Linköping University
INSTITUTE OF TECHNOLOGY

Memo

ANTIK: Effect of switch on-resistance on integrator

Description

This document contains a couple of formulas and simulation results concerning an SC circuit and the effects of limited on-resistance in switches.

Outline

History

Rev	Date	Comment	Issued/created by
PA1	03/10/11	Created document.	J Jacob Wikner
P2A	2010-03-11	Updated with a figure and some simulation results.	J Jacob Wikner
P2A	2011-03-11		J Jacob Wikner

Distribution

ATIK

1. EFFECT OF NON-ZERO ON-RESISTANCE

1.1 Example

Consider the parasitic sensitive integrator in Figure 1.1. It consists of two switches, which in the nonideal case will have a certain on-resistance, R_{on}.

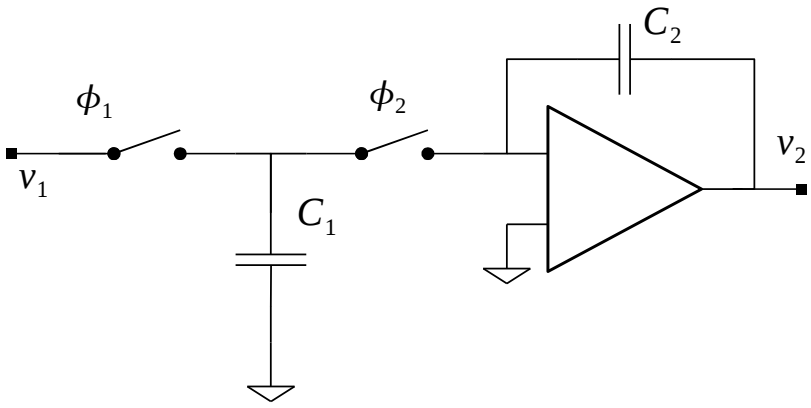

Figure 1.1: The parasitic sensitive integrator.

We can model the effect of this by replacing the switches with resistors having the on-resistance R_{on}. This is shown in Figure 1.2 for the two different phases. Notice now that we have introduced an intermediate node, v_x.

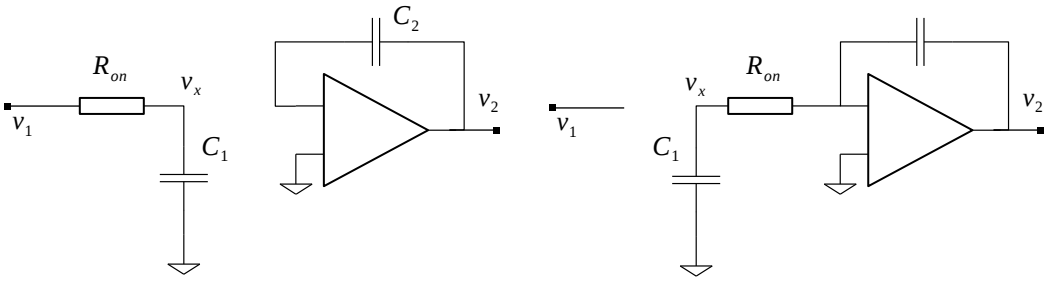

Figure 1.2: The two phases with on-resistance noted for the switches.

In Figure 1.3 a couple of phases have been illustrated. At the top we find the clock phases, the middle graph is for a fairly low on-resistance and the bottom one for a fairly large on-resistance. The graphs essentially show the v_x node for the two cases and the ideal, staircase behavior has also been illustrated.

Notice for the high R_{on} case that the v_x node is not able to settle properly towards the ground level. The average level of the v_x node will shift "upwards" (of course the signal could also be negative and then the average level still could be at ground, that's a special case ...)

