# Lesson 4

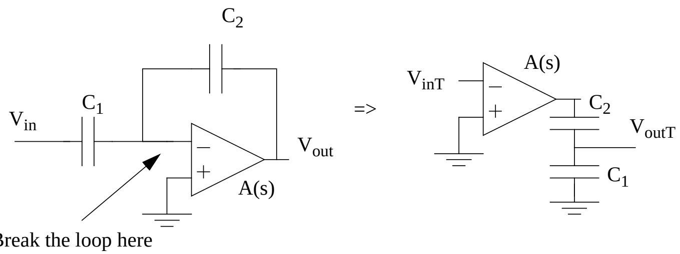
**Lesson Exercises:** **4, B5.1, B5.2, B5.3, B5.4, B5.5**

*Recommended Exercises:* *B5.10*

**Theoretical Issues:** **Opamp**

### Exercise 4

Calculated the loop gain: Break the feedback loop



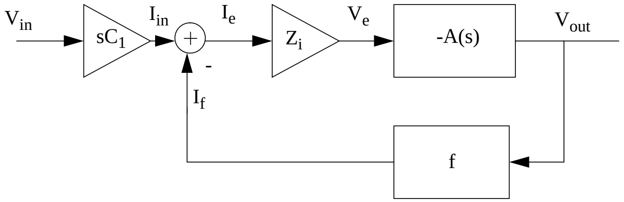

Loop gain:

$$T(s) = A(s)\frac{1/(sC_1)}{1/(sC_1) + 1/(sC_2)} = \frac{C_2}{C_1 + C_2}A(s) = \beta A(s)$$

The feedback factor:

$$\beta = \frac{C_2}{C_1 + C_2}$$

The closed loop system can be modelled as below (see lesson 1)




where $f = -sC_2$ and $Z_i = \dfrac{1}{sC_1 + sC_2}$.

The closed loop transfer function is thus given by

$$A_{CL}(s) = \frac{V_{out}}{V_{in}} = sC_1\frac{Z_i(-A(s))}{1 + Z_i(-A(s))f} = sC_1\frac{Z_i}{\frac{1}{-A(s)} + Z_i f} \approx \frac{sC_1}{f} = -\frac{C_1}{C_2}$$

The loop gain calculated from the system model:



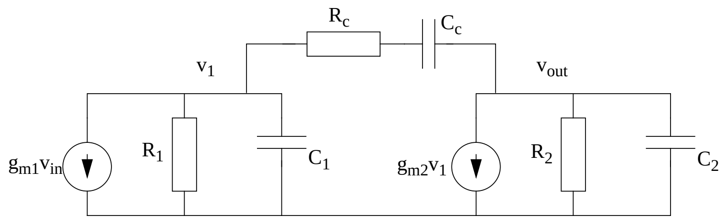


$$T(s) = Z_i(-A(s))f = \frac{1}{sC_1 + sC_2}(-sC_2)(-A(s)) = \frac{C_2}{C_1 + C_2}A(s) \Rightarrow \beta = \frac{C_2}{C_1 + C_2}$$

Conclusion: When calculating the loop gain we can break the loop to directly get the loop gain and feedback factor $\beta$, but the total transfer function is **not** given by $1/\beta$ when using shunt-shunt feedback.

### Exercise B5.1

Small Signal Model for the two stage opamp.




where

$$R_1 = r_{ds4} \| r_{ds2}$$
$$C_1 = C_{db2} + C_{db4} + C_{gs7}$$
$$R_2 = r_{ds6} \| r_{ds7}$$
$$C_2 = C_{db7} + C_{db6} + C_l$$

a) The pole caused by the first stage (this is also the dominant pole) will be low due to the Miller effect:

$|p_1| = \omega_{-3dB} = \dfrac{1}{R_1 C_{L1}}$ where the equivalent capacitive load in the first stage is given by

$$C_{L1} \approx C_c(1 + A_2) = C_c(1 + g_{m2}R_2)$$

$$r_{ds4} = \frac{8000 \cdot L}{I_{D4}} = \frac{8000 \cdot 1.2}{0.05} = 192k\Omega \quad r_{ds2} = \frac{12000 \cdot L}{I_{D2}} = \frac{12000 \cdot 1.2}{0.05} = 288k\Omega$$

$$r_{ds7} = \frac{8000 \cdot L}{I_{D7}} = \frac{8000 \cdot 1.2}{0.1} = 96k\Omega \quad r_{ds6} = \frac{12000 \cdot L}{I_{D6}} = \frac{12000 \cdot 1.2}{0.1} = 144k\Omega$$

