Lesson 6

Lesson Exercises: **K9, K11, K12, K15, K16**

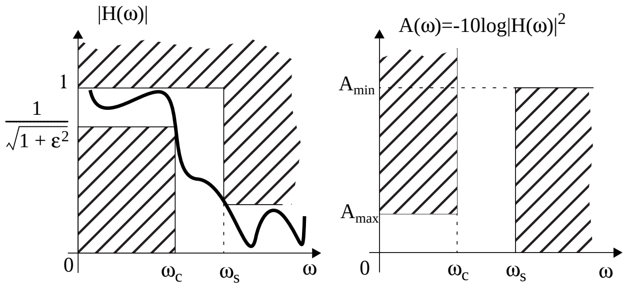
Recommended Exercises: *K8, K13, K14, K17, K18, K23, B15.16*

Theoretical Issues: **Filter synthesis. Signal Flowchart. Integrators using Operational Amplifiers.**

Theoretical

• Filter Synthesis

Filter specification



Component values, R_n, L_n, C_n, and structure (Filter order, filter type, etc.) are chosen, using tables or computer tools. Naturally, several different methods can be used. Component values are denormalized with

$$R_{i,n} = R_0 \cdot R_{i,n},\ L_i = \frac{R_0}{\omega_0} L_{i,n},\ C_i = \frac{1}{\omega_0 R_0} C_{i,n},\ \omega = \omega_0 \cdot \omega_n$$

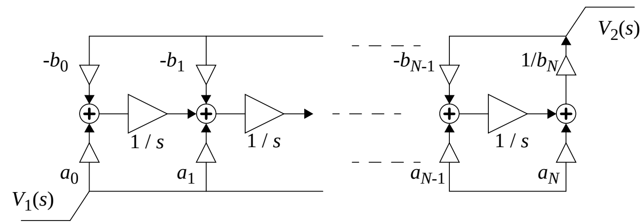
Where R_0 is the load resistance. ω_0 is the pass band stop angluar frequency.

• Continous-Time Filters

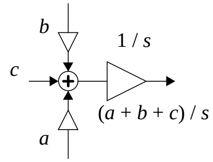
Consider the common transfer function for a linear system

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{a_0 + a_1 s + \ldots + a_{N-1} s^{N-1} + a_N s^N}{b_0 + b_1 s + \ldots + b_{N-1} s^{N-1} + b_N s^N}$$

Suppose that the number of poles is equal to the number of zeros. This does not affect the following discussion. The transfer function can be described by a signal flow chart as in the figure below.



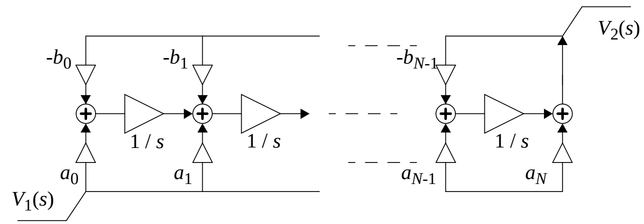
In the flow chart a number of amplifiers is identified, b_0, a_0, …, as well as summating integrators, $1/s$.



The $1/b_N$ coefficient can be eliminated in a number of different ways, i.e., be propagated forward or backward in the flow chart. The latter corresponds to the transformation of the transfer function as

$$H(s) = \frac{a_0 + a_1 s + \ldots + a_N s^N}{b_0 + b_1 s + \ldots + b_N s^N} = \frac{\frac{a_N}{b_N} \cdot s^N + \ldots + \frac{a_0}{b_N}}{s^N + \ldots + \frac{b_0}{b_N}} = \frac{a'_N \cdot s^N + \ldots + a'_0}{s^N + \ldots + b'_0}$$

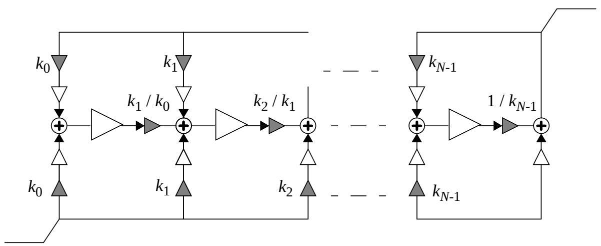
Which is shown in the flow chart as



The negative coefficients, $-b_i$, are kept in the graph until the true coefficient values are known (simulations or tables). After that the flow chart may be transformed or relaxed further.