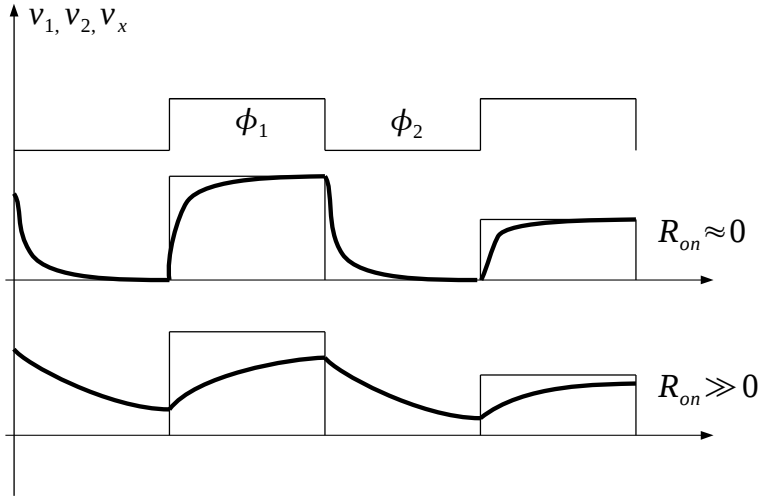

Figure 1.3: Effect of limited settling behavior due to too high on resistance.

1.1.1 Calculation steps

At the first phase, ϕ_1, (since we have a settling behavior, we now have to look at the end of the settling period (see Figure 1.3) –previously we have looked at the instantaneous change of value, since $R_{on}=0$).

$$v_x(t+\tau)=v_x(t)+\left(v_1(t+\tau)-v_x(t)\right)\cdot\underbrace{\left(1-e^{-\tau/T_0}\right)}_{1-\alpha} \tag{1.1}$$

Notice how we use the old value found at the v_x node, $v_x(t)$, which is the start value of the capacitance charging (once again see Figure 1.3). The capacitance on the output, C_2, is isolated and thus it must preserve its voltage from the previous phas:

$$v_2(t+\tau)=v_2(t) \tag{1.2}$$

At the second phase, ϕ_2, we are discharging the first capacitor and the voltage across it has to look like:

$$v_x(t+T)=v_x(t+\tau)\cdot\underbrace{e^{-\tau/T_0}}_{\alpha}\Rightarrow v_x(t)=v_x(t-\tau)\cdot\alpha \tag{1.3}$$

To find the v_2 voltage, we have to examine the charge preservation.

$$\begin{aligned} &q_1(t+\tau)-q_2(t+\tau)=q_1(t+T)-q_2(t+T)\\ &q_1(t+\tau)-q_1(t+T)=q_2(t+\tau)-q_2(t+T)=C_2\cdot\left(v_2(t+\tau)-v_2(t+T)\right)\end{aligned} \tag{1.4}$$

Notice now that the charge on the first capacitor is determined by v_x and not v_1. We can write

$$v_x(t+\tau)-v_x(t+T)=v_x(t+\tau)-v_x(t+\tau)\cdot\alpha=\frac{C_2}{C_1}\cdot\left(v_2(t+\tau)-v_2(t+T)\right)\text{, i.e.,}$$
$$v_x(t+\tau)\cdot(1-\alpha)=\frac{C_2}{C_1}\cdot\left(v_2(t)-v_2(t+T)\right) \quad (1.5)$$

Now we need to replace v_x with v_1 somehow... From above we see that at the instantaneous change of value, since $R_{on}=0$).

$$v_x(t+\tau)=v_x(t-\tau)\cdot\alpha+\left(v_1(t+\tau)-v_x(t-\tau)\cdot\alpha\right)\cdot\left(1-\alpha\right)\text{ which gives}$$
$$v_x(t+\tau)=v_x(t-\tau)\cdot\alpha^2+v_1(t+\tau)\cdot\left(1-\alpha\right) \quad (1.6)$$

