Lecture 4, Noise

Noise and distortion

What did we do last time?

Operational amplifiers

- Circuit-level aspects
- Simulation aspects
- Some terminology

Some practical concerns

- Limited current
- Limited bandwidth
- etc

What will we do today?

Noise

- Circuit noise
- Thermal noise
- Flicker noise

Distortion

- What sets the (non)linearity in our CMOS devices?

The "741 amplifier"

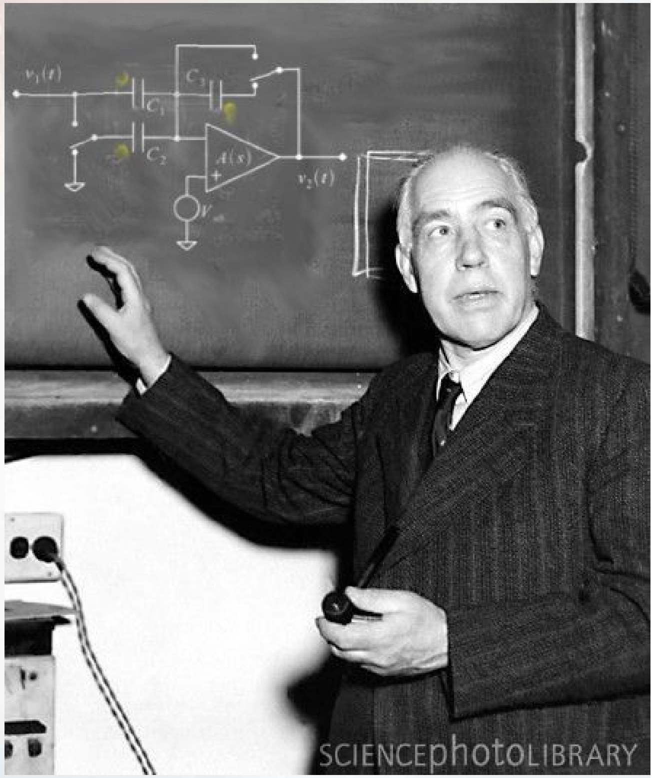
Texas instruments

opa 336 - what is the bandwidth?

opa 358 - what is the DC gain?

Analog Devices

AD854x - what is the DC gain, or what is the open-loop bandwidth?



Operational amplifier architectures

Left-overs

Examples

- Telescopic
- Two-stage
- Folded-cascode
- Current-mirror

Essentially just cascaded stages of different kinds

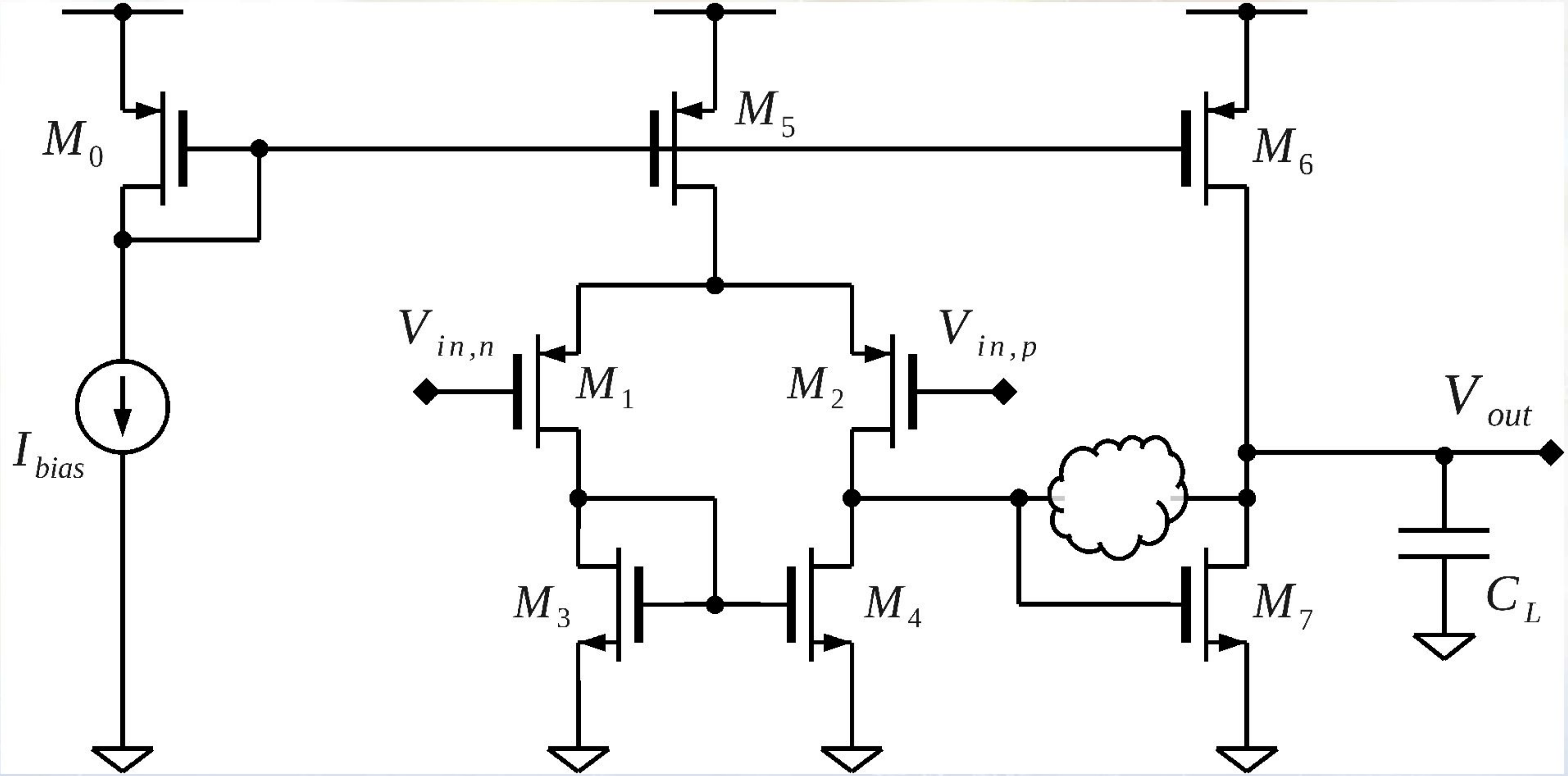
Telescopic OTA

Stack many cascodes on top of each-other and use gain-boosting, etc.

