Lecture 8, ANIK

Noise and distortion

LiU EXPANDING REALITY

What did we do last time?

Operational amplifiers

- Circuit-level aspects
- Simulation aspects
- Some terminology

What will we do today?

Some left-overs

Noise

- Circuit noise
- Thermal noise
- Flicker noise

Distortion

- What sets the (non)linearity in our CMOS devices?

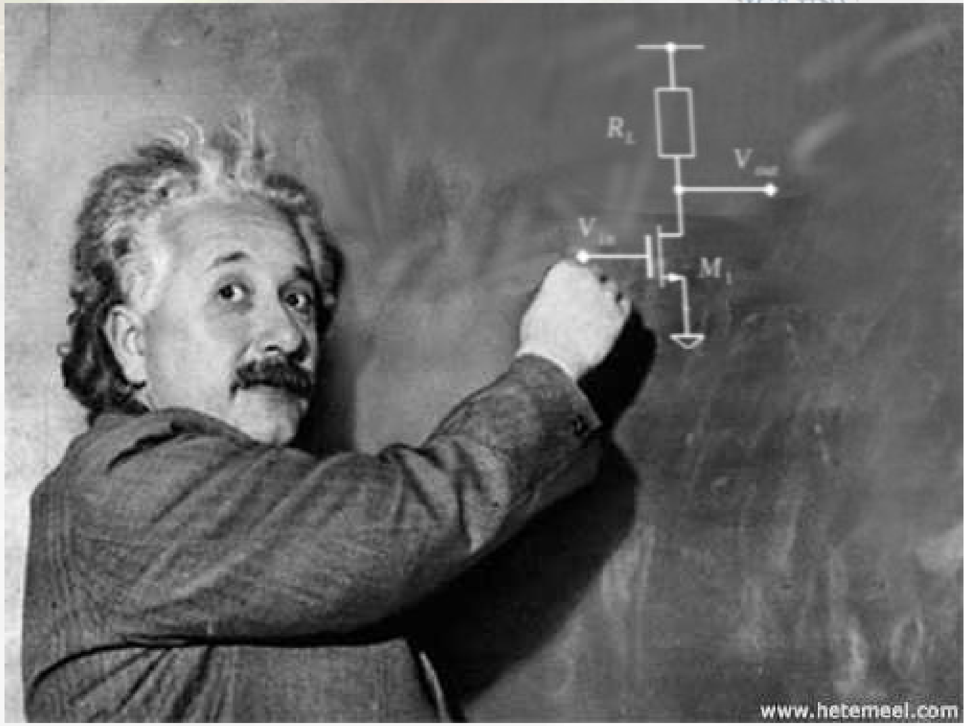
Other practical concerns wrt. current

Feedback with resistors

An OP given with a certain current drive capability.

What is the maximum swing?

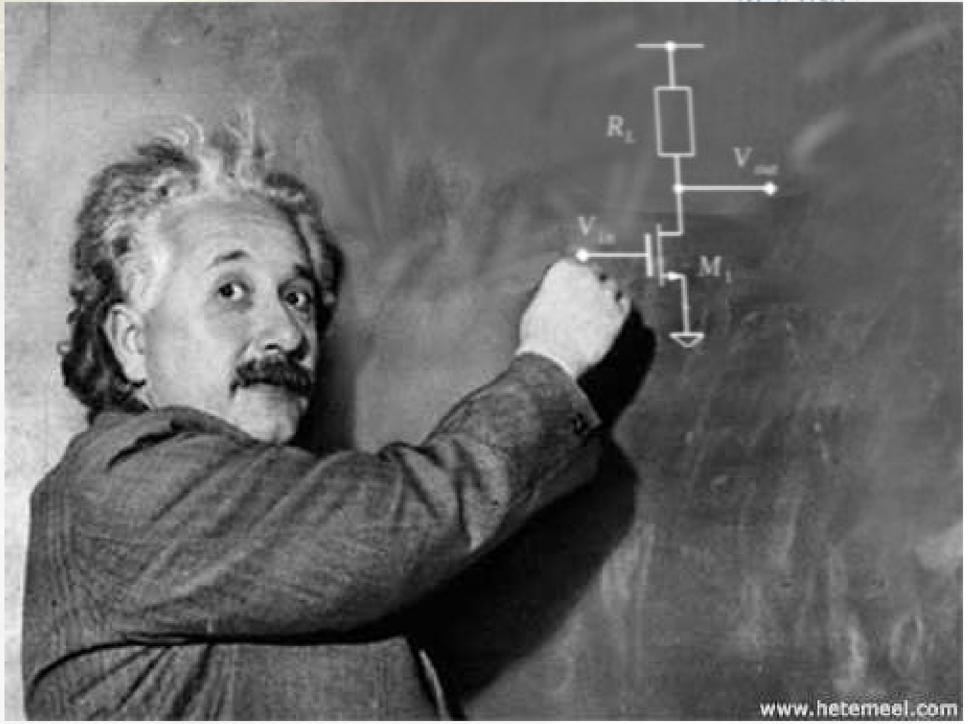
What is the DC level?



Other practical concerns wrt. bandwidth

Various unwanted effects

- Limited gain
- Offset error
- Bandwidth



Noise

Any circuit has noise and you as a designer have to reduce it or minimize the impact of it.

> "A disturbance, especially a random and persistent disturbance, that obscures or reduces the clarity of a signal."