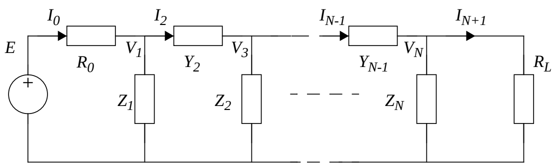
Scaling I

It will show that it is useful to be able to scale the signal level (or the peak value of the signal) at the output or input of each integrator. In a real implementation we want the integrators to work in their proper linear region. This helps the designer to estimate noise, distortion, bandwidth, etc. By introducing a scaling constant, k_i, to the input of each integrator or summation node and a unscaling factor, $1/k_i$ at the output, the level at a certain node can be changed, and the transfer characteristics is still kept due to the linear behaviour of these kinds of circuits.



• Leapfrog Filters

Consider a ladder fillter with impedances and admittances. Suppose the source has an inner (output) resistance of R_0, and that the load is a terminating resistance of R_L.



Note that the structure of the ladder filter is dependent on N. If N is odd the load resistance is connected in parallel with Z_N. When N is even the load resistance is connected in series with Y_N. In the examples given below, N is chosen to be odd.

KCL and KVL result in a number of equations for the currents in the circuit

$$I_0 = (E - V_1)/R_0,\ I_2 = Y_2(V_1 - V_3),\ \ldots,$$
$$I_{N-1} = Y_{N-1}(V_{N-2} - V_N),\ I_{N+1} = V_N/R_L$$

and analogously for the voltage

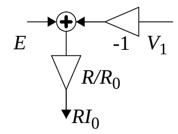
$$V_1 = Z_1(I_0 - I_2),\ V_3 = Z_2(I_2 - I_4),\ \ldots,\ V_N = Z_N(I_{N-1} - I_{N+1})$$

By introducing a normating constant (resistance), R, all equations can be rewritten as

$$RI_0 = \frac{R}{R_0}(E - V_1),\ RI_2 = RY_2(V_1 - V_3),\ \ldots,$$
$$RI_{N-1} = RY_{N-1}(V_{N-2} - V_N),\ RI_{N+1} = \frac{R}{R_L}V_N$$

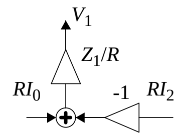
and

$$V_1 = \frac{Z_1}{R}(RI_0 - RI_2),\ V_2 = \frac{Z_2}{R}(RI_2 - RI_4),\ \ldots,$$
$$V_N = \frac{Z_N}{R}(RI_{N-1} - RI_{N+1})$$

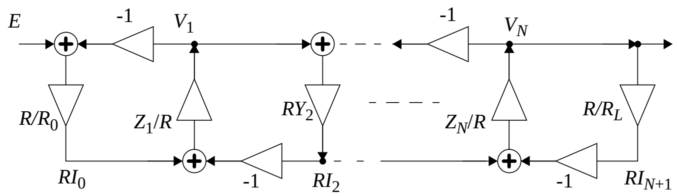


By introducing 'voltage nodes' (or variables);

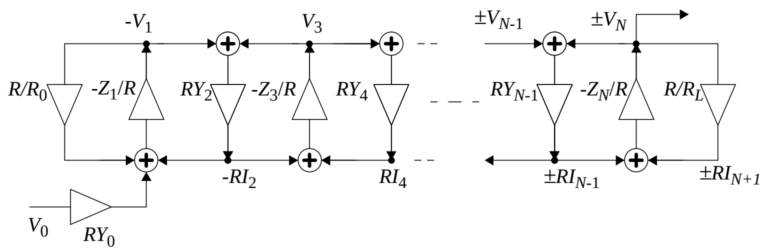
$$RI_0,\ V_1,\ RI_2,\ V_3,\ \ldots,\ V_N,\ RI_{N+1}$$



The equation system can be described with a signal flow chart. Compare the expressions for RI_0 and V_1 with the flow charts to the right. For the whole ladder filter this becomes



The flow chart describes a number of summating and amplifying elements. By introducing the sign of the nodes and eliminate all inverters and some small notation changes, the flow chart is transformed into



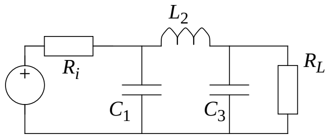
Note the change of sign of the voltage nodes and the amplifiers. If $(N+1)/2$ is odd, the right-most nodes (at the load of the ladder) are given by $-V_N$ and $-RI_{N+1}$ ($+V_{N-1}$, $+RI_{N-1}$ etc.). If $(N+1)/2$ is even, the opposite is true.