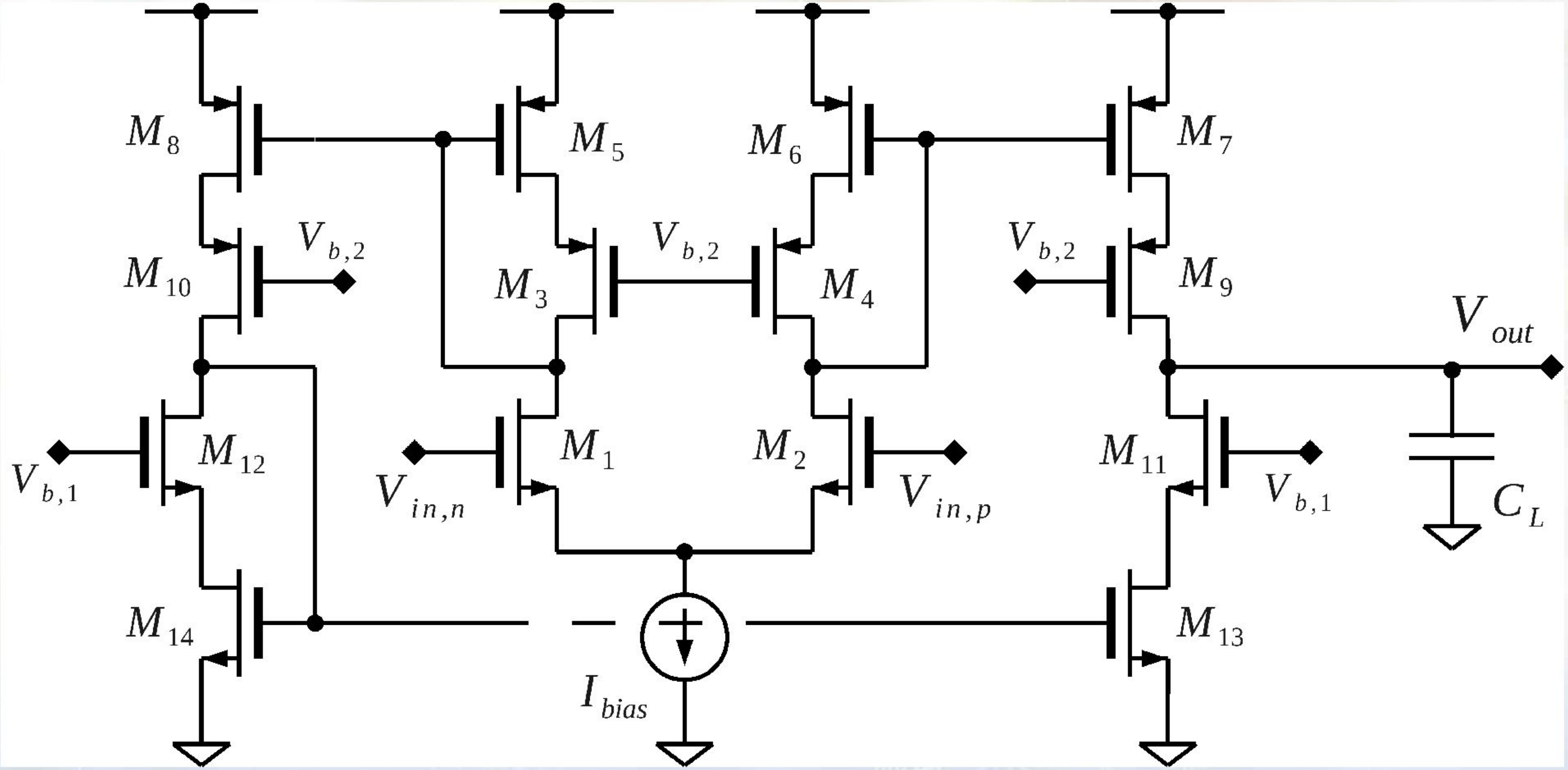
Omitted, since it is not applicable for modern processes.

The swing is eaten up.

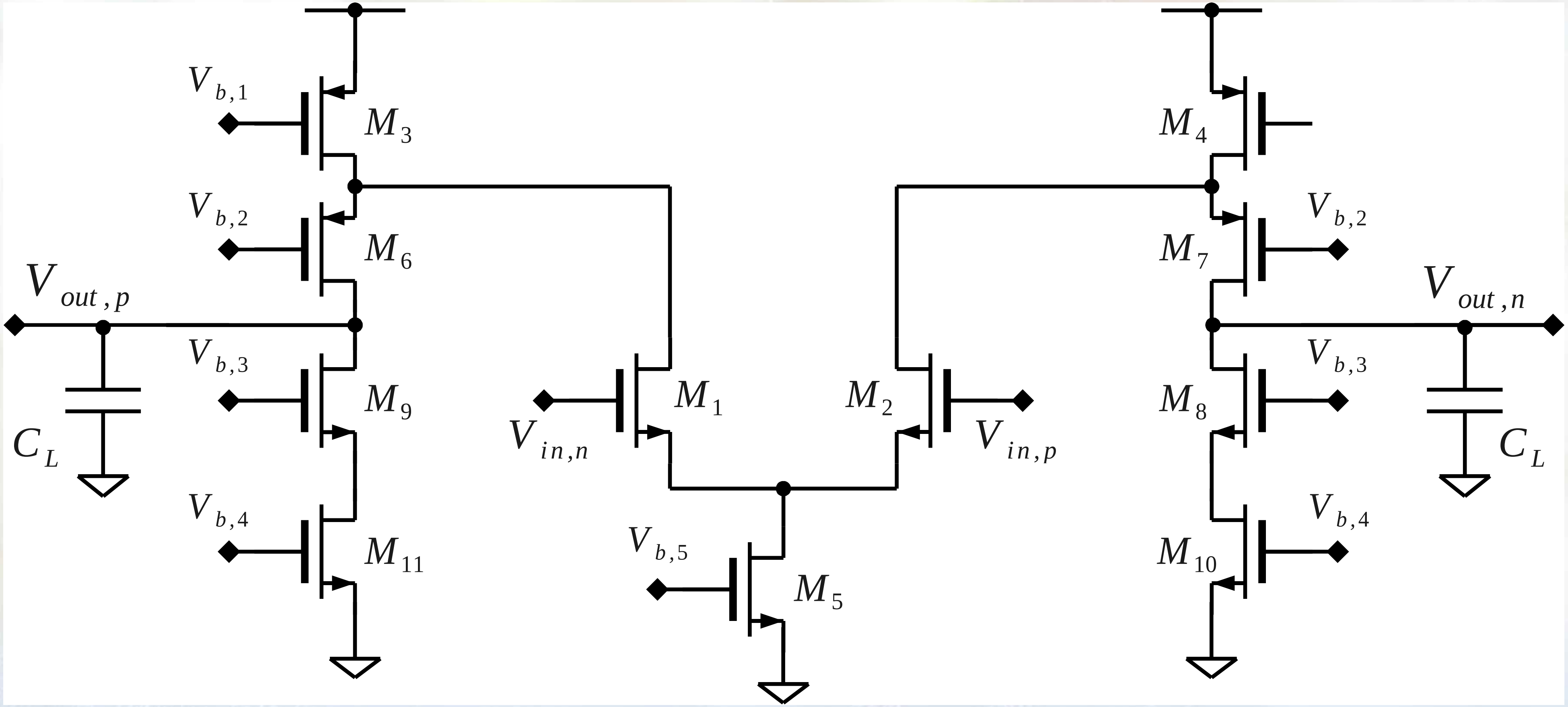
Two-stage OP/OTA



Current-mirror OP/OTA



Folded-cascode OP/OTA



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OP/OTA Compilation

Cookbook recipes

Hand-outs with step-by-step explanation of the design of OP/OTAs

http://www.es.isy.liu.se/courses/ANDA/download/opampRef/ANTIK_0NNN_LN_opampHandsouts_A.pdf

Compensation techniques

http://www.es.isy.liu.se/courses/ANDA/download/opampRef/ANTIK_0NNN_LN_opampCompensationTable_A.pdf

Amplifier classes

Not really covered in this course.

Different classes, such as

Class A, B, AB, C, D, E, F, G, H, I, K, S, T, Z, etc.

Class A

Essentially the common-source stage

Class AB

Essentially a push-pull configured class A

Noise

Any circuit has noise and you, as a designer, have to reduce it or minimize the impact of it

"A disturbance, especially a random and persistent disturbance, that obscures or reduces the clarity of a signal."

Consequences

We need to use stochastic variables and power spectral densities, expectation values, etc.