Consequences

- We need to use stochastic variables and power spectral densities, expectation values, etc.
- We need to make certain assumptions (models) of our noise sources in order to calculate

Superfunction and spectral densitites

Frequency domain

Spectral density (PSD)

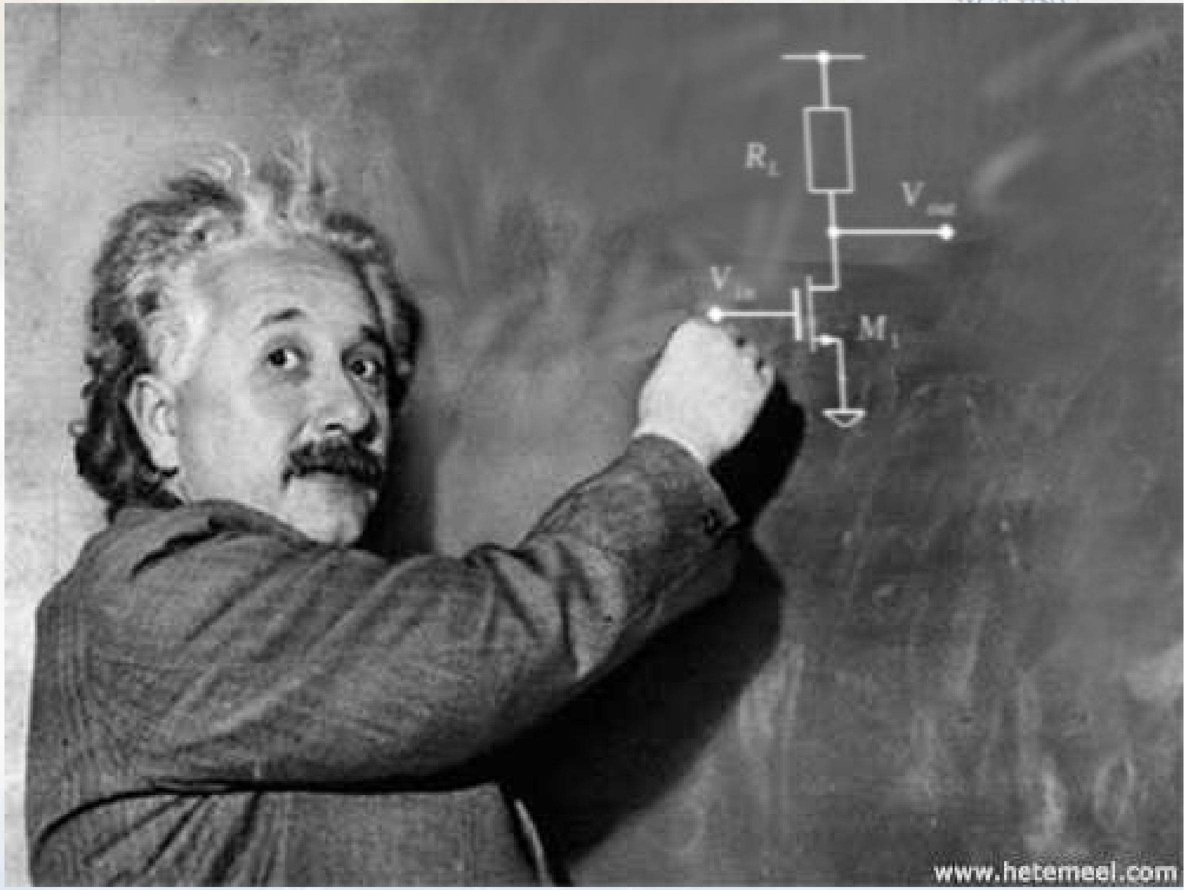
Superfunction

$$S_0(f) = \sum |A_i(f)|^2 \cdot S_i(f)$$

Total noise

$$V_{tot}^2 = \int v_n^2(f)\, df$$

$$V_{tot}^2 = v_n^2(0) \cdot \frac{p_1}{4}$$



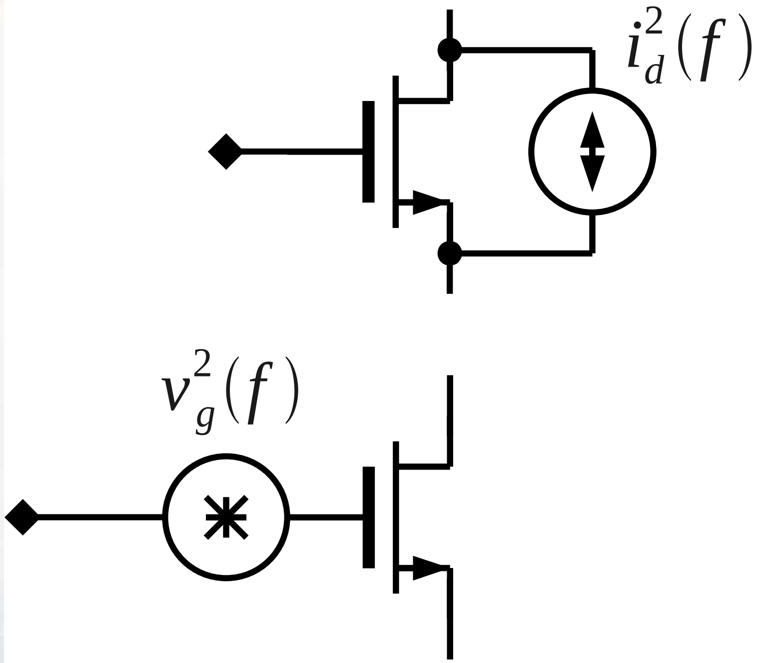
Thermal noise, white noise

Resistor

$$v_n^2 = 4kTR \text{ or } i_n^2 = \frac{v_n^2}{R^2} = \frac{4kT}{R}$$

Transistor

$$v_g^2 = \frac{4kT\gamma}{g_m} \text{ or } i_d^2 = v_g^2 \cdot g_m^2 = 4kT\gamma g_m$$