Scaling II

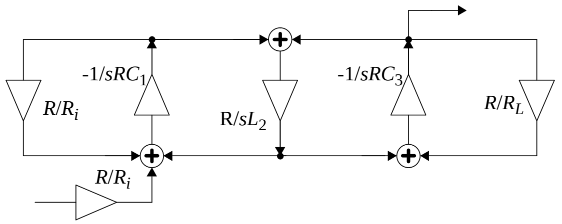
Divide the net into a number of subnets. Every subnet will have a number of inputs and outputs. By scaling all the inputs with a factor k, all internal nodes of the subnet will be scaled with k as well. The outputs also have to be scaled by a factor $1/k$ to "reset".

Ladder filters, contd.

Suppose that all $Z_i = 1/sC_i$ are kapacitances and all $Y_i = 1/sL_i$ are inductances. Example:

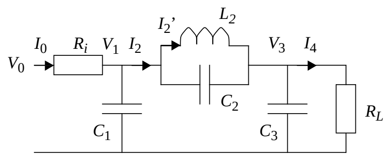


All amplifiers in the flow chart are now functioning as integrators (except those who are corresponding to the output resistance of the source and the load resistance).



Elliptic Leapfrog Filters

It is wanted to rewrite the signal flow grap so that only summating integrators are used. When synthesizing elliptic leapfrog filters the flow graph has to be slightly transformed. Consider the network below.



In the circuit we find the current I_2' through the inductor L_2. The nodal equations are

$$V_1 = \frac{1}{sRC_1}(RI_0 - RI_2),\ V_3 = \frac{1}{sRC_3}(RI_2 - RI_4)$$

and

$$RI_0 = \frac{R}{R_i}(V_0 - V_1),\ RI_2 = \frac{R}{\left(sL_2 \parallel \frac{1}{sC_2}\right)}(V_1 - V_3),\ RI_4 = \frac{R}{R_L}V_3$$

The RI_2 expression is rewritten as

$$RI_2 = RI_2' + sRC_2(V_1 - V_3) \text{ and } RI_2' = \frac{R}{sL_2}(V_1 - V_3)$$

which gives the well-known expressions. By eliminating I_1 in the original equations, we have

$$V_1 = \frac{1}{sRC_1}(RI_0 - RI_2' - sRC_2(V_1 - V_3)) \text{ and}$$

$$V_3 = \frac{1}{sRC_2}(RI_2' + sRC_2(V_1 - V_3) - RI_2)$$

which gives

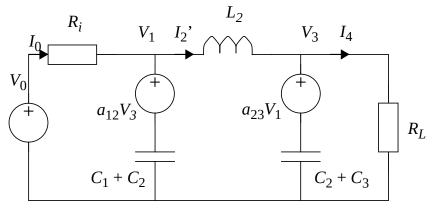
$$V_1 = \frac{1}{sR(C_1 + C_2)}(RI_0 - RI_2') + a_{12}V_3 \text{ and}$$

$$V_3 = \frac{1}{sR(C_2 + C_3)}(RI_2' - RI_4) + a_{23}V_1$$

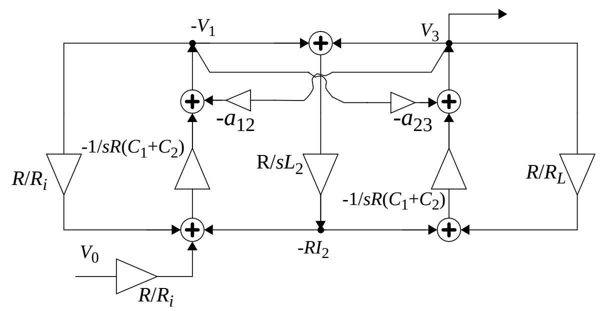
The constants a_{12} and a_{23} are given by

$$a_{12} = \frac{C_2}{C_1 + C_2} \text{ and } a_{23} = \frac{C_2}{C_2 + C_3}$$

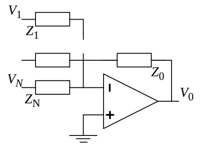
This is equivalent to the circuit below. An extra pair of voltage sources is used.