We need to make certain assumptions (models) of our noise sources in order to calculate

Superfunction and spectral densitites

Power spectral density (PSD)

Superfunction

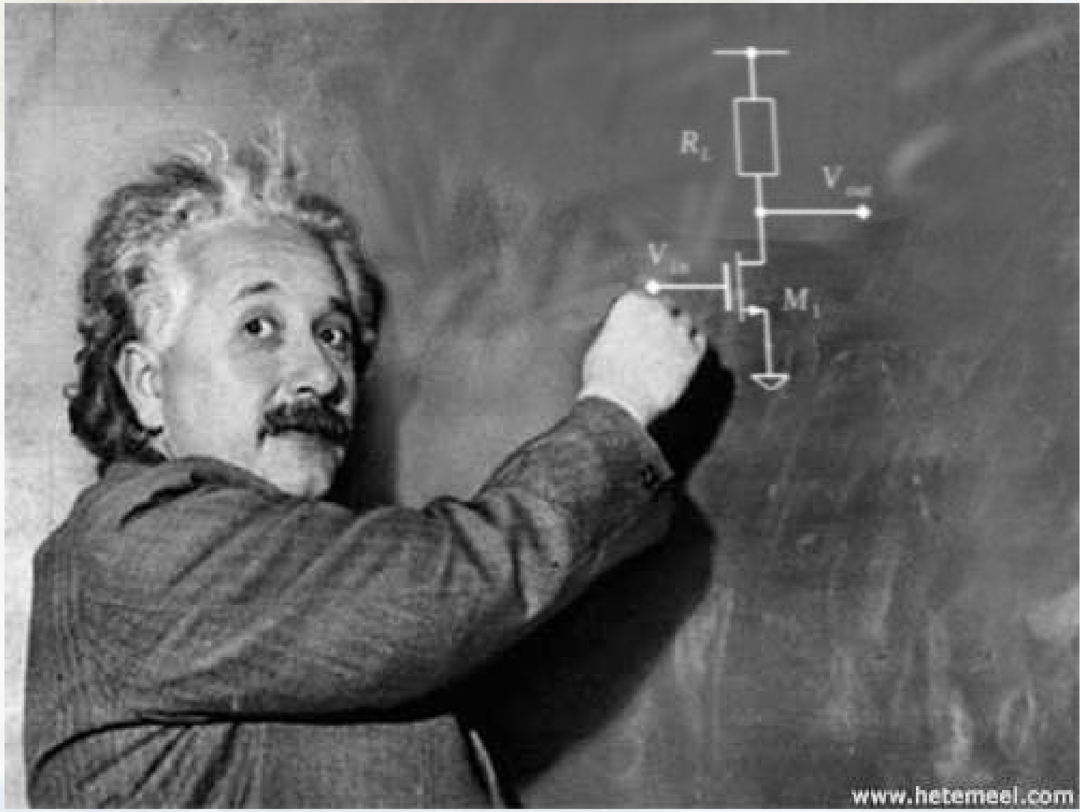
$$S_0(f) = \sum |A_i(f)|^2 \cdot S_i(f)$$

Total noise

$$V_{tot}^2 = \int v_n^2(f)\, df$$

Brickwall noise

$$V_{tot}^2 = v_n^2(0) \cdot \frac{p_1}{4}$$



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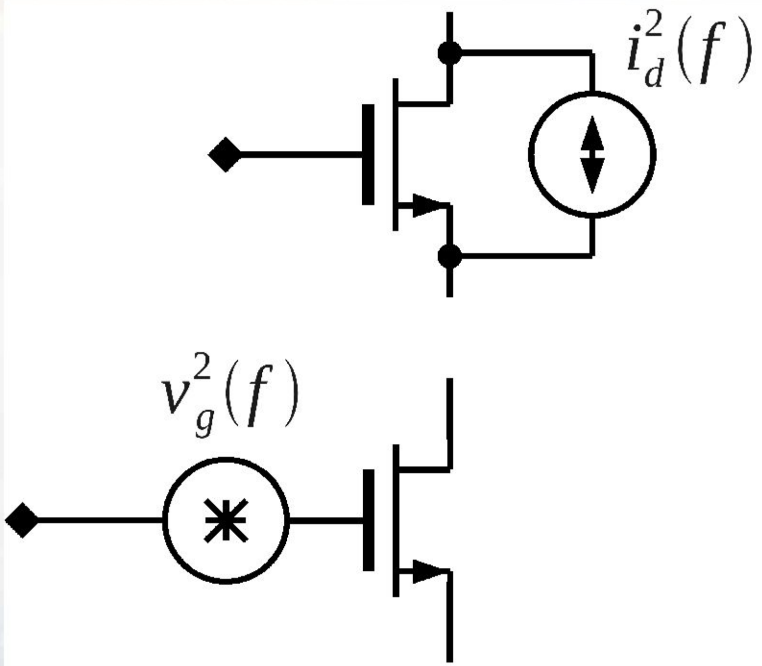
Thermal noise, white noise

Resistor

$$v_n^2 = 4kTR$$ or $$i_n^2 = \frac{v_n^2}{R^2} = \frac{4kT}{R}$$



Transistor

$$v_g^2 = \frac{4kT\gamma}{g_m}$$ or $$i_d^2 = v_g^2 \cdot g_m^2 = 4kT\gamma g_m$$

Opamp

We'll come back to this...

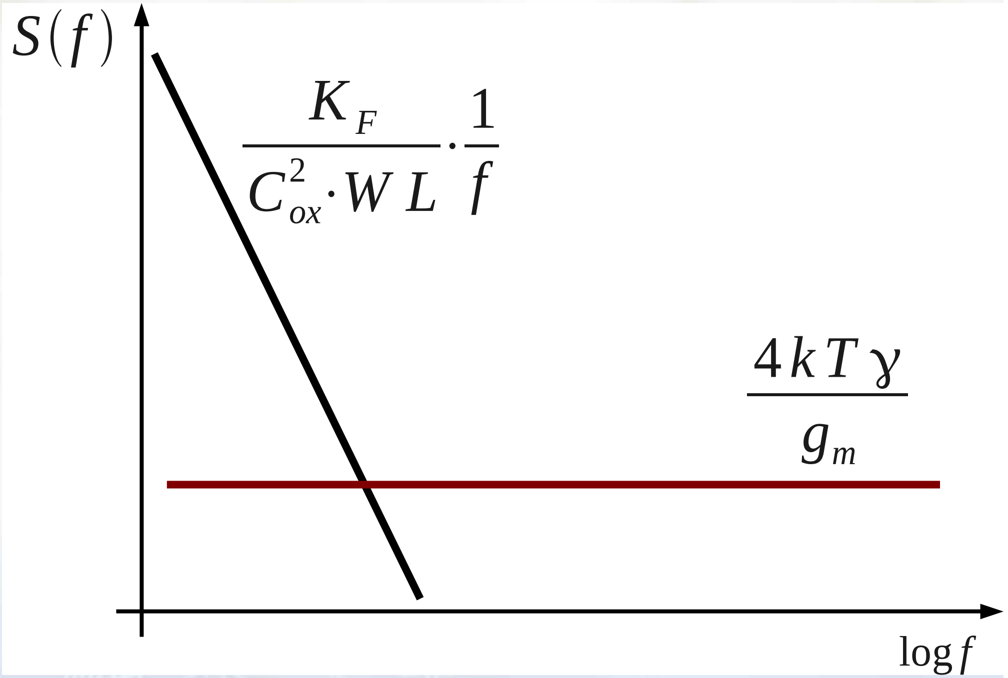
Flicker noise, 1/f-noise, pink noise

Resistor

$$\overline{v_n^2} = \frac{v_{bias}^2 \cdot k}{W\,L \cdot f} \text{ and } \overline{i_n^2} = R^2 \cdot \overline{v_n^2}$$