Now, we can apply the z-transform and we would get

$$v_x(z)\cdot z^{0.5}=v_x(z)\cdot z^{-0.5}\cdot\alpha^2+v_1(z)\cdot z^{0.5}\cdot\left(1-\alpha\right)\Rightarrow v_x(z)\cdot z^{0.5}=\frac{1-\alpha}{1-z^{-1}\cdot\alpha^2}\cdot z^{0.5}\cdot v_1(z) \quad (1.7)$$

And we had

$$v_x(z)\cdot z^{0.5}\cdot(1-\alpha)=\frac{C_2}{C_1}\cdot(1-z)\cdot v_2(z) \quad (1.8)$$

so we can combine them and get

$$\frac{\left(1-\alpha\right)^2}{1-z^{-1}\cdot\alpha^2}\cdot z^{0.5}\cdot v_1(z)=\frac{C_2}{C_1}\cdot(1-z)\cdot v_2(z) \quad (1.9)$$

We can now form the transfer function

$$\frac{v_2(z)}{v_1(z)}=\frac{\left(1-\alpha\right)^2}{1-z^{-1}\cdot\alpha^2}\cdot z^{0.5}\cdot\frac{C_1/C_2}{1-z}=-\frac{C_1}{C_2}\cdot\frac{1}{1-z^{-1}}\cdot\frac{\left(1-\alpha\right)^2}{1-z^{-1}\cdot\alpha^2}\cdot z^{-0.5} \quad (1.10)$$

α is a value that should ideally be 0. For this case the equation above boils down to

$$\frac{v_2(z)}{v_1(z)}=-\frac{C_1}{C_2}\cdot\frac{1}{1-z^{-1}}\cdot\frac{\left(1-0\right)^2}{1-z^{-1}\cdot 0}\cdot z^{-0.5}=-\frac{C_1}{C_2}\cdot\frac{z^{-0.5}}{1-z^{-1}} \quad (1.11)$$

which is our desired integrator function.

If we go for the other extreme, $\alpha=1$, we get an output $v_2(z)=0$, which also makes sense. If $\alpha=1$ we would have infinite resistance in the switch and there would not be any charge transferred to the output. For $\alpha=0.5$, we have a gain of $(1-\alpha)^2=0.25$ rather than 1 and there would also be an additional pole at $p_1=0.25$ (this pole wanders from $p_1=0$ towards $p_1=1$ along the real axis).

1.1.2 Impact on lab

One of the questions in the lab was to find the largest R_{on} that we can tolerate for a certain sample frequency. For this purpose we could look at the term $(1-\alpha)^2$ and we could say we want to look at when this is equal to say $\eta=0.95$ or some level we like. This gives us (for $z=1$, i.e., DC in a discrete-time system):

$$\frac{(1-\alpha)^2}{1-\alpha^2}=\eta \Rightarrow \alpha=\frac{1-\eta}{1+\eta} \tag{1.12}$$

i.e., we get

$$R_{on}<-\frac{2}{f_s\cdot C_1}\cdot\frac{1}{\ln\frac{1-\eta}{1+\eta}} \text{ or maybe } f_s<\frac{2}{R_{on}\cdot C_1}\cdot\frac{1}{\ln\frac{1+\eta}{1-\eta}} \tag{1.13}$$

If we look for the 3-dB frequency, the η would be equal to $\sqrt{2}$. The higher the clock frequency we have the smaller the R_{on} has to be.

1.2 Some simulation results

There are a couple of MATLAB files available in the ANTIK lab area, for example `switCapRon.m`.

The desired simulated transfer characteristics is simulated in Figure 1.4. In the lower graph, the input and the v_x node is overlayered, but we can actually not separate them from eachother. The capacitors are $C_1=C_2=1$ pF. The sample frequency is 300 MHz and the signal frequency is 1 MHz.

