1 — What is Mathematica

Mathematica is a math program handling symbolic expression. You can for example solve system of equations, solve integrals and plotting graphs.

2 — Starting Mathematica

Mathematica is started by first adding the module

```
module add mathematica
```

or by adding the TSEI30 module in the course administrator tool. The Mathematica program is started by writing

```
mathematica &
```

The notebook will now open on your screen. The magnification of the symbols can be changed by using the menu alternative `Format->Magnification`. The help to Mathematica is good so check the help if you get stuck.

3 — Basic Commands

To execute a command shift+Enter is used some where in that row.

Solving a system of equations

When you have a one of a system of equations that you like to solve the following syntax is recommended:

```
A=Solve[{eq1, eq2,...},{vars},{variables to eliminate}]
```

where all equations look like *lhs==rhs*. The vars are the output variables. A is assigned the output.

Example

```
Solve[{gm1 Vin+(gds1+gds2)Vx==0, gm2 Vx+s CL Vout==0},{Vout},{Vx}]
```

The outcome of this will be

$$\left\{\left\{Vout \text{ -> } \frac{gm1gm2Vin}{CL(gds1+gds2)s}\right\}\right\}$$

Simplifications

There are a lot of ways to simplify expressions. The list below contains some of them.

Command	Description
Simplify[expr]	Simplifies expr with time limit
FullSimplify[expr]	Simplify expr
Expand[expr]	Expand the expr

Cancel[expr]	Cancels out common terms in the numerator and denominator
PowerExpand	expands all powers of products and powers

Get specific parts of an expression

`First[]` and `Last[]` returns the first and last element of a list respectively. Further the denominator and the numerator is found by the commands `Denominator` and `Numerator` respectively. All these command can be used when you like to get a specific part of an expression you have received from the command `Solve`.

Example

$$A = \left\{\left\{Vout \text{ -> } \frac{gm1gm2Vin}{CL(gds1+gds2)s}\right\}\right\}$$

The command

```
Numerator[Last[First[First[A]]]]
```

will return the expression

$$gm1gm2Vin$$

while as the command

```
Denominator[Last[First[First[A]]]]
```

will return the denominator.

4 — An example

We like to calculate the transfer function of a common source gain stage with nmos input transistors where we like to take into account the gate-drain parasitic capacitance of the input transistor. The circuit is shown in Figure 1.

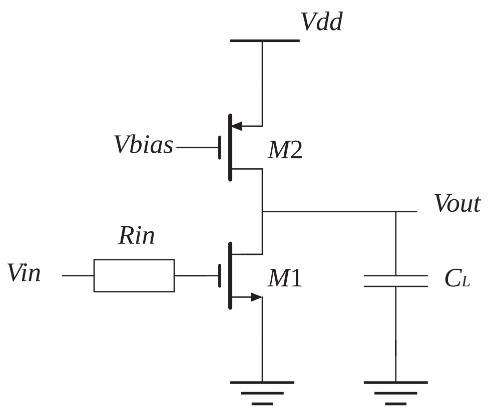

Figure 1: The schematic of a common source amplifier

The Mathematica notebook is shown below.

```
In[1]:= CommonSource = Solve[{(Vin – V1) Gin + (Vout – V1) s Cgd1 == 0, gm1 V1 +
            Vout (gds1 + gds2 + s CL) + (Vout – V1) s Cgd1 == 0}, {Vout}, {V1}]

Out[1]= {{Vout → –(Gin (gm1 – Cgd1 s) Vin) / (gds1 Gin + gds2 Gin + Cgd1 gds1 s +
            Cgd1 gds2 s + Cgd1 Gin s + CL Gin s + Cgd1 gm1 s + Cgd1 CL s^2)}}

In[2]:= CSSimplified = FullSimplify[Last[First[First[CommonSource]]]]

Out[2]= –(Gin (gm1 – Cgd1 s) Vin) / ((gds1 + gds2) Gin +
            (CL Gin + Cgd1 (gds1 + gds2 + Gin + gm1)) s + Cgd1 CL s^2)
```

We like to find the expression for the zeros and for an approximation of the poles.

```
In[3]:= num = Numerator[CSSimplified];

In[4]:= zero = Last[First[First[Solve[num == 0, s]]]]
```

$$\text{Out[4]=}\quad \frac{\text{gm1}}{\text{Cgd1}}$$

```
In[5]:= den = Denominator[CSSimplified]

Out[5]= (gds1 + gds2) Gin + (CL Gin + Cgd1 (gds1 + gds2 + Gin + gm1)) s + Cgd1 CL s^2
```

We like to convert the denomintor to the form a+b s+c s^2

```
In[6]:= a = (gds1 + gds2) Gin;
        b = CL Gin + Cgd1 (gds1 + gds2 + Gin + gm1); c = Cgd1 CL;
```

If the poles are assumed to be well separated then could the poles approximately be given by (1+s/p1)(1+s/p2)=1+s(1/p1+1/p2)+s^2/(p1 p2) approximately 1+s/p1+s^2/(p1+p2). Identifying with the denominator above.

```
In[7]:= p1 = a / b
```

$$\text{Out[7]=}\quad \frac{(\text{gds1} + \text{gds2})\ \text{Gin}}{\text{CL Gin} + \text{Cgd1}\ (\text{gds1} + \text{gds2} + \text{Gin} + \text{gm1})}$$

If gds<<gm, gds<<Gin, Cgd<<CL

$$\text{In[8]:=}\quad \text{p1Simplified} = \frac{(\text{gds1} + \text{gds2})\ \text{Gin}}{\text{CL Gin}}$$

$$\text{Out[8]=}\quad \frac{\text{gds1} + \text{gds2}}{\text{CL}}$$

```
In[9]:= p2 = a / c / p1Simplified
```

$$\text{Out[9]=}\quad \frac{\text{Gin}}{\text{Cgd1}}$$

The DC voltage gain

```
In[10]:= DCgain = FullSimplify[CommonSource /. s → 0]
```

$$\text{Out[10]=}\quad \left\{\left\{\text{Vout} \to -\frac{\text{gm1 Vin}}{\text{gds1} + \text{gds2}}\right\}\right\}$$