## Lecture 9, ATIK

## Data converters 2

## What did we do last time?

Data converters

Fundamentals

Overview of different types of data converters

A first glance at oversampling

## What will we do today?

Recapture the oversamping data converters
Interpolating/Decimating

Sigma-delta modulator

DACs
Designing a current-steering DAC

## Current-steering DAC

All current directed to the output implies high efficiency
No buffer required, implies high speed
Short design time
Current output and no buffer implies now slow rate limitations

$$
I_{o u t}(X)=X \cdot I_{u n i t}=\sum x_{k} \cdot 2^{k-1} \cdot I_{u n i t}
$$

with

$$
X=X_{0}+\Delta X
$$


gives

$$
V_{\text {out }}(X)=Z_{L} \cdot I_{\text {out }}(X)=X \cdot Z_{L} \cdot I_{\text {unit }}=Z_{L} \cdot \sum x_{k} \cdot 2^{k-1} \cdot I_{\text {unit }}
$$

## Current-steering DAC, MOS implementation

Switches are implemented by NMOS

Transmission gates normally does not make any impact

Current sources are typically cascoded transistors

C.f. current mirror

Each weight consists of $k$ unit sources in parallel


## Current-steering DAC, main design tasks

Designing a good differential pair

Designing accurate current mirrors

Optimization

Trade-offs


## Output impedance



## Output impedance

Form the differential output

$$
V_{\text {out }}(\Delta X)=\frac{Z_{L} \cdot I_{\text {unit }} \cdot\left(X_{0}+\Delta X\right)}{1+Z_{L} \cdot Y_{\text {unit }} \cdot\left(X_{0}+\Delta X\right)}-\frac{Z_{L} \cdot I_{\text {unit }} \cdot\left(X_{0}-\Delta X\right)}{1+Z_{L} \cdot Y_{\text {unit }} \cdot\left(X_{0}-\Delta X\right)}
$$

Scale and introduce helping variables

$$
\begin{aligned}
& \frac{V_{\text {out }}(\Delta X)}{Z_{L} \cdot I_{\text {unit }} \cdot X_{0}}=\frac{1+\delta x}{1+\eta \cdot(1+\delta x)}-\frac{1-\delta x}{1+\eta \cdot(1-\delta x)} \\
& \frac{V_{\text {out }}(\Delta X)}{Z_{L} \cdot I_{\text {unit }} \cdot X_{0}}=\frac{2 \delta x}{(1+\eta \cdot(1+\delta x)) \cdot(1+\eta \cdot(1-\delta x))}
\end{aligned}
$$

and approximate

$$
\frac{V_{\text {out }}(\Delta X)}{Z_{L} \cdot I_{\text {unit }} \cdot X_{0}}=\frac{2 \delta x /(1+\eta)^{2}}{1-\left(\frac{\eta}{1+\eta}\right)^{2} \cdot \delta x^{2}} \approx \frac{2 \delta x}{(1+\eta)^{2}} \cdot\left(1+\left(\frac{\eta}{1+\eta}\right)^{2} \cdot \delta x^{2}\right)
$$

## Output impedance, linearity requirement

Harmonic distortion is the power ratio between fundamental and distortion term:

$$
\mathrm{HD}_{3} \sim \frac{1}{8 \cdot\left|\frac{\eta}{1+\eta}\right|^{4}}=\frac{1}{8}+\frac{1}{8 \cdot \eta^{4}} \approx \frac{1}{8 \cdot \eta^{4}}=\frac{1}{8 \cdot\left(Z_{L} \cdot Y_{\text {unit }} \cdot X_{0}\right)^{4}}=\frac{1}{8} \cdot\left(\frac{Z_{\text {unit }} / Z_{L}}{2^{N-1}}\right)^{4}
$$

For a given output impedance, $Z_{u n i t}$ :
With more bits, the harmonic distortion gets worse
With a larger load resistance, the distortion gets worse
Example:

$$
\begin{gathered}
Z_{\text {unit }}=1 \mathrm{M} \Omega, Z_{L}=50 \Omega, N=10 \text { gives } \mathrm{HD}_{3} \approx 55 \mathrm{~dB} \text { such that } \\
\mathrm{ENOB}=\frac{\mathrm{HD}_{3}-1.76}{6.02} \approx 9 \text { bits }
\end{gathered}
$$

## Output impedance - what does this mean?



The higher up in frequency, the worse impedance ratio!