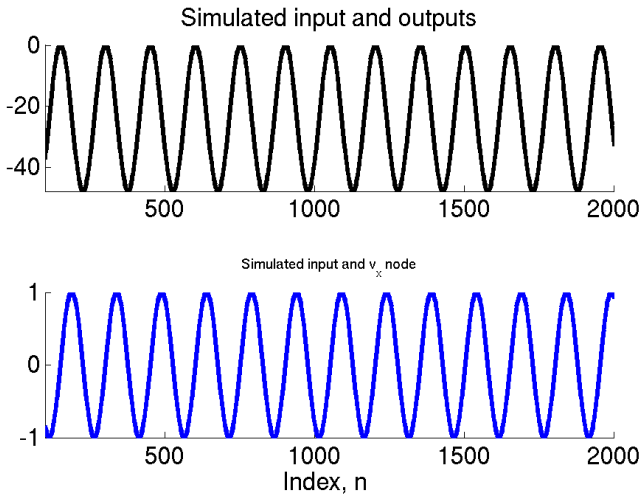

Figure 1.4: Simulated "ideal" behavior, with $R_{on}=1$.

If we increase the on-resistance to $R_{on}=1000$ Ohms we get the behavior displayed in Figure 1.5 and here we can see some more effects. First of all, we can v_x does not track very well and that the amplitude at the output has changed. To further illustrate this we show in Figure 1.6 the effect of having a very high resistance. The output is essentially equal to zero and the v_x tracks the v_1 poorly.

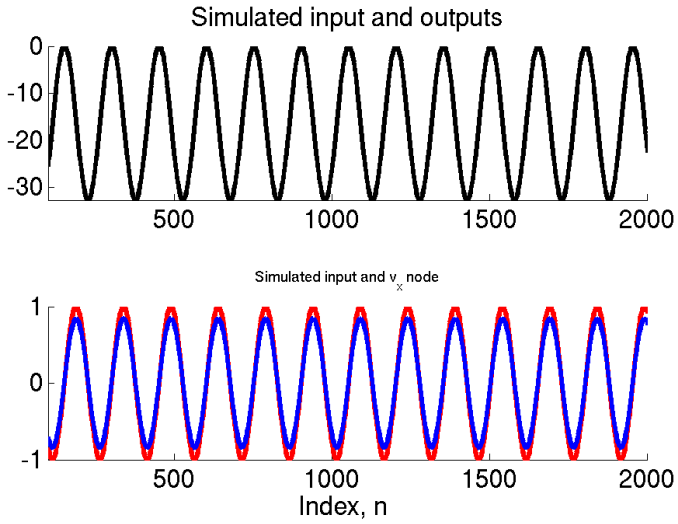

Figure 1.5: Simulated behavior with $R_{on}=1000$.

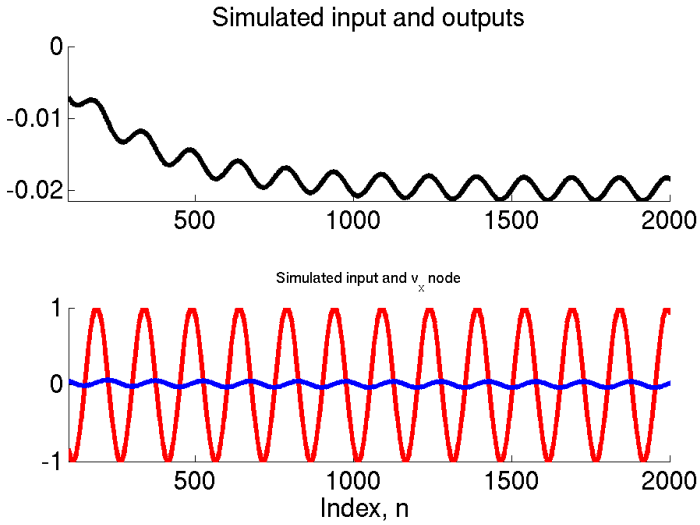

Figure 1.6: Simulated behavior with $R_{on}=1$ MOhm

We can sweep the on-resistance for a certain sample frequency and measure the amplitude at the output (max - min) and plot them. To get a more elaborate measure, we could plot the signal power at the output and find for which R_{on} we obtain the 3-dB limit. In Figure 1.7 we find this transfer function and the trained eye could see that the 3-dB "bandwidth" is at approximately 1 kOhm.