$R_1 = r_{ds2} \| r_{ds4} = 115k\Omega$ and $R_2 = r_{ds6} \| r_{ds7} = 58k\Omega$

$$g_{m7} = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_{D7}} = \sqrt{2 \cdot 92 \cdot 10^{-6} \cdot \frac{300}{1.2} \cdot 0.1m} = 2.15\,\text{mA/V}$$

$$A_2 = g_{m7}R_2 = 124 \Rightarrow$$

$$|p_1| = \omega_{-3dB} = \frac{1}{R_1 C_{L1}} = \frac{1}{115k \cdot 10p \cdot 125} = 2\pi \cdot 1.1\,\text{kHz}$$

b)

$$g_{m1} = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_{D1}} = \sqrt{2 \cdot 92 \cdot 10^{-6} \cdot \frac{300}{1.2} \cdot 0.05m} = 0.866\,\text{mA/V}$$

$|A_0| = |A_1 A_2 A_3| \approx g_{m1}R_1 \cdot g_{m7}R_2 \cdot 1$ and for the single pole amplifier we have

$$\omega_u = |A_o| \cdot |p_1| = g_{m1}R_1 \cdot g_{m7}R_2 \cdot \frac{1}{R_1 C_c(1 + g_{m7}R_2)} \approx \frac{g_{m1}}{C_c} = 2\pi \cdot 14\,\text{MHz}$$

c)

$$SR = \frac{I_{D5}}{C_c} = \frac{100u}{10p} = 10\,\text{V/us}$$

**Exercise B5.2**

$I_{D1} = 50\mu\text{A}$, $C_c = 4\,\text{pF}$, $(W/L)_1 = 300u/1.2u$ and $(W/L)_5 = 300u/1.2u$.

The slew rate is limited by $SR = I_{D5}/C_c$. To increase the SR without changing $C_c$ we must increase $I_{D5}$ by a factor 2. This is done by increasing the size of $M_5$ to $(W/L)_5 = 600u/1.2u$. The unity gain frequency is given by

$$\omega_u = \frac{g_{m1}}{C_c} = \frac{\sqrt{2\mu_n C_{ox}(W/L)_1 I_{D1}}}{C_c}$$

To keep the unity gain frequency constant ($I_{D5} \cdot 2 \Rightarrow I_{D1} \cdot 2$) we must decrease the size of $M_1$ and $M_2$ by a factor 2, i.e. $(W/L)_1 = 150u/1.2u$.

**Exercise B5.3**

To avoid systematic offset the currents in the output stage must be equal, i.e. $I_{D6} = I_{D7}$. Since $M_3$ and $M_4$ are equal and have the same drain current and gate-source voltage they must also have the same drain-source voltage. => $V_{DS4} = V_{DS3} = V_{GS3} = V_{GS4}$.

From the schematic we have:

$V_{GS5} = V_{GS6}$, $V_{GS7} = V_{DS4} = V_{GS4}$.

$$I_{D4} = \frac{I_{D5}}{2},\ I_{D6} = \frac{(W/L)_6}{(W/L)_5}I_{D5}$$

$$V_{GS4} = \sqrt{\frac{2I_{D4}}{\beta_4}} + V_T = \sqrt{\frac{I_{D5}}{\beta_4}} + V_T,\quad V_{GS7} = \sqrt{\frac{2I_{D7}}{\beta_7}} + V_T = \sqrt{\frac{2 \cdot \frac{(W/L)_6}{(W/L)_5}I_{D5}}{\beta_7}} + V_T$$

=>

$\frac{\beta_7}{\beta_4} = 2 \cdot \frac{(W/L)_6}{(W/L)_5} \Rightarrow \frac{(W/L)_7}{(W/L)_6} = 2 \cdot \frac{(W/L)_4}{(W/L)_5} = 2 \cdot \frac{150u}{600u} = \frac{1}{2}$ where the transistor sizes from 5.2. have been used. =>





$$\begin{cases} W_6 = 300u,\ W_7 = 150u \Rightarrow I_{D7} = 100uA \\ W_7 = 600u,\ W_7 = 300u \Rightarrow I_{D7} = 200uA \end{cases}$$