In the signal flow chart this is easily rewritten as



• Integrators with Operational Amplifiers



Consider a common set up consisting of an operational amplifier and a number of impedances. Suppose that the operational amplifier is ideal, hence infinite high input impedance.

With KCL we have

$$\frac{V_0}{Z_0} = -\left(\frac{V_1}{Z_1} + \frac{V_2}{Z_2} + \ldots + \frac{V_N}{Z_N}\right)$$

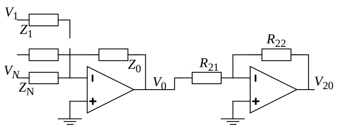
which gives the voltage at the output to

$$V_0 = -\frac{Z_0}{Z_1}V_1 - \frac{Z_0}{Z_2}V_2 - \ldots - \frac{Z_0}{Z_N}V_N$$

Suppose that among the impedances, Z_0 is a capacitance and the others are chosen to be resistances. This gives

$$V_0 = -\frac{1}{sR_1C_0}V_1 - \frac{1}{sR_2C_0}V_2 - \ldots - \frac{1}{sR_NC_0}V_N$$

By varying R_i and C_0 we have an inverting, summating (and scaling) integrating link. A non-inverting integrator is achieved by cascading the integrator with an additional invering buffer.



Transistors as resistors (MOSFET-C filter)

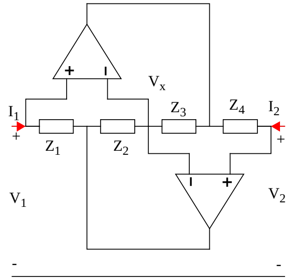
The resistance of a transistor in the linear region can be written as

$$R = \frac{1}{\mu_0 C_{ox}(W/L)(V_G - V_T - V_D)}$$

By changing size and control voltage, the resistance value is changed. Note that the resistance is signal dependent.

Exercises

Exercise K9



GIC - *Generalized impedance converter*.

The gic can be used to realize on-chip inductances. However, the circuit is area consuming and there are some other drawbacks.

Derive the K-matrix

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where

$$A = \left.\frac{V_1}{V_2}\right|_{-I_2 = 0},\ B = \left.\frac{V_1}{-I_2}\right|_{V_2 = 0},\ C = \left.\frac{I_1}{V_2}\right|_{-I_2 = 0},\ D = \left.\frac{I_1}{-I_2}\right|_{V_2 = 0}$$

(If N links with K-matrix K_i are cascaded, the total system can be described by the matrix product $K_{TOT} = K_1K_2\ldots K_N$.)

Assume that the OPamps are ideal. This implies that the voltage over the input must be zero. This forces the potential in V_x to be equal to V_1 **and** V_2, and further $V_1 = V_2$. With KCL, and denoting the currents through Z_2 and Z_3 with I_{Z2} and I_{Z3}, respectively, we have:

$$V_1 - I_1Z_1 - I_{Z2}Z_2 = V_x = V_1 \text{ which gives } I_{Z2} = -I_1Z_1/Z_2$$
$$V_2 - I_2Z_4 - I_{Z3}Z_3 = V_x = V_2 \text{ which gives } I_{Z3} = -I_2Z_4/Z_3$$

Due to the infinite input impedance, there can be no current flowing into the OPamps, and:

$$I_{Z2} = -I_{Z3} \text{ which gives } -I_1Z_1/Z_2 = I_2Z_4/Z_3\text{, or } I_1 = -I_2\frac{Z_2Z_4}{Z_1Z_3}$$

From this we get

$$A = 1,\ B = 0,\ C = 0,\ D = \frac{Z_2Z_4}{Z_1Z_3}$$

If an impedance, Z, is terminating port two, the relation between output current and voltage is given by

$$V_2 = -I_2Z$$

Equations $V_1 = V_2$ and $I_1 = \frac{Z_2Z_4}{Z_1Z_3}\cdot\frac{V_2}{Z}$ give

$$Z_{in} = \frac{V_1}{I_1} = \frac{V_2}{\frac{Z_2Z_4}{Z_1Z_3}\cdot\frac{V_2}{Z}} = Z\frac{Z_1Z_3}{Z_2Z_4}$$

Suppose

$$Z = Z_1 = Z_3 = Z_2 = R \text{ and } Z_4 = \frac{1}{sC}$$

we have

$$Z_{in} = sCR^2 = sL$$

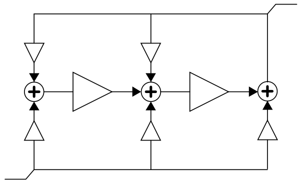
which simulates an inductor.

