\frac{R_1+R_2}{A})^2 \right)} \tag{47}$$

If we evaluate (47) at $A = \infty$ (ideal opamp), we get

$$S_{R_2 (A=\infty)}^T = \frac{R_2}{T} \cdot \frac{-R_1}{R_1^2} = \frac{R_2}{-R_2/R_1} \frac{-1}{R_1} = 1 \tag{48}$$

These results show that a 1% change in the resistors R_1 or R_2 causes respectively a -1% and 1% change in G_0. Since accurate and stable resistors are available, this value of sensitivity is acceptable. To evaluate sensitivity to the opamp gain A we compute

$$S_A^T = \frac{A}{T} \frac{\partial T}{\partial A} = \frac{A}{T} \frac{\partial}{\partial A} \left(\frac{-G_0}{1+(1+G_0)/A} \right) = \frac{(1+G_0)}{(A+1+G_0)} \tag{49}$$

Since $A \gg (1+G_0)$ in the frequency range of interest, the sensitivity in (49) is approximately

$$S_A^T = \frac{1}{A}(1+G_0) \tag{50}$$

For large values of A, (50) is a very small number. It demonstrates that the closed-loop gain of the amplifier is $\approx G_0 = \frac{R_2}{R_1}$, independent of A as long as the opamp DC gain A is large. However, if R_2 is removed ($R_2 = \infty$) so that there is no feedback, the sensitivity given in (49) becomes

$$S^T_{A\,(R_2=\infty)} = \frac{(1 + G_0)}{(A + 1 + G_0)} = \frac{R_1 + R_2}{(A + 1)R_1 + R_2} \approx 1 \tag{51}$$

From (51), it is evident that in the absence of feedback, changes in A directly translate to changes in G_0 (R_2/R_1). Whereas sensitivities of the order of unity are acceptable for passive components, such values are normally too large for opamp gains, because the active parameters in a circuit must be expected to vary widely.

6.3   Comparison of the two cases

By comparing the results of the sensitivity analyses of the inverting amplifier and the passive RLC low-pass circuit, we arrive at the following conclusions

- Sensitivity of a circuit to active parameters such as opamp gain should be much lower than that for passive components such as resistors, capacitors etc.

- Since accurate passive components can be made available, sensitivity requirements of a circuit for passive components are relaxed

- Circuit parameters which depend on product of component values are more adversely affected by component tolerance than those circuit parameters which depend on the ratio of the components. This means that filter structures which use capacitor-ratios (as in switched-capacitor circuits) provide more accurate coefficients/filter-response compared to those using RC constants.

7   First-order Gm-C filter

This exercise is identical to Example 15.2 in pp 581-582 of [2]. A first-order filter with a zero at $40\ MHz$ and a pole at $20\ MHz$ is given by

$$H(s) = \frac{k(s + 2\pi \cdot 40\ MHz)}{(s + 2\pi \cdot 20\ MHz)} \tag{52}$$

Setting $s = 0$ in (52) gives $H(0) = 2k$. Since the DC gain is given as 0.5, we have $2k = 0.5$ resulting in $k = 0.25$. Substituting $k = 0.25$ in (52), we have

$$H(s) = \frac{0.25s + 2\pi \cdot 10\ MHz}{(s + 2\pi \cdot 20\ MHz)} \tag{53}$$

The transfer function of a general first-order continuous-time filter is given by

$$H(s) = \frac{k_1 s + k_0}{s + \omega_0} \tag{54}$$

Comparing (53) with (54), we have $k_1 = 0.25$, $k_0 = 2\pi \cdot 10^7$, $\omega_0 = 4\pi \cdot 10^7$.

$$C_x = \left(\frac{k_1}{1 - k_1}\right) C_A = \frac{0.25}{0.75} \cdot 2\ pF = 0.667\ pF \tag{55}$$

$$G_{m1} = k_0(C_A + C_x) = 2\pi \cdot 10^7(2\ pF + 0.667\ pF) = 0.168\ mA/V \tag{56}$$

$$G_{m2} = \omega_0(C_A + C_x) = 4\pi \cdot 10^7(2\ pF + 0.667\ pF) = 0.335\ mA/V \tag{57}$$

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8 Second-order Gm-C filter

The transfer function of a general second-order (biquad) continuous-time filter is given by

$$H(s) = \frac{k_2 s^2 + k_1 s + k_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0{}^2} \tag{58}$$