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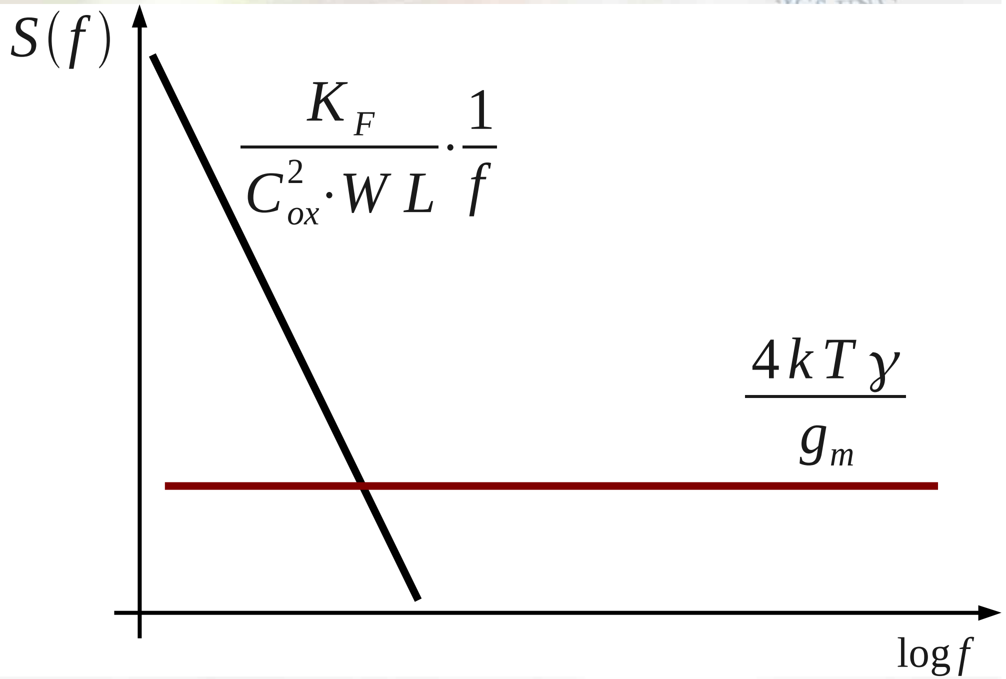
Flicker noise, 1/f-noise, pink noise

Resistor

$$v_n^2 = \frac{v_{bias}^2 \cdot k}{W\,L \cdot f} \quad \text{and} \quad i_n^2 = R^2 \cdot v_n^2$$

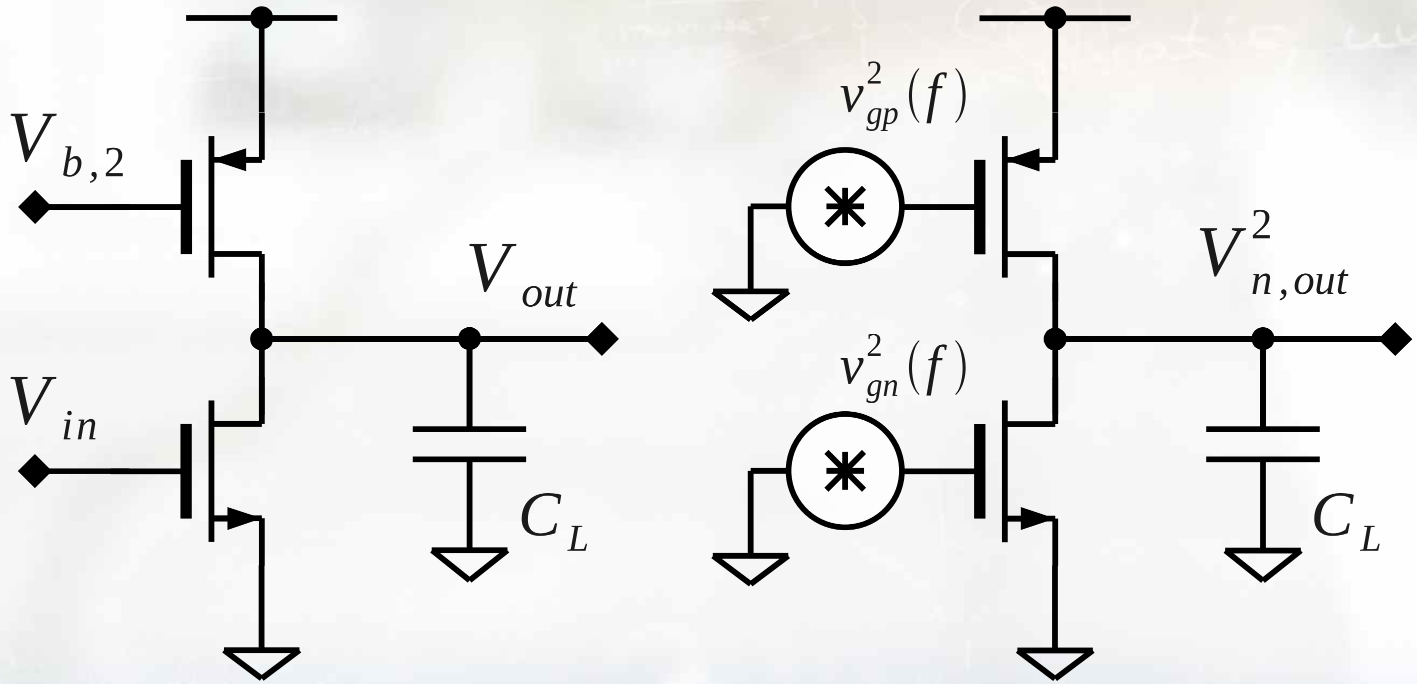
Transistor

$$v_g^2 = \frac{K_F}{C_{ox}^2 \cdot W\,L} \cdot \frac{1}{f} \quad \text{and} \quad i_d^2 = g_m^2 \cdot v_n^2$$