Exercise Extra

Find a signal flow chart which describes a 4th order leapfrog filter. Or generally, an even order leapfrog filter.

Exercise K11

Realize the filter having the transfer function

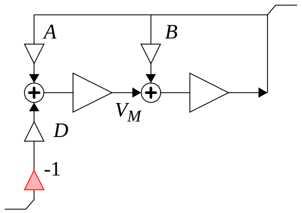


$$H(s) = \frac{1\times10^6}{s^2 + 3\times10^5 \cdot s + 6\times10^6}$$

We rewrite the function as

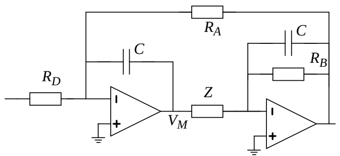
$$Y(s)\cdot[s^2 + 3\times10^5\cdot s + 6\times10^6] = 1\times10^6\cdot X(s)$$

Or



$$Y(s) = -\frac{3\times10^5}{s}Y(s) - \frac{6\times10^6}{s^2}Y(s) + \frac{1\times10^6}{s^2}X(s)$$

The flow graph is transformed. $A = -6\times10^6$, $B = -3\times10^5$ and $D = -1\times10^6$. Note the insertion of the inverter. This can now be used to implement the active filter.



(Note that there is no inversion included in the active RC filter implementation). Now the component values have to be determined. Assume all capacitances to be equal. We can see that the intermediate node can be written as

$$V_M = \frac{A}{s}\cdot Y(s) + \frac{D}{s}\cdot X(s)$$

Identifying this from the active implementation, we have

$$V_M = -\frac{1}{sR_AC}\cdot Y(s) - \frac{1}{sR_CC}\cdot X(s)$$

This gives

$$A = -\frac{1}{R_AC} = -6\times10^6 \text{ and } D = -\frac{1}{R_DC} = -1\times10^6$$

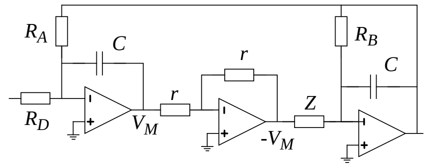
We also see that the output can be written as

$$Y(s) = \frac{1}{s}\cdot V_M + \frac{B}{s}\cdot X(s)$$

Identifying this from the active implementation, we have

$$B = -\frac{1}{R_BC} = -3\times10^5$$

We also see that the implementation above is impossible. The intermediate node V_M is **not** transformed correctly. In fact, we have to inverse the voltage with a buffer.



Then we identify

$$\frac{1}{sZC} = \frac{1}{s} \text{ or } Z = \frac{1}{C}$$

Set the capacitance to say $C = 1\mu\text{F}$. The equations give

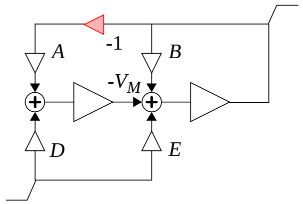
$$R_A = \frac{1}{6},\ R_D = \frac{1}{1},\ R_B = \frac{10}{3} \text{ and}$$

$$Z = R = 1\times10^6.$$

We do some notations. The structure could be changed by letting A be positive using an inverting buffer on the output signal instead. This decreases the number of opamps with one. We thereby also conclude that we do not find the simplest structure by forcing all input arguments to the summation nodes to be negative. We also see that we can use the inverting buffers to scale signal levels and relaxing the size on Z. There are numerous way to implement the filter. One can also soon realize that R_A, R_D and Z can dependently be scaled and still maintain true transfer function, see next exercise.