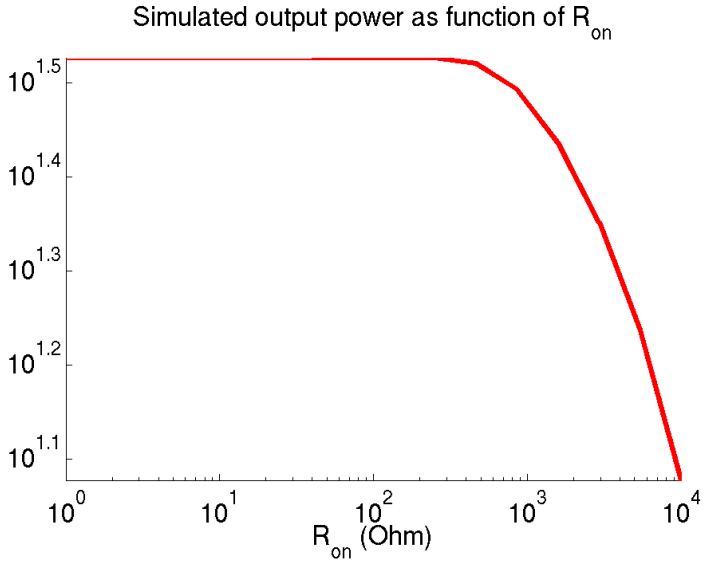

Figure 1.7: Simulated transfer characteristics as function of on-resistance.

So that was kind of simple. However, now the problem is that the sample frequency **will also change the amplitude** of the output signal (why?). If we increase the sample frequency (and maintain the signal frequency) the output power also increases. Remember the continuous-time equivalent

$$\frac{v_2(s)}{v_1(s)} = -\frac{C_1}{C_2}\cdot\frac{1}{sT} = -\frac{C_1}{C_2}\cdot\frac{f_s}{s} \tag{1.14}$$

With higher frequencies we increase the output signal power (more voltage is accumulated). In fact, we can use our MATLAB code to simulate this too. These results are displayed in Figure 1.8 where we find a fine straight line just as is predicted by the equations.

So the exercise in the lab is: if I change the sample frequency and measure the "3-dB" R_{on} - how are they related? What does the theory say? Well, first of all, we need to kind of compensate for the loss of amplitude from above. This means that - for a given frequency, the desired gain is the one given by the $R_{on}=1$ case, so we cannot use e.g. the maximum sample frequency as reference (infinite sample frequency would give you infinite gain!).

This means we can sweep the sample frequency and we can sweep the on-resistance and then plot the relative output power for the given frequency. This would look something like the result in Figure 1.9. Notice that all start at 0 dB. If we now draw a straight line through -3 dB and then note which frequency we get for which resistance (and vice versa). For these simulation results we

sweep the sample frequency from 10 MHz up to 1 GHz. The on-resistance is swept from 1 to 10 kOhm.

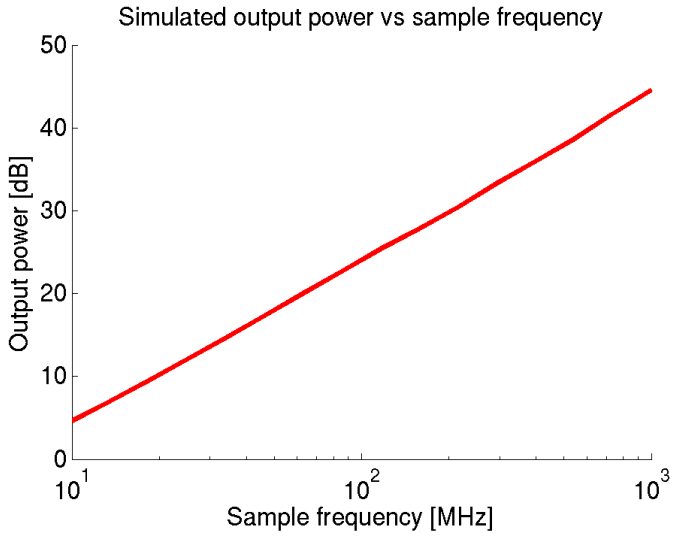

Figure 1.8: Output power vs sample frequency.