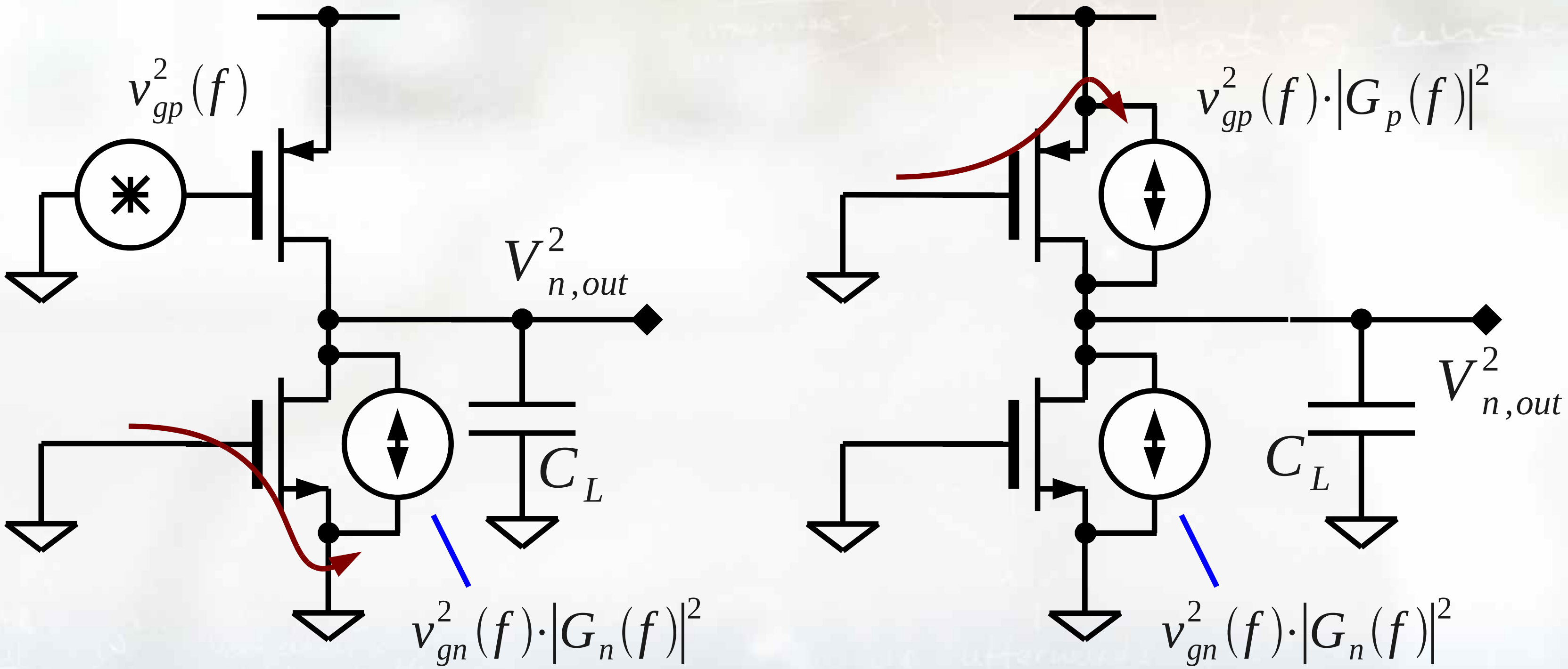
Noise compiled in one example

Common-source with noisy transistors



Noise compiled in one example, cont'd

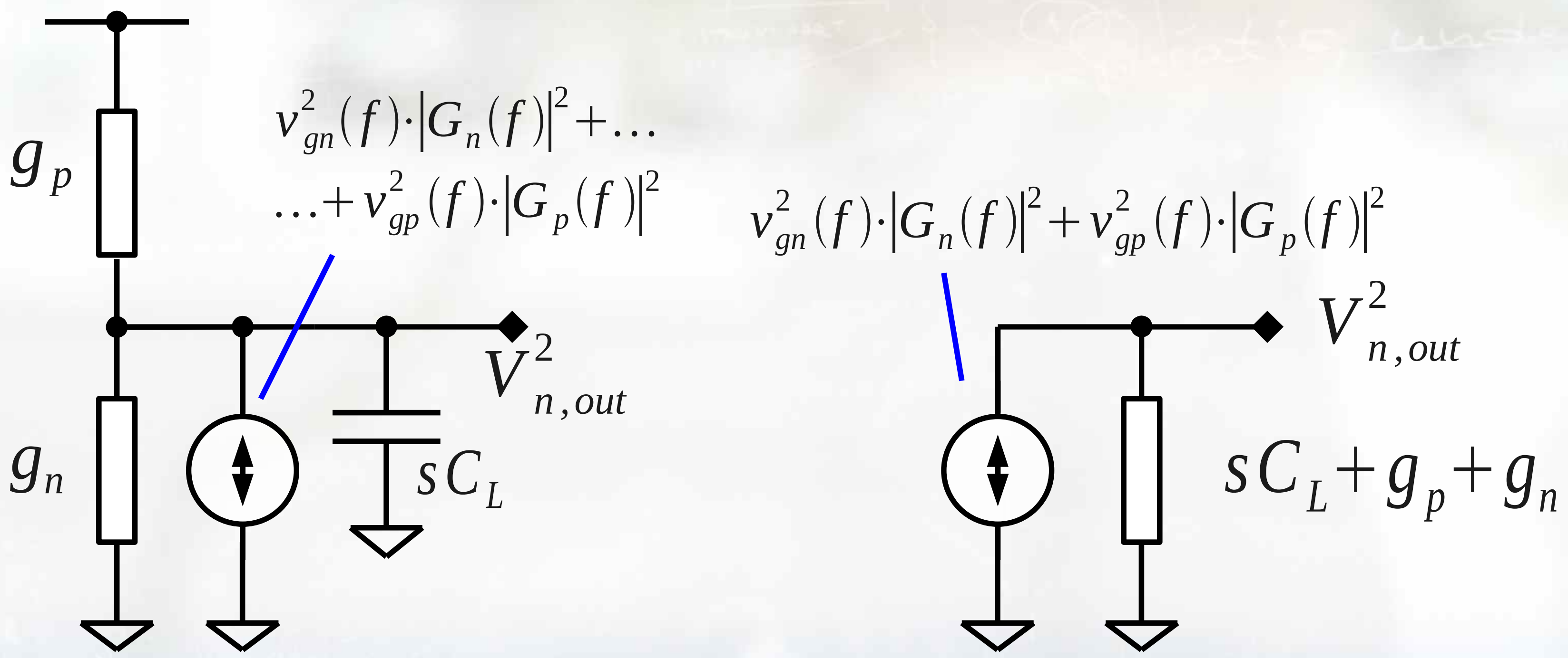
Potentially reorder the sources for convient calculations