Exercise K12

Realize the filter having the transfer function



$$H(s) = \frac{-1\times10^6(s-1)}{s^2 + 3\times10^5\cdot s + 6\times10^6}$$

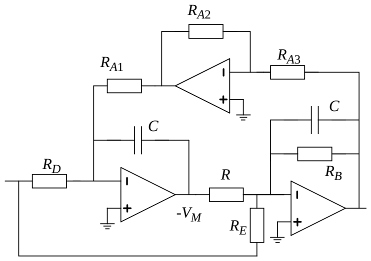
The function is rewritten in the same manner as in the previous exercise. In this case we however have a slightly different structure. We assume that we feed back the positive output (constant A) and we construct with a negative intermediate node, $-V_M$:

$$Y(s) = -\frac{3\times10^5}{s}Y(s) - \frac{6\times10^6}{s^2}Y(s) - \frac{1\times10^6}{s}X(s) + \frac{1\times10^6}{s^2}X(s)$$

With

$$A = 6\times10^6,\ B = -3\times10^5,\ D = 1\times10^6 \text{ and } E = -1\times10^6$$

The active filter structure becomes



If we now directly set up the transfer function for the active filter implementation, we have

$$-V_M = -\frac{1}{sCR_D}\cdot X + \frac{1}{sCR_{A1}}\cdot\frac{R_{A2}}{R_{A3}}\cdot Y$$

$$Y = -\frac{1}{sCR}(-V_M) - \frac{1}{sCR_E}X - \frac{1}{sCR_B}Y$$

which gives

$$Y = -\frac{1}{sCR_B}Y - \frac{1}{s^2C^2R_{A1}R}\cdot\frac{R_{A2}}{R_{A3}}\cdot Y - \frac{1}{sCR_E}X + \frac{1}{s^2C^2RR_D}X$$

We now identify the terms

$$\frac{1}{R_BC} = 3\times10^5,\ \frac{1}{C^2R_{A1}R}\cdot\frac{R_{A2}}{R_{A3}} = 6\times10^6,\ \frac{1}{CR_E} = 1\times10^6,\ \frac{1}{C^2RR_D} = 1\times10^6$$

Suppose $C = 1\mu$ F, and $R_{A2} = R_{A3}$. Then

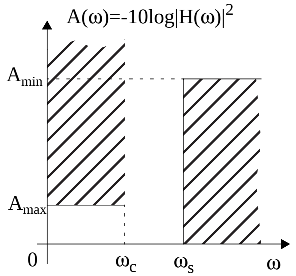
$$R_B = \frac{10}{3},\ R\cdot R_{A1} = \frac{1\times10^6}{6},\ R_E = 1,\ R\cdot R_D = 1\times10^6$$

Choose $R_{A1} = 1000$ which gives $R_{A1} = \frac{1000}{6}$ and $R_D = 1000$

Exercise K15

Pass Band: $0 < f < 3.5$ kHz and $A_{max} = 1$ dB

Stop Band $f > 10$ kHz and $A_{min} = 20$ dB



Passive Butterworthfilter, order is found in table to be $N = 3$.

We choose a voltage driven π-net with reflection $r = 1$ for symmetry. Resistances are chosen to be

$$R_i = R_L = R_0 = 1k\Omega$$

Component values are found in table to be:

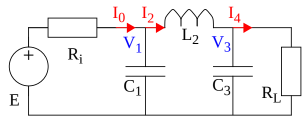
$$C_{3n} = 1,\ L_{2n} = 2 \text{ and } C_{1n} = 1$$

These values are denormalized according table to

$$C = \frac{C_n}{\omega_0R_0},\ L = \frac{R_0L_n}{\omega_0},\ \text{with } \omega_0 = \omega_c\varepsilon^{-1/N} = \omega_c[\sqrt{10^{0.1A_{max}}-1}]^{-1/N} \approx 27.55 \text{ krad/s.}$$