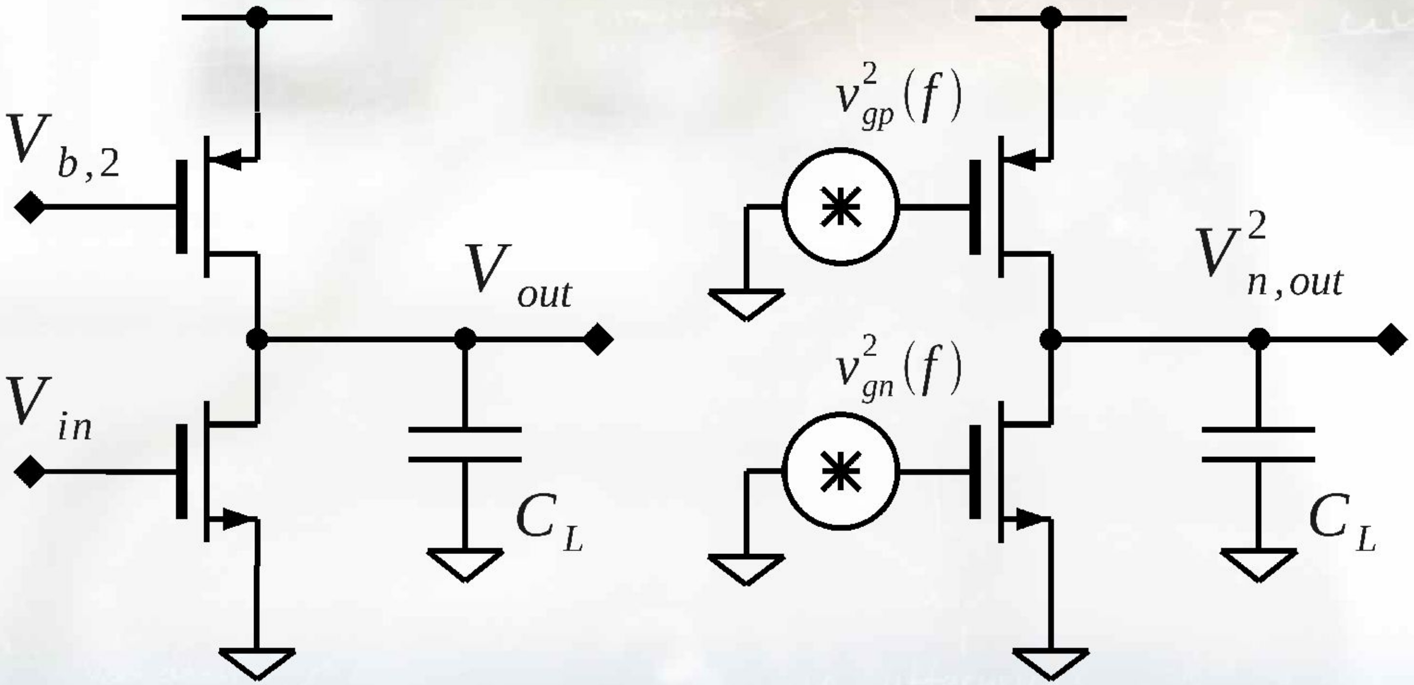
Transistor

$$\overline{v_g^2} = \frac{K_F}{C_{ox}^2 \cdot W\,L} \cdot \frac{1}{f} \text{ and } \overline{i_d^2} = g_m^2 \cdot \overline{v_n^2}$$



Noise compiled in one example

Common-source with noisy transistors



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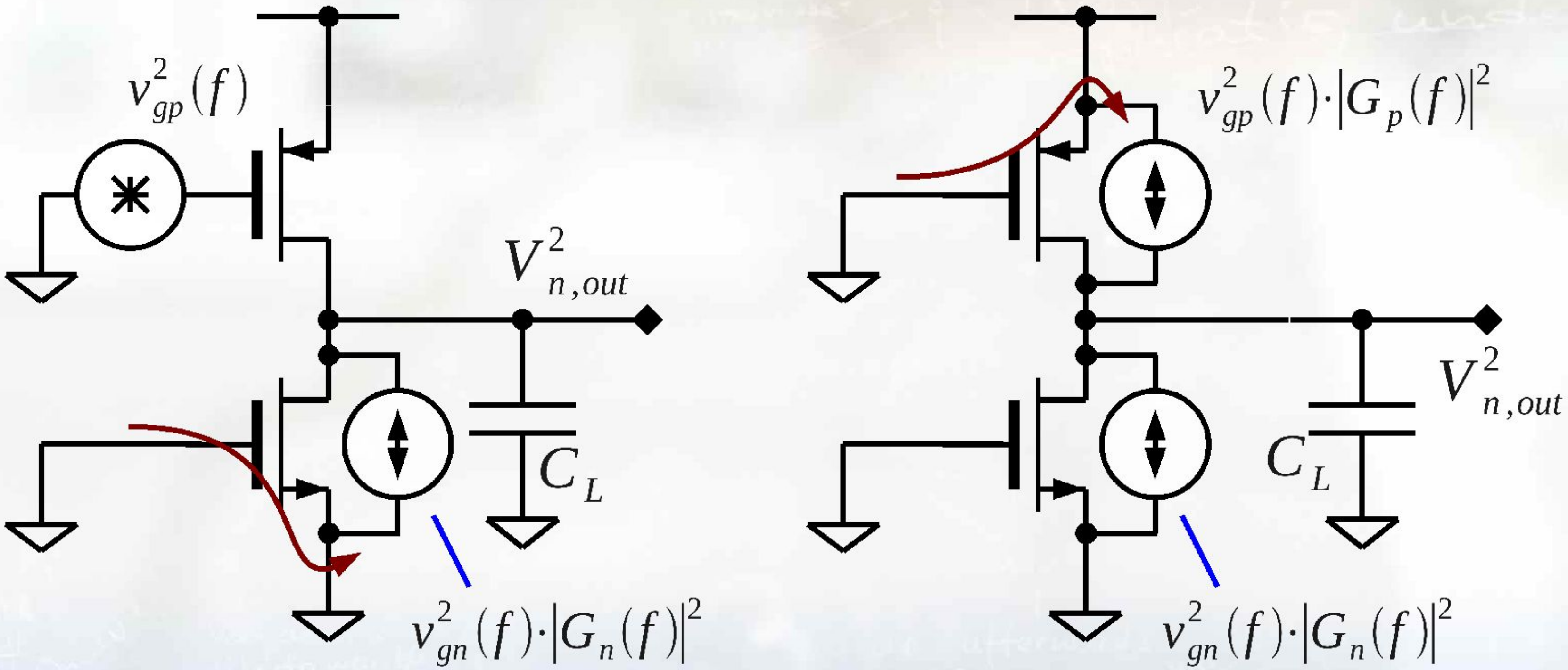
Noise compiled in one example, cont'd

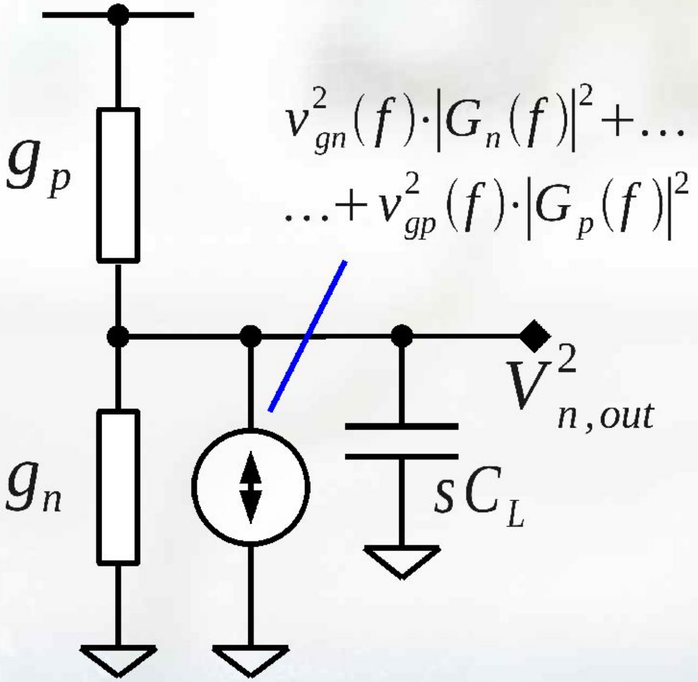
Potentially reorder the sources for convient calculations