Notice the use of transconductance from voltage to current.

Noise compiled in one example, cont'd

Equivalent small-signal schematics (ESSS)



Noise compiled in one example, cont'd

The general transfer function to the output is given by

$$V^2_{n,out}(f)=\frac{v^2_{gn}(f)\cdot\left|G_n(f)\right|^2+v^2_{gp}(f)\cdot\left|G_p(f)\right|^2}{\left|sC_L+g_p+g_n\right|^2}$$

Insert the values

$$V^2_{n,out}(f)=4kT\gamma\frac{\frac{g^2_{mn}}{g_{mn}}+\frac{g^2_{mp}}{g_{mp}}}{(g_p+g_n)^2\cdot\left|1+\frac{s}{\frac{g_p+g_n}{C_L}}\right|}=4kT\gamma\frac{\frac{g_{mn}+g_{mp}}{(g_p+g_n)^2}}{\left|1+\frac{s}{\frac{g_p+g_n}{C_L}}\right|^2}$$

Noise compiled in one example, cont'd

Use the brickwall approach

$$V^2_{n,tot} = \int V^2_{n,out}(f) = V^2_{n,out}(0) \cdot \frac{p_1}{4}$$

Insert the expressions

$$V^2_{n,tot} = 4kT\gamma \frac{g_{mn}+g_{mp}}{(g_p+g_n)^2} \cdot \frac{g_p+g_n}{4C_L} = \frac{kT\gamma}{C_L} \cdot \frac{g_{mn}+g_{mp}}{g_p+g_n}$$

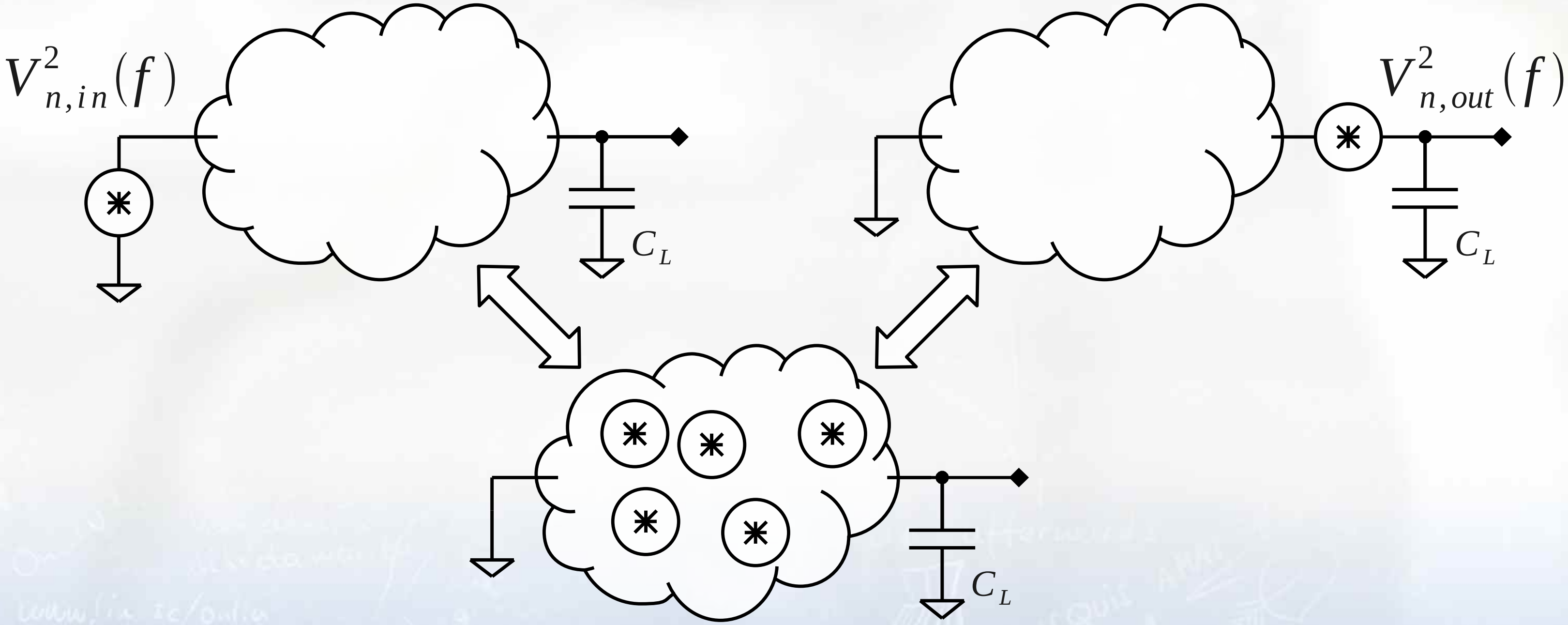
and conclude

$$V^2_{n,tot} = \frac{kT\gamma}{C_L} \cdot A_0 \cdot \left(1 + \frac{g_{mp}}{g_{mn}}\right)$$