**Exercise B5.4**

For no systematic offset $V_{gs7}$ should be

$$V_{gs7} = \sqrt{\frac{2 \cdot I_{D6}}{\mu_n C_{ox}(W/L)_7}} + V_T = \sqrt{\frac{2 \cdot 100u}{92 \cdot 10^{-6} \cdot (300/1.6)}} + V_T = 0.1077 + V_T$$

The actual value is

$$V_{gs7} = V_{gs4} = V_{gs3} = \sqrt{\frac{2 \cdot I_{D3}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_3}} + V_T = \sqrt{\frac{2 \cdot 50u}{92 \cdot 10^{-6} \cdot \frac{50}{1.6}}} + V_T = 0.1865 + V_T$$

The error in the gate voltage is thus $\Delta V_{gs7} = 78.8\,\text{mV}$. The equivalent input offset voltage is given by $V_{os} = \frac{\Delta V_{gs7}}{A_1} = \frac{\Delta V_{gs7}}{g_{m1}(r_{ds4} \| r_{ds2})}$.

$$r_{ds4} = \frac{8000 \cdot 1.6}{0.05} = 256k\Omega,\ r_{ds2} = \frac{12000 \cdot 1.6}{0.05} = 384k\Omega,$$

$$g_{m1} = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_{D1}} = \sqrt{2 \cdot 30 \cdot 10^{-6} \cdot \frac{300}{1.6} \cdot 0.05m} = 0.75\,\text{mA/V} \Rightarrow$$

$$V_{os} = \frac{\Delta V_{gs7}}{A_1} = \frac{\Delta V_{gs7}}{g_{m1}(r_{ds4} \| r_{ds2})} = 0.7\,\text{mV}.$$

**Exercise B5.5**

$$V_{out,min} = V_{eff9} + V_{ss},\ V_{out,max} = V_{dd} - V_{eff6} - V_{gs8} = V_{dd} - V_{eff6} - V_{eff8} - V_{Tn}$$

$$V_{in,max} = V_{dd} - V_{eff5} - V_{gs1} = V_{dd} - V_{eff5} - V_{eff1} - |V_{Tp}|$$

$$V_{in,min} = V_{ss} + V_{gs3} + V_{eff1} - V_{gs1} = V_{ss} + V_{eff3} + V_{Tn} - |V_{Tp}|$$

By using $V_{eff} = \sqrt{\frac{2I_D}{\mu_n C_{ox} W/L}}$:

$V_{eff1} = V_{eff2} = 0.133V$, $V_{eff3} = V_{eff4} = 0.108V$ and $V_{eff8} = V_{eff9} = 0.108V$

$V_{eff5} = V_{eff6} = 0.189V$ =>

$V_{out,max} = 5 - 0.189 - 0.108 - 0.8 = 3.9\,\text{V}$, $V_{out,min} = -5 + 0.108 = -4.9\,\text{V}$

$V_{in,max} = 5 - 0.189 - 0.133 - 0.9 = 3.8\,\text{V}$,

$V_{in,min} = -5 + 0.108 + 0.8 - 0.9 = -4.99\,\text{V}$

**Exercise B5.10**

$g_m = 0.775\,\text{mA/V}$, $|p_2| = 60\,\text{MHz}$

From p. 237 we have

$$\omega_t = \tan(90° - PM)|p_2| = \tan(90° - 55°)|p_2| = 0.7|p_2| = 2\pi \cdot 42\,\text{MHz}$$

where $\omega_t$ is the unity gain frequency of the loop gain ($T(s) = A(s)\beta$). For high frequencies the loop gain is given by (p. 237):

$$T(s) = \beta A(s) = \frac{\beta \cdot A_0|p_1|}{s\left(1 + \frac{s}{|p_2|}\right)} = \frac{\beta \cdot g_m/C_c}{s\left(1 + \frac{s}{|p_2|}\right)} \text{ where } \beta = \frac{R_1}{R_1 + R_2} = \frac{1}{6}$$

At the unity gain frequency:

$$T(j\omega_t) = 1 = \frac{\beta \cdot g_m/C_c}{\omega_t\sqrt{1 + \left(\frac{\omega_t}{|p_2|}\right)^2}} = \frac{\beta \cdot g_m/C_c}{\omega_t\sqrt{1 + 0.7^2}} \Rightarrow C_c = \frac{\beta \cdot g_m}{\omega_t\sqrt{1 + 0.7^2}} = 0.4\,\text{pF}$$