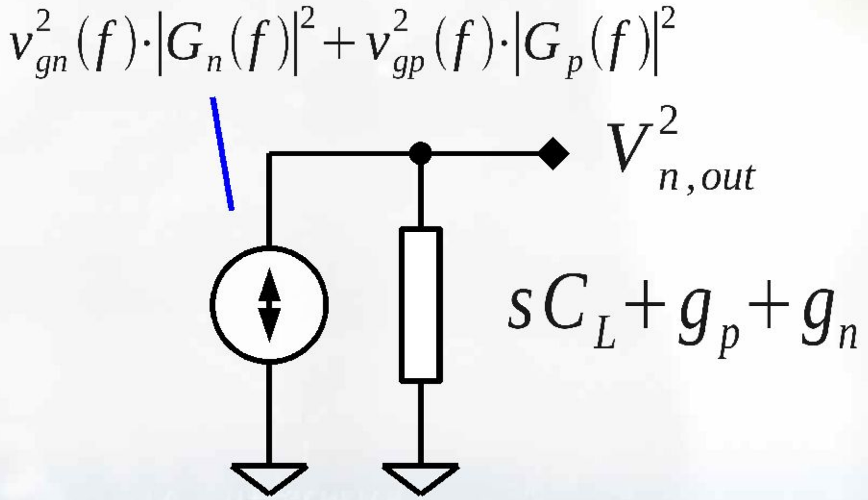


Notice the use of transconductance from voltage to current.

Noise compiled in one example, cont'd

Equivalent small-signal schematics (ESSS)





Noise compiled in one example, cont'd

The general transfer function to the output is given by

$$V^2_{n,out}(f)=\frac{v^2_{gn}(f)\cdot|G_n(f)|^2+v^2_{gp}(f)\cdot|G_p(f)|^2}{|sC_L+g_p+g_n|^2}$$

Insert the values

$$V^2_{n,out}(f)=4kT\gamma\frac{\frac{g^2_{mn}}{g_{mn}}+\frac{g^2_{mp}}{g_{mp}}}{(g_p+g_n)^2\cdot\left|1+\frac{s}{\frac{g_p+g_n}{C_L}}\right|}=4kT\gamma\frac{\frac{g_{mn}+g_{mp}}{(g_p+g_n)^2}}{\left|1+\frac{s}{\frac{g_p+g_n}{C_L}}\right|^2}$$

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Noise compiled in one example, cont'd

Use the brickwall approach

$$V_{n,tot}^2 = \int V_{n,out}^2(f) = V_{n,out}^2(0) \cdot \frac{p_1}{4}$$

$$V_{n,tot}^2 = 4kT\gamma \frac{g_{mn}+g_{mp}}{(g_p+g_n)^2} \cdot \frac{g_p+g_n}{4C_L} = \frac{kT\gamma}{C_L} \cdot \frac{g_{mn}+g_{mp}}{g_p+g_n}$$

Conclude

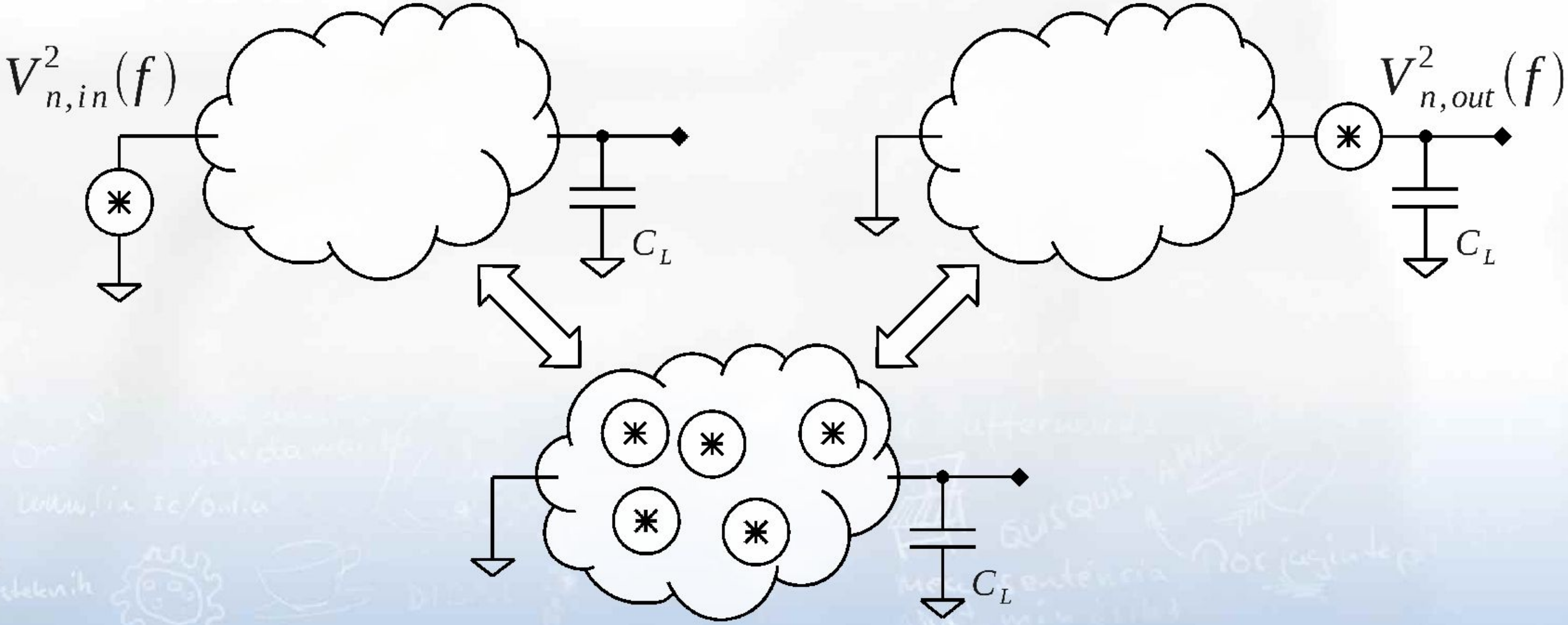
$$V_{n,tot}^2 = \frac{kT\gamma}{C_L} \cdot A_0 \cdot \left(1 + \frac{g_{mp}}{g_{mn}}\right)$$

kT/C noise!

Input-referred noise

Revert the output noise back to the input:

$$V^2_{n,in}(f) = \frac{V^2_{n,out}(f)}{\left|A_{in}(f)\right|^2}$$



The common-source example

Input-referred noise

$$V_{n,in}^2(f) = \left[4kT\gamma \frac{\frac{(g_{mn}+g_{mp})}{(g_p+g_n)^2}}{\left|1+\frac{s}{\frac{(g_p+g_n)}{C_L}}\right|^2} \right] \cdot \left[\frac{\left|1+\frac{s}{\frac{(g_p+g_n)}{C_L}}\right|^2}{\frac{g_{mn}^2}{(g_p+g_n)^2}} \right] = \frac{4kT\gamma}{g_{mn}} \cdot \left(1+\frac{g_{mp}}{g_{mn}}\right)$$

What does this mean?