Input-referred noise

Revert the output noise back to the input:

$$V^2_{n,in}(f) = \frac{V^2_{n,out}(f)}{|A_{in}(f)|^2}$$



The common-source example

Input-referred noise

$$V_{n,in}^2(f)=\left[4kT\gamma\frac{\dfrac{(g_{mn}+g_{mp})}{(g_p+g_n)^2}}{\left|1+\dfrac{s}{\dfrac{(g_p+g_n)}{C_L}}\right|^2}\right]\cdot\left[\frac{\left|1+\dfrac{s}{\dfrac{(g_p+g_n)}{C_L}}\right|^2}{\dfrac{g_{mn}^2}{(g_p+g_n)^2}}\right]=\frac{4kT\gamma}{g_{mn}}\cdot\left(1+\frac{g_{mp}}{g_{mn}}\right)$$

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What does this mean?

Bias transistor should be made with low transconductance!

Visible from the formula

Gain transistors should be made with high transconductance!

Visible from the formula

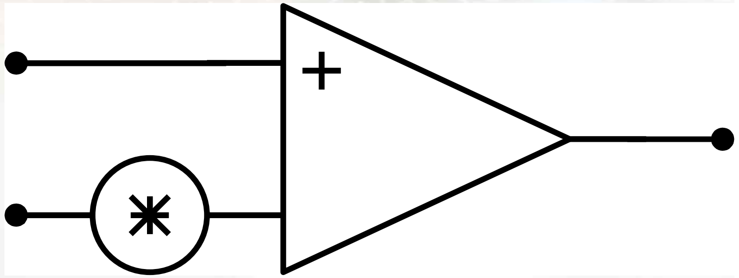
Gain should be distributed between multiple stages, c.f., Friis.!

Left as an exercise

Noise in operational amplifiers

CMOS opamps can be assumed to have an input referred noise source on one of the inputs.

Input referred noise can be calculated according to previous principles and will be given by a spectral density



Noise in OP, example

Assume noisy resistors and noisy opamp.

$$\frac{V_2(s)}{V_1(s)} = \frac{-1}{1+sRC}$$

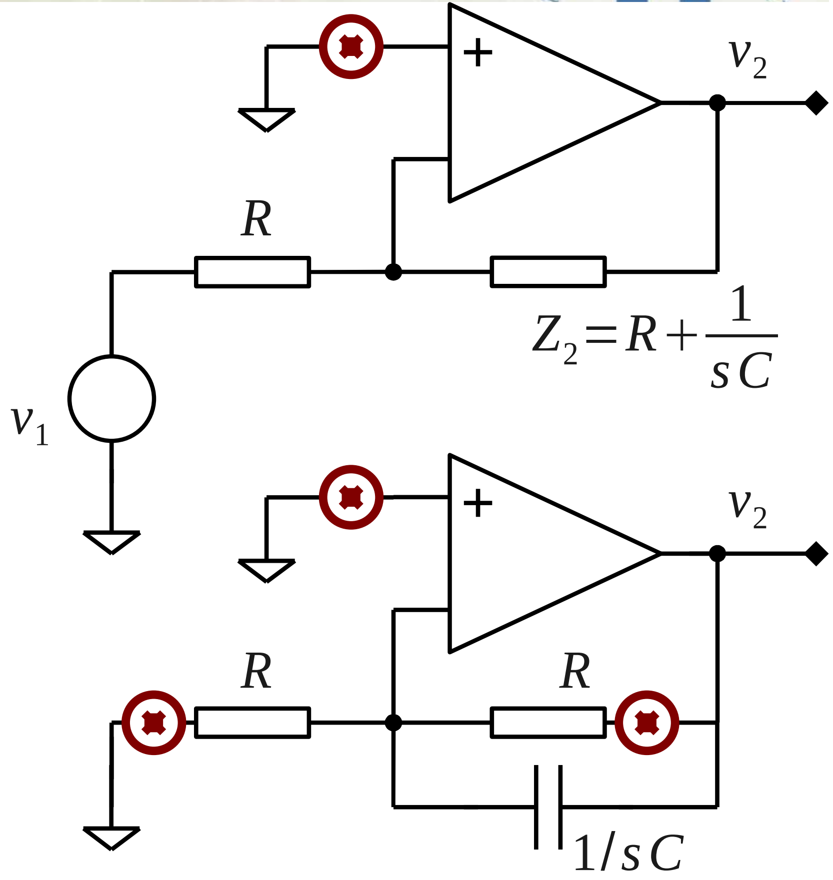
And the noise sources

Opamp noise

$$\frac{V_2(s)}{V_n(s)} = 1 + \frac{1}{1+sRC}$$

Resistor noise

$$\frac{V_2(s)}{V_{r2}(s)} = \frac{1}{1+sRC} \quad \text{and} \quad \frac{V_2(s)}{V_{r1}(s)} = \frac{-1}{1+sRC}$$



Superposition and superfunction yields the result.

