

Lesson 5

Lesson Exercises: 6, B6.10, B6.11

Recommended Exercises: B6.24

Theoretical Issues: Opamp

Exercise 6

1. Determine the transfer functions, $H_i = \frac{V_o}{V_{ni}}$, from all the noise sources in the amplifier (one for each transistor) to the output.

2. Calculate the total output noise by $S_{o,tot}(f) = \sum_{i=1}^7 |H_i| S_i(f)$

3. calculate the equivalent input noise by $S_{i,tot}(f) = \frac{S_{o,tot}(f)}{A_{tot}}$

The noise sources in the differential amplifier have the following transfer functions to the gate of M₇ (the output of stage 1: V_{o1}).

$$\left| \frac{V_{o1}}{V_{n1}} \right| = \left| \frac{V_{o1}}{V_{n2}} \right| = g_{m1} R_{out1}, \quad \left| \frac{V_{o1}}{V_{n3}} \right| = \left| \frac{V_{o1}}{V_{n4}} \right| = g_{m3} R_{out1} \quad \text{and} \quad \left| \frac{V_{o1}}{V_{n5}} \right| = \frac{g_{m5}}{g_{m3}}$$

where $R_{out1} = r_{ds4} \parallel r_{ds2}$ and $R_{out2} = r_{ds6} \parallel r_{ds7}$.

To get the total transfer function to the output the noise voltages are amplified by the gain of stage 2:

$$|H_1| = |H_2| = g_{m1} R_{out1} \cdot g_{m7} R_{out2}, \quad |H_3| = |H_4| = g_{m3} R_{out1} \cdot g_{m7} R_{out2} \quad \text{and}$$

$$|H_5| = g_{m5}/g_{m3} \cdot g_{m7} R_{out2}.$$

The transfer functions for the transistors in stage 2 are:

$$|H_6| = g_{m6} R_{out2} \quad \text{and} \quad |H_7| = g_{m7} R_{out2}.$$

The total equivalent input noise spectral density is thus given by

$$\begin{aligned} S_{i,tot}(f) &= \sum_{i=1}^7 \frac{|H_i| S_i(f)}{A_{tot}} = S_1(f) + S_2(f) + \left(\frac{g_{m3}}{g_{m1}} \right)^2 (S_3(f) + S_4(f)) + \\ &+ \left(\frac{g_{m5}}{g_{m3} A_1} \right)^2 S_5(f) + \left(\frac{1}{A_1} \right)^2 S_7(f) + \left(\frac{g_{m6}}{g_{m7} A_1} \right)^2 S_5(f) \approx \\ &\approx 2 \cdot S_1(f) + \left(\frac{g_{m3}}{g_{m1}} \right)^2 2 \cdot S_3(f) \end{aligned}$$

$$\text{where } S_i(f) = 4kT \frac{1}{3g_{mi}} + \frac{K}{(W/L)_i C_{ox} f}$$

Conclusions:

- Noise contributions from M₅-M₇ are small
- Large area for the transistors => small 1/f-noise
- $g_{m1} > g_{m3}$ => small contribution from M₃-M₄.
- Large g_{m1} => Small thermal noise in M₁-M₂.

Exercise B6.10

$$P = (I_{D4} + I_{D3})(V_{dd} - V_{ss}) = 2 \cdot I_{D3} \cdot (V_{dd} - V_{ss}) \Rightarrow I_{D3} = \frac{P}{2(V_{dd} - V_{ss})} = 125 \mu\text{A}$$

$$\text{From p. 271: } I_B = 4 \cdot I_{D5} \Rightarrow I_{D5} = I_{D6} = 25 \mu\text{A}, I_{D1} = I_{D1} = 100 \mu\text{A}$$

$$g_{m1} = \sqrt{2\mu C_{ox}(W/L)_1 I_{D1}} = 1.9 \text{ mA/V}, A_0 = g_{m1} \cdot r_{out},$$

$$|p_1| = \frac{1}{r_{out} C_L} \Rightarrow \omega_u = A_0 |p_1| = \frac{g_{m1}}{C_L} = \frac{1.9 \text{ m}}{10 \text{ p}} = 2\pi \cdot 30 \text{ MHz}$$

$$SR = \frac{I_{out,max}}{C_L} = \frac{I_{D3}}{C_L} = \frac{125 \mu}{10 \text{ p}} = 12.5 \text{ V/us}$$

Exercise B6.11

The second pole is approximately given by

$$|p_2| = \frac{g_{m5}}{C_{p2}}$$

where C_{p2} is the total capacitance at the drain of M₂.

$$\begin{aligned} C_{p2} &= C_{dg2} + C_{dg4} + C_{sg4} = C_{gd(overlap)}(W_2 + W_4 + W_5) + \frac{2}{3} W_5 L C_{ox} = \\ &= 0.2 \text{ f}(300 + 300 + 60) + \frac{2}{3} \cdot 60 \cdot 1.6 \cdot 1.9 \text{ f} = 254 \text{ fF} \end{aligned}$$

$$g_{m5} = \sqrt{2\mu C_{ox}(W/L)_1 I_{D1}} = 0.237 \text{ mV} \Rightarrow |p_2| = g_{m5}/C_{p2} = 2\pi \cdot 148.5 \text{ MHz}$$

From p. 237:

$\frac{\omega_t}{|p_2|} = \tan(90^\circ - PM) = \tan(20^\circ) = 0.364 \Rightarrow \omega_t = 2\pi \cdot 54.05 \text{ MHz}$, where ω_t is the unity gain frequency of the loop gain. In this case we are only interested in the transfer function of the opamp which corresponds to $\beta = 1$. For high frequencies the loop gain is given by (p. 237):

$$T(s) = \beta A(s) = \frac{\beta \cdot A_0 |p_1|}{s \left(1 + \frac{s}{|p_2|} \right)} = \frac{g_{m1}/C_L}{s \left(1 + \frac{s}{|p_2|} \right)}$$

At the unity gain frequency:

$$T(j\omega_t) = 1 = \frac{g_{m1}/C_L}{\omega_t \sqrt{1 + \left(\frac{\omega_t}{|p_2|}\right)^2}} = \frac{g_{m1}/C_L}{\omega_t \sqrt{1 + 0.364^2}} \Rightarrow C_L = \frac{g_{m1}}{\omega_t \sqrt{1 + 0.364^2}} = 5.3 \text{ pF}$$

where the value on g_{m1} was calculated in 6.10.

$$SR = \frac{I_{D4}}{C_L} = \frac{125 \mu}{5.3 \text{ p}} = 22.4 \text{ V/us}$$

Exercise B6.24

By breaking the feedback loop we see that the total load capacitance at the output is

$$C_L = C_0 + \frac{(C_1/M) \cdot (C_1 + C_p)}{C_1/M + C_1 + C_p} \text{ and the feedback factor } \beta = \frac{C_1/M}{C_1/M + C_1 + C_p}.$$

The unity gain frequency is given by $\omega_u = g_{m1}/C_L$ and according to p. 234

$$\tau = \frac{1}{\omega_{-3dB}} = \frac{1}{|p_1|} = \frac{1}{\beta \omega_u} = \frac{1}{g_{m1}} \left((M+1)C_0 + C_p + \frac{MC_0C_p}{C_1} + C_1 \right)$$

$$\frac{\partial \tau}{\partial C_1} = 0 \Rightarrow -\frac{C_0C_p}{C_1} + M = 0 \Rightarrow C_{1,opt} = \sqrt{MC_0C_p}$$