Bias transistor should be made with low transconductance!

Visible from the formula

Gain transistors should be made with high transconductance!

Visible from the formula

Gain should be distributed between multiple stages (Friis)

Left as an exercise

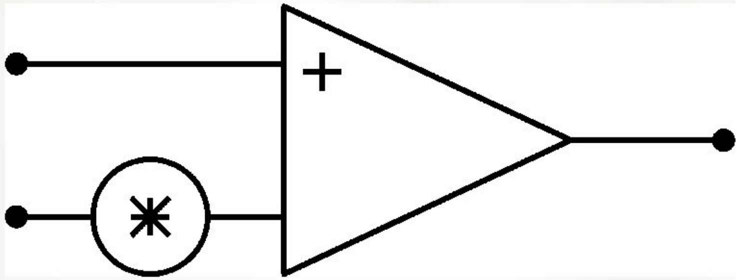
Noise in operational amplifiers

Opamps assumed to have input referred noise sources on its inputs

A voltage source

Two current sources

(Often ignored in CMOS opamps)



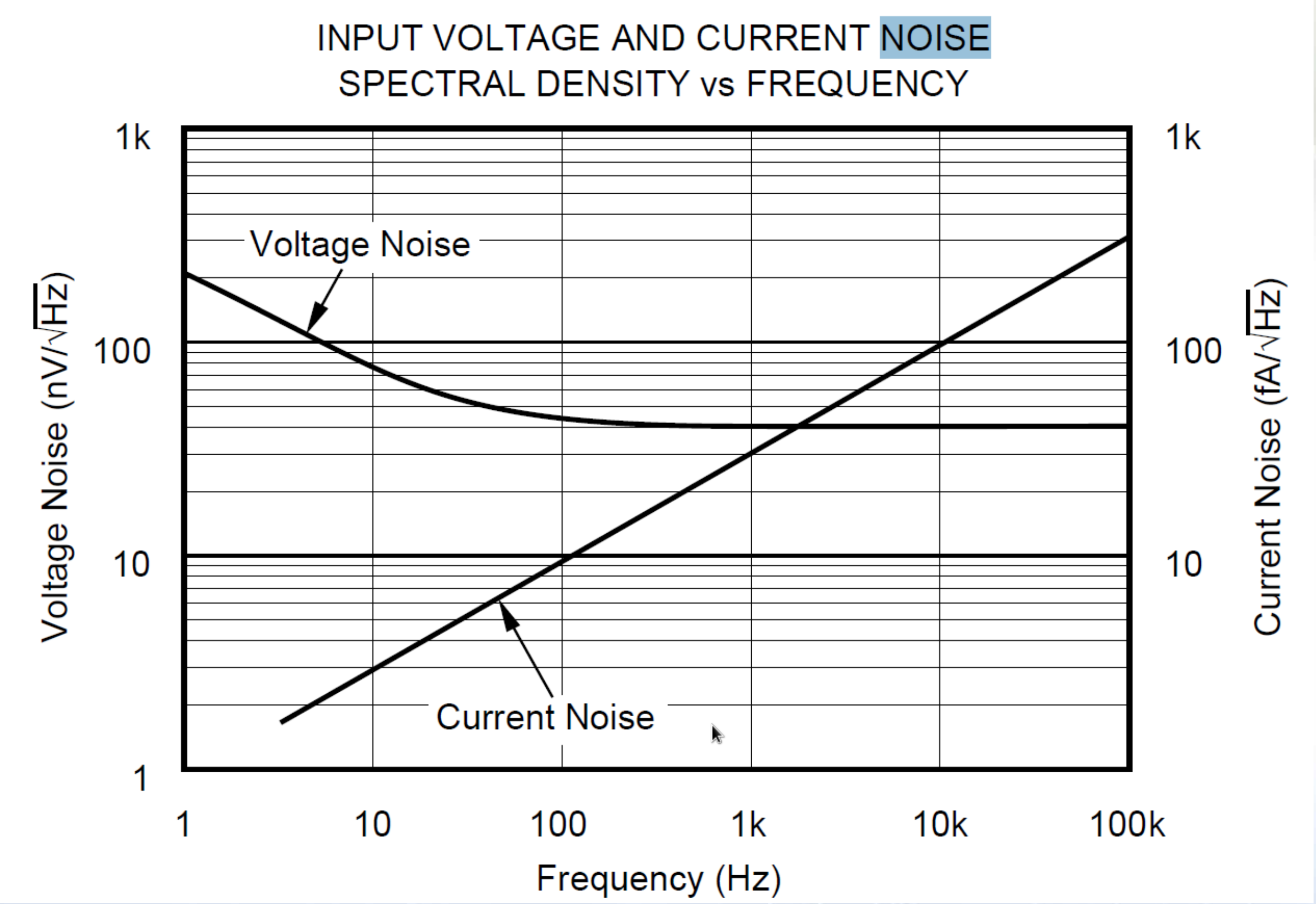
Input referred noise can be calculated according to previous principles and will be given by a spectral density

Example from "741" opamp

OPA336N (Texas Instruments)

Input Voltage Noise, f = 0.1 to 10 Hz	3 µVp-p
Input Voltage Noise Density, f = 1 kHz $\left(e_n\right)$	40 nV/√Hz
Current Noise Density, f = 1 kHz $\left(i_n\right)$	30 fA/√Hz

Example from "741" opamp



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Noise in OP, example

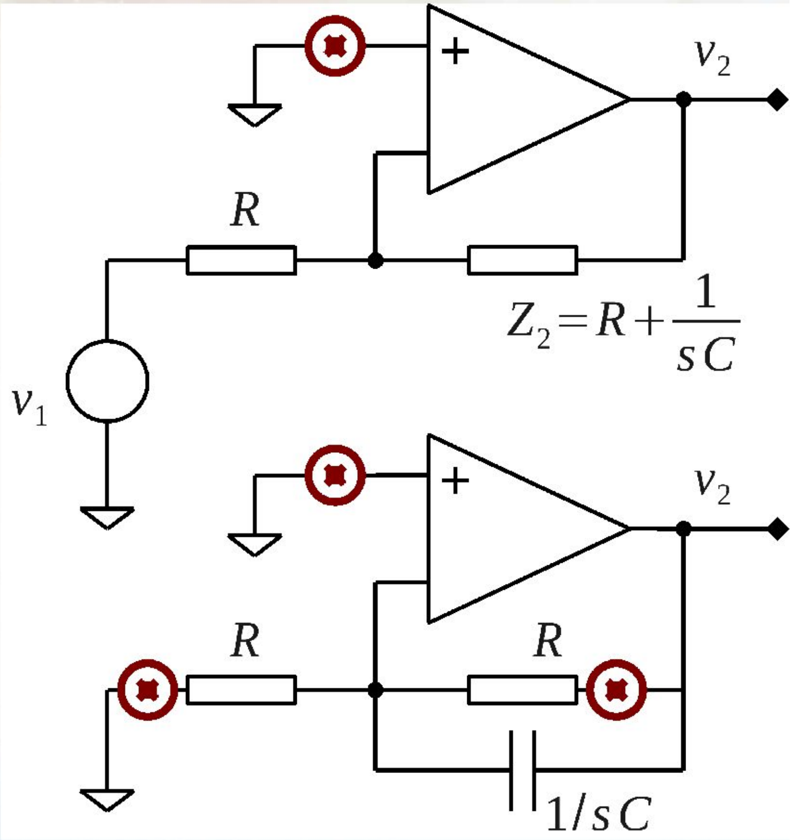
Noisy resistors and noisy opamp

$$\frac{V_2(s)}{V_1(s)} = \frac{-1}{1+sRC}$$

The noise sources, opamp

$$\frac{V_2(s)}{V_n(s)} = 1 + \frac{1}{1+sRC}$$

Ignore the current for now



Noise in OP, example cont'd

PSD

$$S_n(f) = \left| 1 + \frac{1}{1 + j2\pi f \cdot RC} \right|^2$$

A nasty transfer function - the noise never reaches zero!