Distortion

No circuit is fully linear... in reality something like

$$Y = \alpha_0 + \alpha_1 \cdot X + \alpha_2 \cdot X^2 + \alpha_3 \cdot X^3 + \alpha_4 \cdot X^4 + \ldots$$

Example

$X = \sin\omega t$ and $\alpha_1 = 1, \alpha_2 = 0.01$

$$Y = \sin\omega t + \alpha_2 \cdot \sin^2\omega t = \sin\omega t + \alpha_2 \cdot \frac{1 - \cos 2\omega t}{2}$$

which results in a DC shift, and a distortion term:

$$Y = \underbrace{\frac{\alpha_2}{2}}_{\text{DC shift}} + \underbrace{\sin\omega t}_{\text{desired}} \underbrace{- \frac{\alpha_2}{2} \cdot \cos 2\omega t}_{\text{distortion}}$$

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Distortion

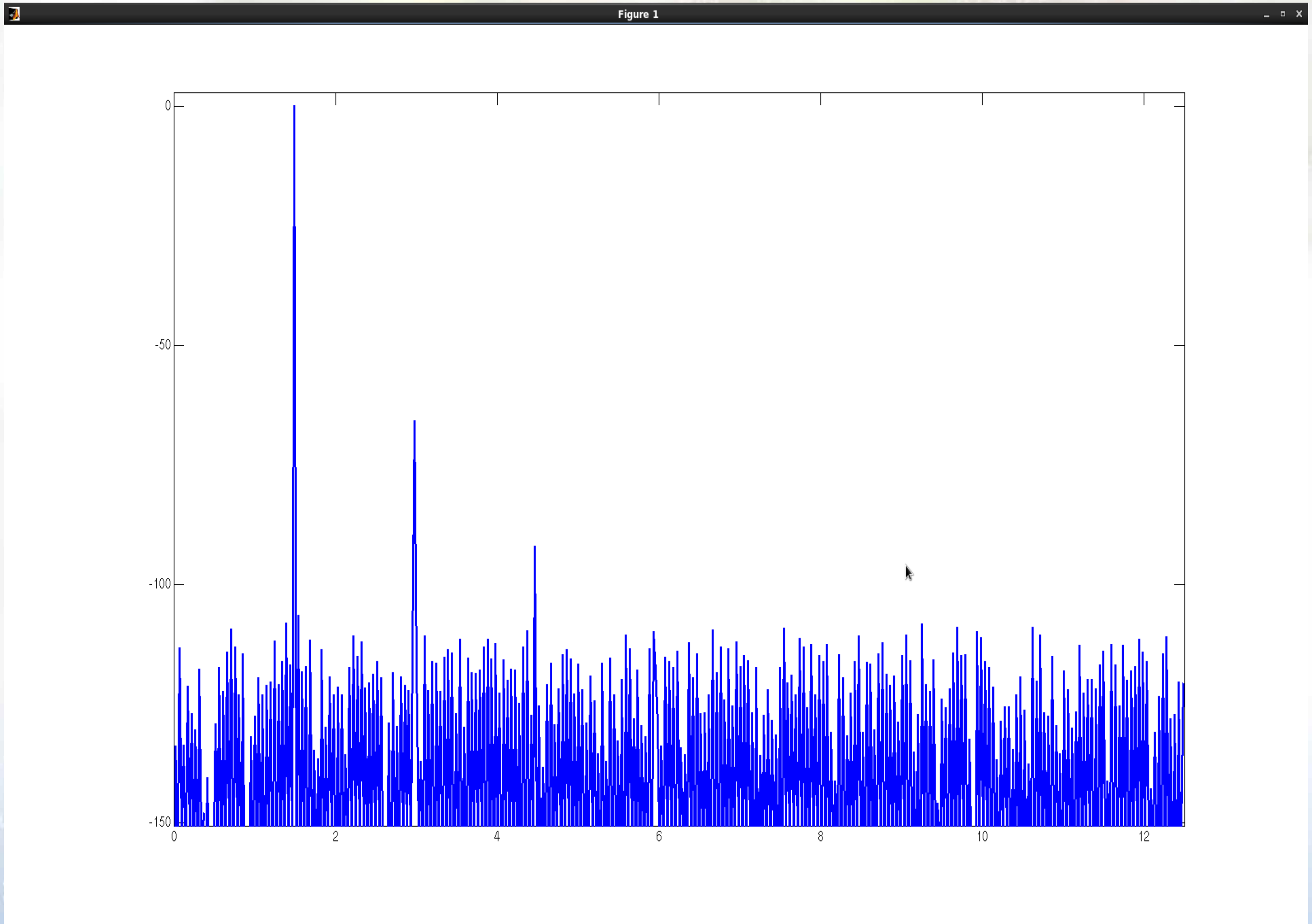
Frequency-domain measures

- Spurious-free dynamic range, SFDR
- Harmonic distortion, HD
- Signal-to-noise-and-distortion ratio, SNDR

Amplitude domain measures

- Compression
- Clipping
- Gain-boosting
- Offset

Frequency domain measures



Distortion, fully differential circuits

Assume distortion is identical in both branches

$$Y_p = \alpha_0 + \alpha_1 \cdot X_p + \alpha_2 \cdot X_p^2 + \alpha_3 \cdot X_p^3 + \alpha_4 \cdot X_p^4 + \ldots$$ and

$$Y_n = \alpha_0 + \alpha_1 \cdot X_n + \alpha_2 \cdot X_n^2 + \alpha_3 \cdot X_n^3 + \alpha_4 \cdot X_n^4 + \ldots$$

Difference:

$$\Delta Y = Y_p - Y_n = (\alpha_0 - \alpha_0) + \alpha_1 \cdot (X_p - X_n) + \alpha_2 \cdot \left(X_p^2 - X_n^2\right) + \ldots$$