This gives

$$C_1 = C_3 = 36.3nF \text{ and } L_2 = 72.6mH$$



A number of filter components have been found. Create a signal flow chart for the circuit. The nodal voltages for the circuit is found to be (normalized with R):

$$I_0 = \frac{E-V_1}{R_i} \qquad RI_0 = \frac{R}{R_i}(E-V_1)$$

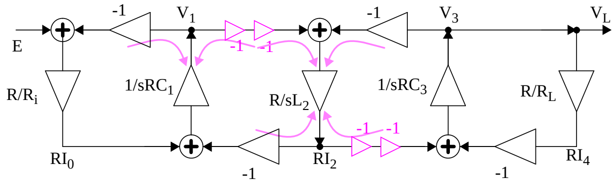
$$V_1 = \frac{1}{sC_1}(I_0-I_2) \qquad V_1 = \frac{1}{sC_1R}(RI_0-RI_2)$$

$$I_2 = \frac{V_1-V_3}{sL_2} \qquad RI_2 = \frac{R}{sL_2}(V_1-V_3)$$

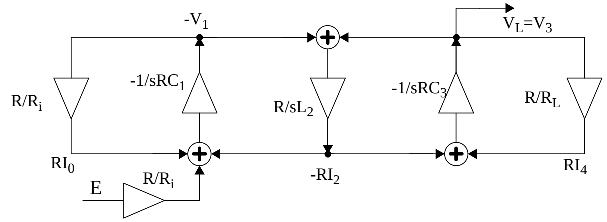
$$V_3 = \frac{1}{sC_3}(I_2-I_4) \qquad V_3 = \frac{1}{sRC_3}(RI_2-RI_4)$$

$$I_4 = \frac{V_3}{R_L} \qquad RI_4 = \frac{R}{R_L}V_3$$

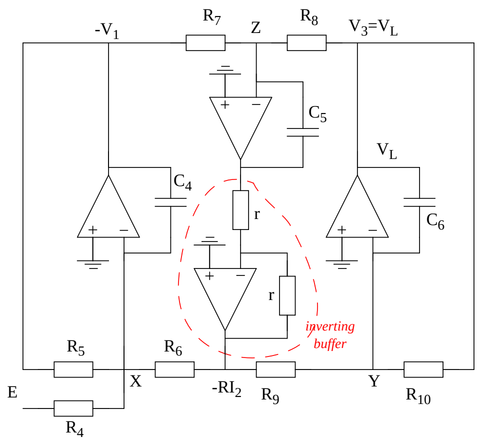
From these equations we find:



Modify the graph by propagating the inverters and denote negative voltage nodes:



We now have a number of integrators, $K\frac{1}{s}$. Realized with operational amplifiers we have a structure according to



We now have to determine the component values for this realization, This is done by comparing the signal flow chart with the OPamp net:

Leapfrog	Signal flow graph	Result
$[-V_1]_E = -\frac{1}{sC_4}\frac{E}{R_4}$	$[-V_1]_E = \frac{R}{R_i}\cdot-\frac{E}{sRC_1} = -\frac{E}{sR_iC_1}$	$C_4R_4 = C_1R_i$
$[-V_1]_{-V_1} = -\frac{1}{sC_4}\cdot\frac{V_1}{R_5}$	$[-V_1]_{-V_1} = \frac{R}{R_i}\cdot-\frac{-V_1}{sRC_1} = -\frac{-V_1}{sR_iC_1}$	$C_4R_5 = C_1R_i$
$[-V_1]_{-RI_2} = -\frac{1}{sC_4}\frac{-RI_2}{R_6}$	$[-V_1]_{-RI_2} = -\frac{1}{sRC_1}(-RI_2)$	$C_4R_6 = C_1R$
$[-RI_2]_{-V_1} = -\frac{(-1)}{sC_5}\cdot\frac{-V_1}{R_7}$	$[-RI_2]_{-V_1} = \frac{R}{sL_2}(-V_1)$	$C_5R_7 = L_2/R$
$[-RI_2]_{V_3} = -\frac{(-1)}{sC_5}\cdot\frac{V_3}{R_8}$	$[-RI_2]_{V_3} = \frac{R}{sL_2}V_3$	$C_5R_8 = L_2/R$
$[V_3]_{-RI_2} = -\frac{1}{sC_6}\frac{-RI_2}{R_9}$	$[V_3]_{-RI_2} = -\frac{1}{sRC_3}(-RI_2)$	$C_6R_9 = C_3R$
$[V_3]_{V_3} = -\frac{1}{sC_6}\frac{V_3}{R_{10}}$	$[V_3]_{V_3} = -\frac{1}{sRC_3}\frac{R}{R_L}V_3 = -\frac{V_3}{sC_3R_L}$	$C_6R_{10} = C_3R_L$