A second-order band-pass transfer function is given by

$$H(s) = \frac{Gs\frac{\omega_0}{Q}}{s^2 + s\frac{\omega_0}{Q} + \omega_0{}^2} \tag{59}$$

The gain at the centre frequency $G = 1$. We have $\omega_0 = 2\pi \cdot 20\ MHz$ and $Q = 5$. By comparing (59) and (58) we find

$$k_1 = G\frac{\omega_0}{Q} = 2.513 \cdot 10^7\ rad/s \tag{60}$$

Since $k_0 = 0$ and $k_2 = 0$, we have

$$k_0 = \frac{G_{m2}G_{m4}}{C_A(C_x + C_B)} = 0 \tag{61}$$

$$k_2 = \frac{C_x}{C_x + C_B} = 0 \tag{62}$$

From (61) and (62), $C_x = 0$ and $G_{m4} = 0$. $G_{m2} \neq 0$ since

$$\omega_0{}^2 = \frac{G_{m1}G_{m2}}{C_A(C_x + C_B)} \neq 0 \tag{63}$$

$$G_{m2} = \omega_0(C_B + C_x) = 2\pi \cdot 20\ MHz(2\ pF + 0) = 0.2513\ mA/V \tag{64}$$

$$G_{m1} = \omega_0 C_A = 0.2513\ mA/V \tag{65}$$

For $G = 1$, $k_1 = \omega_0/Q$, we have

$$G_{m3} = \frac{\omega_0(C_B + C_x)}{Q} = G_{m5} = k_1(C_B + C_x) \tag{66}$$

Thus,

$$G_{m3} = G_{m5} = k_1(C_B + C_x) = 2.513 \cdot 10^7(2\ pF + 0) = 50.27\mu A/V \tag{67}$$

9 Active Filters

In order to derive the transfer function of the active filter circuit given in Fig.13. we form nodal equations at the output of each stage of the filter.

At node V_1 we have,

$$V_1 = \frac{-1}{sC_1}\left(\frac{V_{in}}{R_4} + \frac{V_{out}}{R_1}\right) \tag{68}$$

For nodes V_2 and V_{out} we have,

$$V_2 = -R_8\left(\frac{V_1}{R_7} + \frac{V_{out}}{R_2} + \frac{V_{in}}{R_5}\right) \tag{69}$$

$$V_{out} = \frac{-1}{sC_2}\left(\frac{V_{in}}{R_6} + \frac{V_{out}}{R_3} + \frac{V_2}{R_9}\right) \tag{70}$$

Using (68)-(70), the nodes V_1, V_2 can be eliminated to give the transfer function V_{out}/V_{in}.

References

[1] R. Schaumann and M. E. V. Valkenburg, *Design of Analog Filters*. Oxford University Press, 2001.

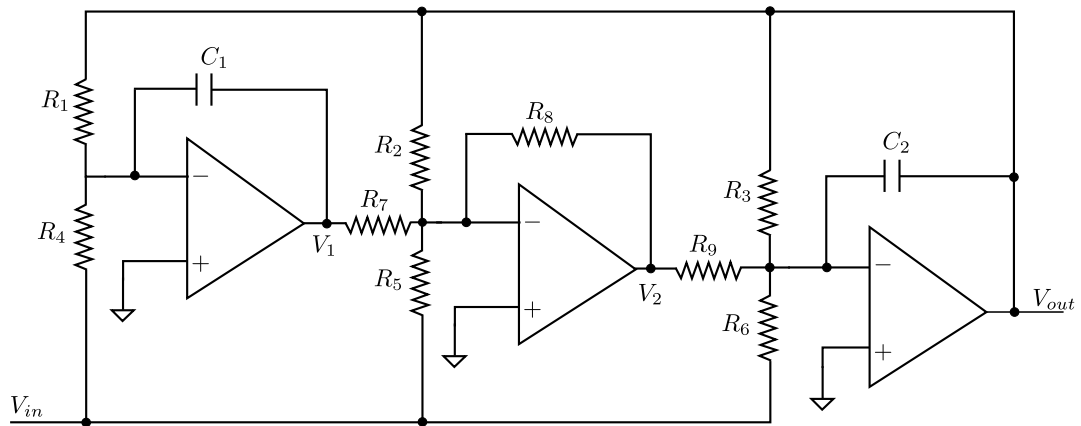[2] D. A. Johns and K. Martin, *Analog Integrated Circuit Design*. Wiley, 1997.

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Figure 13: Active filter circuit