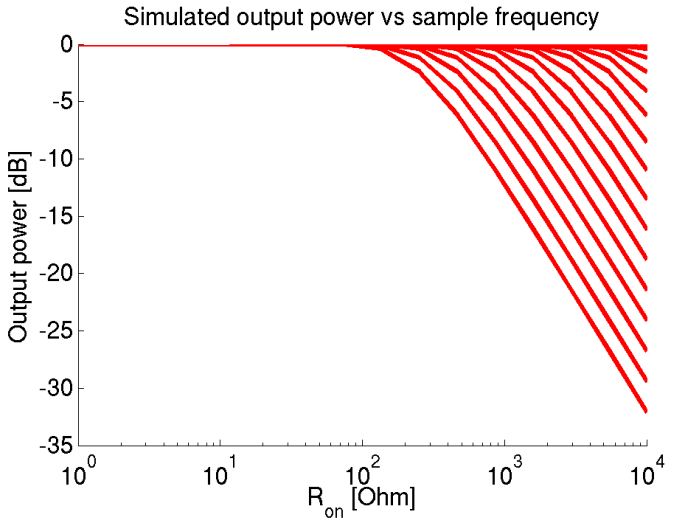

Figure 1.9: Simulated relative gain error as function of frequency and on-resistance.

By clevely writing your script you can find these values and plot them as for example is done in Figure 1.10. Notice that we have a kind of truncation for higher frequencies. (Notice that we run up to 1 GHz in this example and that we have quite few values causing this truncation.) What we can

see from the results is for example that for 30 MHz sample frequency the on-resistance can be 10 kOhm. For the 1-GHz sample frequency, we need to have an on-resistance less than 300 Ohms. This kind of behavior fits very well with the formulas we derived earlier. There is an offset just plugging in the numbers, but they have also been normalized differently.

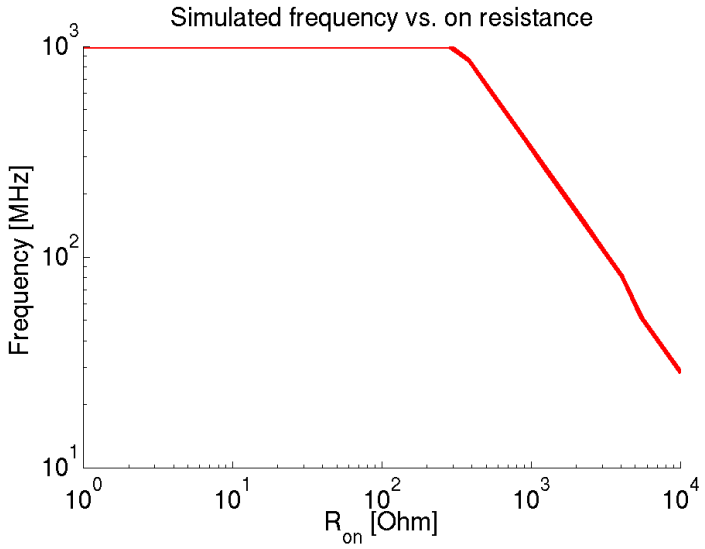

Figure 1.10: Simulated maximum on-resistance for a given sample frequency.

✗ *So, with lower sample frequency, the acceptable R_{on} increases, which kind of makes sense. Try to demonstrate this with Cadence in the lab. Use the parametric sweeps and pick a few points.*

✗ *The amplitude can be measured in e.g. the transient window or using PSS analysis. You can then plot the amplitude as a function of the swept sample frequency and/or the swept on-resistance.*