## Impact of capacitances

Single-transistor has pole/zero at

$$
p_{1}=\frac{g_{d s}}{C}, z_{1}=\frac{g_{m}}{C}
$$

DC

$$
R_{\text {out }}=\frac{g_{m}}{g_{P}^{2}} \sim \frac{L_{s r c} \cdot \sqrt{(W L)_{s w}}}{I_{\text {unit }}^{1.5}}
$$

Low current, (very!!!) long source transistor, and large switches.


The latter not feasible due to speed and the fact that the capacitance increases.
Also here we get a requirement on the current and transistors sizes.

## Architectural choices, binary

$$
x_{0}(n T) \quad x_{1}(n T) \quad x_{2}(n T) \quad x_{3}(n T)
$$



$$
I_{\text {out }}(X)=X \cdot I_{\text {unit }}=\sum x_{k} \cdot 2^{k-1} \cdot I_{\text {unit }}
$$

## Impact of binary weighted DACs

Mismatch errors will cause differences in weights
Assume errors in the most significant bit (MSB) only in this example.


## Architectural choices, Unary/Thermometer



$$
I_{\text {out }}(X)=X \cdot I_{\text {unit }}=\sum y_{k} \cdot 1 \cdot I_{\text {unit }}
$$

## Architectural choices, segmented

$$
\begin{gathered}
x_{0}(n T) \quad y_{1}(n T) \quad y_{1}(n T) \quad y_{2}(n T) \\
I_{\text {out }}(X)=X \cdot I_{\text {unit }}=x_{M} \cdot \sum y_{k} \cdot 1 \cdot I_{\text {unit }}+\sum x_{k} \cdot 2^{k-1} \cdot I_{\text {unit }}
\end{gathered}
$$

## Impact of mismatch 1

Go back to the binary and consider the square-wave error

$$
\mathrm{HD}_{3}=\frac{P_{s}}{P_{\epsilon, 3}}=\frac{\frac{2^{2 N}}{8} \cdot I_{u n i t}^{2}}{\frac{2^{N+2} \cdot \sigma_{u n i t}^{2}}{9 \pi^{2}}}=2^{N} \cdot \frac{9 \pi^{2}}{32} \cdot \frac{I_{u n i t}^{2}}{\sigma_{u n i t}^{2}} \approx \frac{3 \cdot 2^{N}}{\sigma_{u n i t}^{2} / I_{u n i t}^{2}}=\frac{3 \cdot 2^{N}}{\sigma_{r}^{2}}
$$

All architectures (including quantization noise)

$$
\mathrm{SNDR}=\frac{3 \cdot 2^{2 N-1}}{1+3 \cdot \sigma_{r}^{2} \cdot 2^{N+1}}
$$

There is a trade-off between linearity and noise! Which is worst for your application?

## Impact of mismatch 2

Consider the effective number of bits instead:

$$
\mathrm{SNDR}=\frac{3 \cdot 2^{2 N-1}}{1+6 \cdot \sigma_{r}^{2} \cdot 2^{N}} \Rightarrow \mathrm{ENOB}=N-\frac{1}{2} \cdot \log _{2}\left(1+6 \sigma_{r}^{2} \cdot 2^{N}\right)
$$

such that (assume 3 -sigma for higher yield)

$$
\sigma_{r}=\frac{1}{3} \cdot \sqrt{\frac{2^{2(N-\mathrm{ENOB})}-1}{6 \cdot 2^{N}}}
$$

Assume the target is 12-bit performance with a nominal 14-bit DAC:

$$
\sigma_{r} \approx 0.4 \%
$$

Is this a big number?

## Impact of mismatch 3

The random, relative error in current is given by

$$
\sigma_{r}^{2}=\sigma^{2}\left(\frac{\Delta I}{I_{u n i t}}\right)=\frac{A_{\beta}^{2}}{W L}+\frac{4 A_{V T}^{2}}{V_{e f f}^{2} \cdot W L} \text { giving } W L=\frac{1}{\sigma_{r}^{2}} \cdot\left|A_{\beta}^{2}+\frac{4 A_{V T}^{2}}{V_{e f f}^{2}}\right|
$$

## Example values:

$$
V_{e f f}=0.5 \mathrm{~V}, A_{\beta}=2 \% \mathrm{um}, A_{V T}=7 \mathrm{mV} \text { um, and } \sigma_{r} \approx 0.4 \%
$$

which gives

$$
W L \approx 70 \mathrm{squm}
$$

## Impact of mismatch 4

But there are also gradients and intradependencies at hand.