Practically

The opamp unity-gain bandwidth will bandlimit the noise for which we can use the brickwall approach:

$$P_{tot,op} \approx 2 \cdot v_{n,op}^2(0) \cdot \frac{\omega_{ug}}{4}$$

Noise in OP, example cont'd

The noise sources, resistors

$$\frac{V_2(s)}{V_{r2}(s)} = \frac{1}{1+sRC} \text{ and } \frac{V_2(s)}{V_{r1}(s)} = \frac{-1}{1+sRC}$$

PSD

$$S_n(f) = \left| \frac{1}{1+j2\pi f \cdot RC} \right|^2$$

A low-pass filter response - use the brickwall approach

$$P_{tot,R} \approx 2 \cdot v_{n,R}^2(0) \cdot \frac{1/RC}{4}$$

Noise in OP, example cont'd

Combined

$$P_{tot} = P_{tot,R} + P_{tot,op} \approx 2 \cdot v_{n,op}^2(0) \cdot \frac{\omega_{ug}}{4} + 2 v_{n,R}^2(0) \cdot \frac{1/RC}{4}$$

$$P_{tot} \approx v_{n,op}^2(0) \cdot \frac{\omega_{ug}}{2} + 4kTR \cdot \frac{1/RC}{2} = v_{n,op}^2(0) \cdot \frac{\omega_{ug}}{2} + \frac{2kT}{C}$$

This is the power normalized over 1 Ohm

We could define the rms voltage as:

$$v_{o,rms} = \sqrt{P_{tot}} \approx \sqrt{v_{n,op}^2(0) \cdot \frac{\omega_{ug}}{2} + \frac{2kT}{C}}$$

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Noise in OP, example cont'd, values

Example:

$C = 1$ nF, $R = 10$ kOhm, i.e., a bandwidth of $f_{bw} \approx 14$-kHz

OPA336:

$v_{n,op}^2(0) \approx 1.6 \cdot 10^{-15}$ sqV/Hz, and $f_{ug} \approx 100$ kHz

$$P_{tot} \approx 1.6 \cdot 10^{-15} \cdot \frac{10^5}{2} + 2 \cdot \frac{4 \cdot 10^{-21}}{10^{-9}} = 88 \cdot 10^{-12}$$

or

$$v_{o,rms} \approx 9.4 \text{ uV}$$

(compare with the reported 3 uV in the data sheet)

Signal-to-noise ratio (SNR)

At the output of our system, we will have a certain signal power. The signal-to-noise ratio (SNR) determines the quality of the system:

$$SNR = \frac{P_{sig}}{P_{noise}} \quad \text{or} \quad SNR = 10 \cdot \log_{10}\left(\frac{P_{sig}}{P_{noise}}\right) \quad \text{or} \quad SNR = 20 \cdot \log_{10}\left(\frac{v_{s,rms}}{v_{n,rms}}\right)$$

OPA336 example

Assume signal swing at output is $v_{rms} = 1$ V.

Then the SNR is

$$SNR \approx 20 \cdot \log_{10}\left(\frac{1}{9.4 \cdot 10^{-6}}\right) \approx 100$$ dB (approximately 16 bits)

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Distortion

Frequency-domain measures

Spurious-free dynamic range, SFDR

Harmonic distortion, HD

Signal-to-noise-and-distortion ratio, SNDR

Amplitude domain measures

Compression (clipping)

Offset

Distortion

No circuit is fully linear...

$$Y = \alpha_0 + \alpha_1 \cdot X + \alpha_2 \cdot X^2 + \alpha_3 \cdot X^3 + \alpha_4 \cdot X^4 + \ldots$$

Example

$X = \sin \omega t$ is the sinusoidal, steady-state signal

$\alpha_1 = 1, \alpha_2 = 0.01$ are the characteristic coefficients of the system

Results in an output as

$$Y(t) = \sin \omega t + \alpha_2 \cdot \sin^2 \omega t = \sin \omega t + \alpha_2 \cdot \frac{1 - \cos 2\omega t}{2}$$

Distortion, cont'd

Results in a DC shift and a distortion term

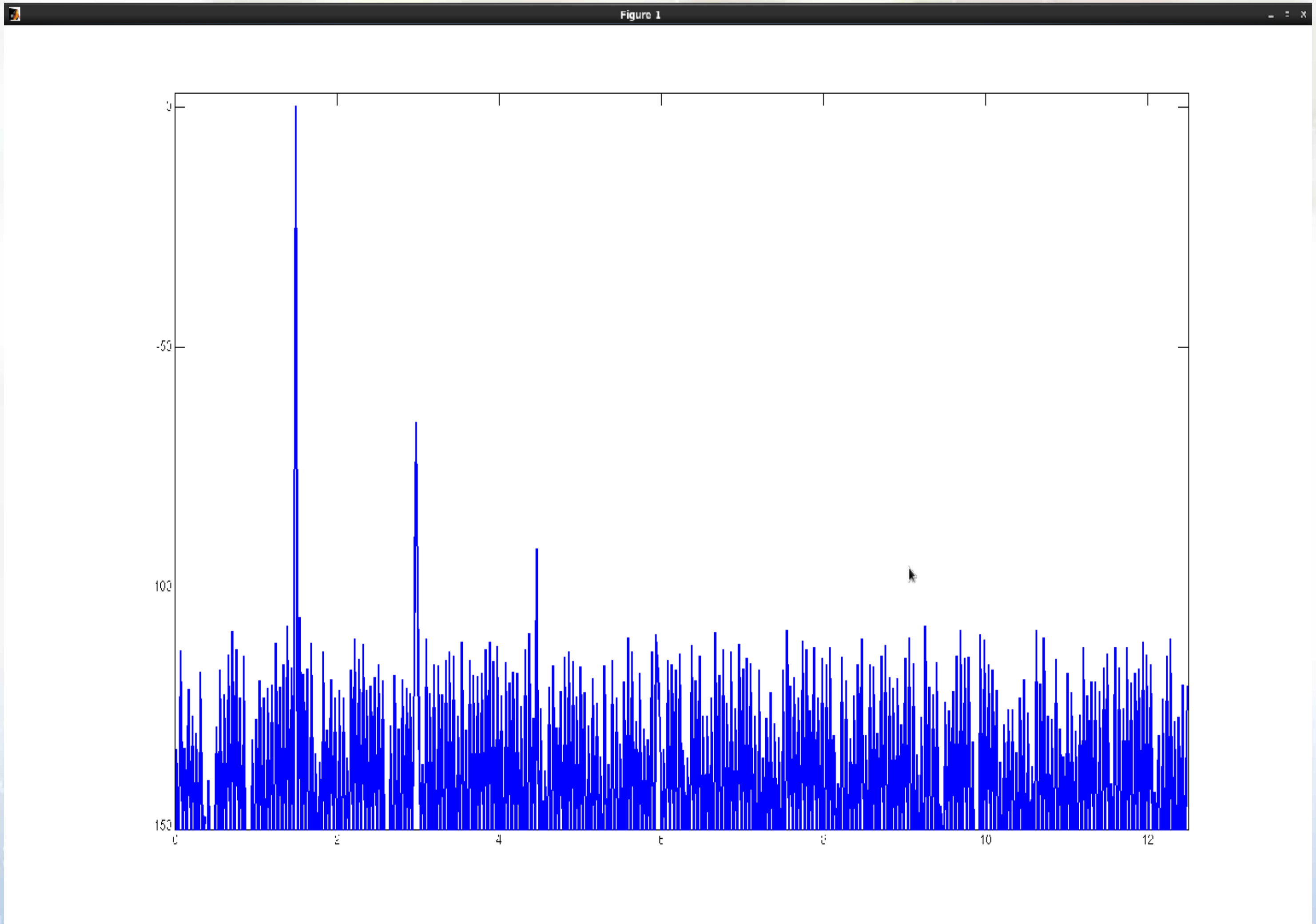
$$Y = \underbrace{\frac{\alpha_2}{2}}_{\text{DC shift}} + \underbrace{\sin \omega t}_{\text{desired}} - \underbrace{\frac{\alpha_2}{2} \cdot \cos 2 \omega t}_{\text{distortion}}$$