Further on:

$$X_p = -X_n = \frac{\Delta X}{2}$$

Results in

$$\Delta Y = \alpha_1 \cdot (X_p - (-X_p)) + \alpha_2 \cdot \left(X_p^2 - (-X_p)^2\right) + \alpha_3 \cdot \left(X_p^3 - (-X_p)^3\right) + \ldots$$

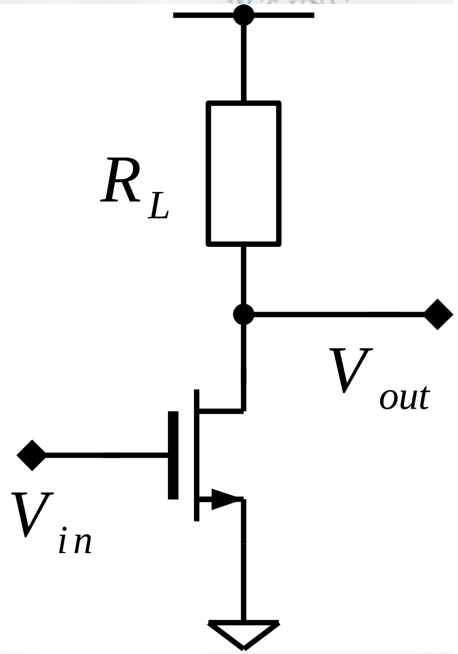
$$\Delta Y = \alpha_1 \cdot \Delta X + \frac{\alpha_3}{4} \cdot \Delta X^3 + \ldots$$ Even-order terms disappear!

Distortion in a common-source

Assume a simple common-source with resistive load

First-order model $I_D = \alpha \cdot V_{eff}^2$

$$V_{out} = V_{DD} - R \cdot I_D = V_{DD} - R \cdot \alpha \cdot V_{eff}^2$$



Assume a limited input signal

$$V_{eff}(t) = V_{eff0} + V_x \cdot \sin \omega t$$

$$V_{out}(t) = V_{DD} - R \cdot \alpha \cdot \left(V_{eff0} + V_x \sin \omega t\right)^2$$

$$V_{out}(t) = V_{DD} - R \cdot \alpha \cdot \left(V_{eff0}^2 + 2 V_{eff0} V_x \cdot \sin \omega t + V_x^2 \sin^2 \omega t\right)$$

$$V_{out}(t) = V_{DD} - R \cdot \alpha \cdot \left(V_{eff0}^2 + 2 V_{eff0} V_x \cdot \sin \omega t + \frac{V_x^2}{2} \cdot (1 - \cos 2\omega t)\right)$$

$$V_{out}(t) = \underbrace{V_{DD} - R \cdot \alpha \cdot V_{eff0}^2 + V_x^2/2}_{\text{DC}} + \underbrace{2 V_{eff0} V_x \cdot R \cdot \alpha \cdot \sin \omega t}_{\text{desired signal}} - \underbrace{(V_x^2/2)}_{\text{distortion}} \cdot \cos 2\omega t$$

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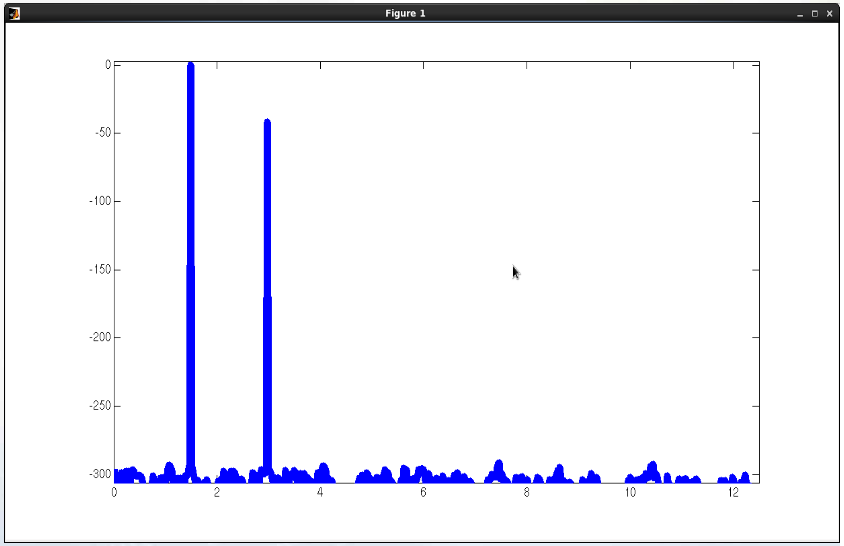
Compression analysis

$$V_{out}(t) = \underbrace{V_{DD} - R \cdot \alpha \cdot V_{eff0}^2 + V_x^2/2}_{\text{DC}} + \underbrace{2 V_{eff0} V_x \cdot R \cdot \alpha \cdot \sin \omega t}_{\text{desired signal}} - \underbrace{(V_x^2/2) \cdot \cos 2 \omega t}_{\text{distortion}}$$

Signal power scales "linearly" with amplitude

Distortion power scales "quadratically"

At some point they will meet.



Intercept points

Output and input referred

IIP2, IIP3

OIP2, OIP3

A common measure of nonlinearity

Distortion vs Noise, concludingly

High signal power gives high signal-to-noise ratio

High signal power gives low signal-to-distortion ratio

This means that you need to distribute the gain between the different stages accordingly and trade-off between the two.

What did we do today?

Noise

- Circuit noise
- Thermal noise
- Flicker noise

Distortion

- What sets the (non)linearity in our CMOS devices?

What will we do next time?

Data converters

- Fundamentals