Bad vs better layout:

| N/A | 0 | 1 | 1 |
| :---: | :---: | :---: | :---: |
| 2 | 2 | 2 | 2 |
| 4 | 4 | 4 | 4 |
| 4 | 4 | 4 | 4 |


| 4 | $\mathrm{~N} / \mathrm{A}$ | 2 | 4 |
| :---: | :---: | :---: | :---: |
| 1 | 4 | 0 | 2 |
| 2 | 4 | 4 | 1 |
| 4 | 2 | 4 | 4 |

Minimizing the cap in the sensitive node 1
Swatch array


## Minimizing the cap in the sensitive node 2

Encoded array structure


Switch is turned on if entire row is selected or if row and column are selected.


## Minimizing the cap in the sensitive node 2

Array, cont'd


## Array-based architecture



Here is where we need the entire row pointer, otherwise this would happen:


## Array-based architecture



## Redundancy implies we can scramble!



## No scrambled structure

The error is strongly signal-dependent (c.f. low number of bits)



## Scrambled structure

Scrambling removes some (most) of the signal-dependency
${ }^{1} \mathrm{~N}_{\mathrm{GS}}$ UNIT"

Simulated sinusoids


Simulated sinusoid error


## The effect in the frequency domain

Notice, though, that the SNDR is the same!
Improve the SNDR by filtering (already in place due to the oversampling)



## Biasing scheme



## Biasing scheme



Trade-off between speed and noise supression!

## Biasing scheme



## Biasing scheme



More branches implies more mismatch and noise. Trade-off!

## Biasing scheme



Sacrifice some of the unit cells and replace with bias components. Let them distribute bias to several rows and columns! (N.b. a row is signal dependent!)

## Biasing scheme, use wide-swing mirrors

Use the unit current source as a macro to generate cascode voltage


## Switching scheme



Overlapping switch signals, such that the switch is never turned off $>$ use some kind of SR latch or so.

Switching must be symmetrical and data independent!
Switches are operating in saturation region!


## Switching levels

Saturation should be guaranteed to obtain high impedance

There is a risk that we slip out

$$
-V_{\text {swing }}>-V_{T} \Rightarrow V_{T}>V_{\text {swing }}
$$ of saturation region

This requirement gives us a maximum swing at the output, ie., a dependency


$$
\begin{aligned}
& V_{o u t, l o}-V_{c m}>V_{s w t, h i}-V_{c m}-V_{T} \\
& V_{D D}-V_{s w i n g}>V_{s w t, h i}-V_{T}
\end{aligned}
$$

 between current and load impedance.

$$
I_{\text {max }}=\frac{V_{\text {swing }}}{R_{\text {load }}} \Rightarrow I_{\text {unit }}=\frac{V_{\text {swing }}}{R_{\text {load }} \cdot 2^{N}}<\frac{V_{T}}{R_{\text {load }} \cdot 2^{N}}
$$

## Concluding remarks

Mismatch

$$
W L=\frac{1}{\sigma_{r}^{2}} \cdot\left(A_{\beta}^{2}+\frac{4 A_{V T}^{2}}{V_{e f f}^{2}}\right) \text { where } \sigma_{r}=\frac{1}{3} \cdot \sqrt{\frac{2^{2(N-\mathrm{ENOB})}-1}{6 \cdot 2^{N}}}
$$

Swing

$$
I_{u n i t}<\frac{V_{T}}{R_{L} \cdot 2^{N}}
$$

Impedance (example single-transistor source)

$$
R_{\text {out }}=\frac{g_{m}}{g_{P}^{2}} \sim \frac{L_{s r c} \cdot \sqrt{(W L)_{s w}}}{I_{\text {unit }}^{1.5}} \text { where } \mathrm{ENOB} \approx 3.2+6.6 \cdot \log _{10} \frac{Z_{\text {unit }}}{Z_{L}}-2 \cdot N(\mathrm{HD} 3)
$$

which then is combined with

$$
I_{u n i t}=\frac{\mu C_{o x}}{2} \cdot \frac{W}{L} \cdot V_{e f f}^{2}
$$

## Concluding remarks (Omitted parts)

Noise from individual current sources:

$$
i_{\text {tot }}^{2}(f)=2^{N} \cdot i_{\text {unit }}^{2}(f)=\frac{4 k T \gamma \cdot 2^{N}}{g_{m}}=\frac{4 k T \gamma \cdot 2^{N}}{\sqrt{2 \alpha I_{\text {unit }}}}
$$

And quite a few others:
Noise from biasing scheme
PSRR
CMRR
Total maximum area
Power consumption (don't forget the digital parts)
Bandwidth requirements

## What did we do today?

Oversampling
Sigma-delta modulator and its principles

Case-study current-steering DAC
A general study of the high-speed current-steering DAC

Some design guidance

Understanding impact of impedance and mismatch

## What will we do next time?

Case-study pipelined ADC
Architecture

Comparator

Sample-and-hold

Wrap-up