Harmonic distortion

$$HD_2 = \frac{1^2}{\left(\alpha_2/2\right)^2} = \frac{1}{\left(0.01/2\right)^2} = 40000,$$

i.e., approximately 46 dB

Frequency domain measures



Distortion, fully differential circuits

Assume distortion is identical in two branches

$$Y_p = \alpha_0 + \alpha_1 \cdot X_p + \alpha_2 \cdot X_p^2 + \alpha_3 \cdot X_p^3 + \alpha_4 \cdot X_p^4 + \ldots$$ and

$$Y_n = \alpha_0 + \alpha_1 \cdot X_n + \alpha_2 \cdot X_n^2 + \alpha_3 \cdot X_n^3 + \alpha_4 \cdot X_n^4 + \ldots$$

Difference

$$\Delta Y = Y_p - Y_n = (\alpha_0 - \alpha_0) + \alpha_1 \cdot (X_p - X_n) + \alpha_2 \cdot \left(X_p^2 - X_n^2\right) + \ldots$$

Further on, assume input is already OK

$$X_p = -X_n = \frac{\Delta X}{2}$$

Distortion, fully differential, cont'd

Results in

$$\Delta Y = \alpha_1 \cdot (X_p - (-X_p)) + \alpha_2 \cdot \left(X_p^2 - (-X_p)^2\right) + \alpha_3 \cdot \left(X_p^3 - (-X_p)^3\right) + \ldots$$

and eventually

$$\Delta Y = \alpha_1 \cdot \Delta X + \frac{\alpha_3}{4} \cdot \Delta X^3 + \ldots$$

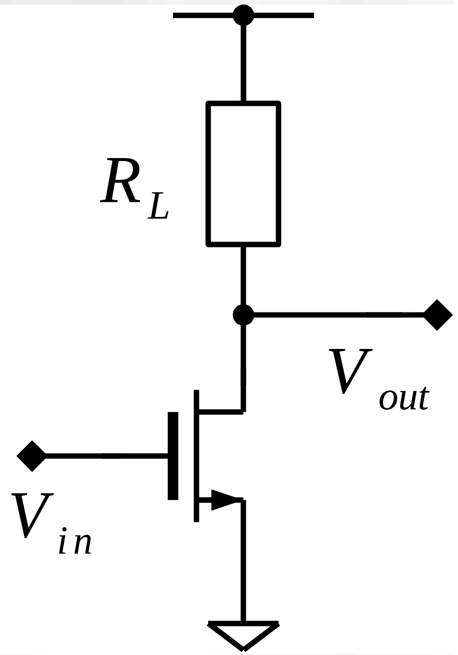
Even-order terms disappear!

Distortion in a common-source

Assume a common-source stage with resistive load

First-order model $I_D = \alpha \cdot V_{eff}^2$

$$V_{out} = V_{DD} - R \cdot I_D = V_{DD} - R \cdot \alpha \cdot V_{eff}^2$$



Assume a limited input signal (no clipping)

$$V_{eff}(t) = V_{eff0} + V_x \cdot \sin \omega t$$

Form the output

$$V_{out}(t) = V_{DD} - R \cdot \alpha \cdot \left(V_{eff0} + V_x \sin \omega t \right)^2$$

$$V_{out}(t) = V_{DD} - R \cdot \alpha \cdot \left(V_{eff0}^2 + 2 V_{eff0} V_x \cdot \sin \omega t + V_x^2 \cdot \sin^2 \omega t \right)$$

Distortion in a common-source, cont'd

Continue to rewrite using trigonometrics

$$V_{out}(t) = V_{DD} - R \cdot \alpha \cdot \left(V_{eff0}^2 + 2 V_{eff0} V_x \cdot \sin \omega t + \frac{V_x^2}{2} \cdot (1 - \cos 2 \omega t) \right)$$

Analyze

$$V_{out}(t) = \underbrace{V_{DD} - R \cdot \alpha \cdot V_{eff0}^2 + \frac{V_x^2}{2}}_{\text{DC}} + \underbrace{2 V_{eff0} V_x \cdot R \cdot \alpha \cdot \sin \omega t}_{\text{desired signal}} - \underbrace{\frac{V_x^2}{2} \cdot \cos 2 \omega t}_{\text{distortion}}$$

Compression analysis

Signal power scales "linearly" with amplitude

$$V_{out}(t) = \underbrace{V_{DD} - R \cdot \alpha \cdot V_{eff0}^2 + \frac{V_x^2}{2}}_{\text{DC}} + \underbrace{2 V_{eff0} V_x \cdot R \cdot \alpha \cdot \sin \omega t}_{\text{desired signal}} - \underbrace{\frac{V_x^2}{2} \cdot \cos 2 \omega t}_{\text{distortion}}$$

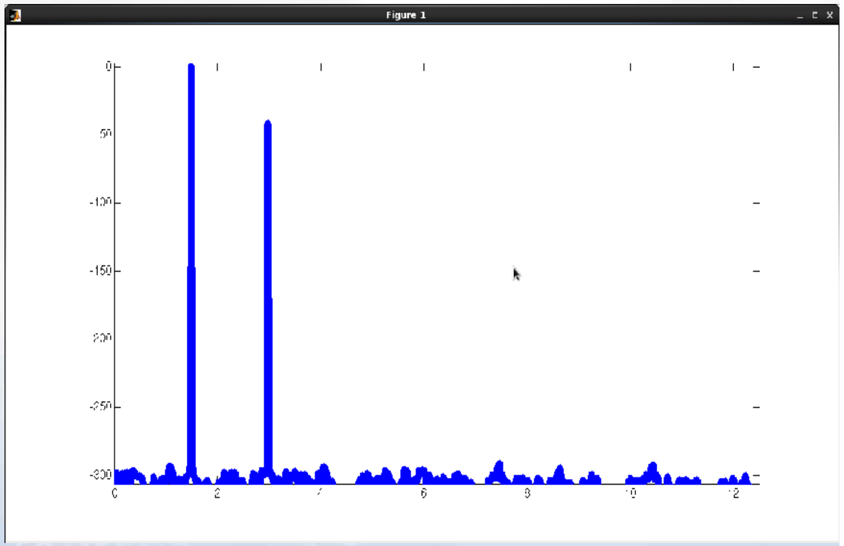
Distortion power scales "quadratically"

At some point they will meet.

Intercept points

Output and input-referred IIP, OIP

Common measures of nonlinearity



Distortion vs Noise, concludingly

High signal power gives high signal-to-noise ratio

High signal power gives low signal-to-distortion ratio

This means that you need to distribute the gain between the different stages accordingly and trade-off between the two.

What did we do today?

Noise

Circuit noise

Thermal noise

Flicker noise

Distortion

What sets the (non)linearity in our CMOS devices?

What will we do next time?

PCB vs silicon

What are the differences when scaling up the geometries

Components

Surface-mounted components

PCB

Some PCB specifics