Now we have several equations. Choose all capacitors to be equally large, maybe:

$$C_4 = C_5 = C_6 = 30nF$$

The values are chosen to be approximately equal to those found in the ladder filter realization, which would give reasonable resistance values. From the equations we also find:

$$R_4 = R_5 = \frac{C_1R_i}{C_4} = \frac{36.3nF\cdot 1k\Omega}{30nF} = 1.21k\Omega \text{ och } R_{10} = \frac{C_3R_L}{C_6} = 1.21k\Omega$$

And this also gives

$$R_6 = R_9 = \frac{C_1R}{C_4} \text{ and } R_7 = R_8 = \frac{L_2}{C_5R}$$

For symmetry, $R_6 = R_7 = R_8 = R_9$ are chosen to be equal, which gives:

$$R^2 = \frac{L_2C_4}{C_1C_5} \Rightarrow R = \sqrt{L_2/C_1} = 1000\sqrt{72.6/36.3} \approx 1.41k\Omega$$

Finally, we have

$$R_6 = R_7 = R_8 = R_9 = 1.21\cdot 1.41k\Omega \approx 1.71k\Omega.$$

The final value that has to be determined is the resistor value used in the invering buffer, r. Choose r to be equal to anyone of the other resistances, i.e., :

$$r = R_6 = 1.71k\Omega.$$

Exercise K16

Synthesize an active elliptic leapfrog filter. Termination resistances are $1k\Omega$. Specification gives:

Pass band: $0 < \omega < 2\pi$ krad/s, $A_{max} = 0.1$ dB

Stop band: $\omega > 4\pi$ krad/s, $A_{min} > 20$ dB

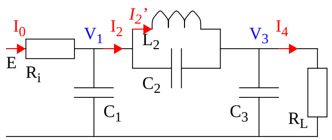
Order is found with table to be $N = 3$.

This gives following filter structure. Component values are found to be

$C_1' = C_3' = 0.8740$, $C_2' = 0.2411$ and

$L_2' = 0.9083$, and $R_i = R_L = 1k\Omega$,

$\kappa^2 = 1$.



Denormalized values are given by

$$C = \frac{C_n}{\omega_0 R_0} \text{ and } L = \frac{R_0}{\omega_0}L_n \text{ give } C_1 = C_3 \approx 139.1nF,\ C_2 \approx 38.4nF,\ L_2 \approx 144.6mH$$

Set up the equations:

$$I_0 = \frac{E-V_1}{R_i} \qquad RI_0 = \frac{R}{R_i}(E-V_1)$$

$$V_1 = \frac{1}{sC_1}(I_0-I_2) \qquad V_1 = \frac{1}{sRC_1}(RI_0-RI_2)$$

$$I_2 = \frac{1}{L_2 \parallel C_2}(V_1-V_3) \qquad RI_2 = \frac{R}{sL_2/(1+s^2L_2C_2)}(V_1-V_3)$$

$$V_3 = \frac{1}{sC_3}(I_2 - I_4) \qquad V_3 = \frac{1}{sRC_3}(RI_2 - RI_4)$$

$$I_4 = \frac{V_3}{R_L} \qquad RI_4 = \frac{R}{R_L}V_3$$

Introuduce a current, I'_2, through the inductor. The equations are modified:

$$RI_2 = RI_2' + sRC_2(V_1 - V_3) \text{ and } RI_2' = \frac{R}{sL_2}(V_1 - V_3)$$

The RI_2 expression can be eliminated, and gives:

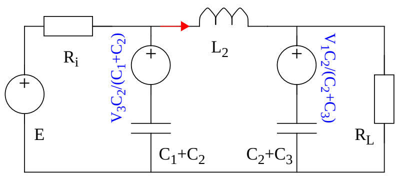
$$V_1 = \frac{1}{sRC_1}(RI_0 - RI_2' - sRC_2(V_1 - V_3)) \Rightarrow$$

$$V_1 = \frac{1}{sR(C_1 + C_2)}(RI_0 - RI_2') + \frac{C_2}{C_1 + C_2}V_3$$

And correspondingly

$$V_3 = \frac{1}{sR(C_2 + C_3)}(RI_2' - RI_4) + \frac{C_2}{C_2 + C_3}V_1$$

This gives the structure with a pair of “helping” voltage sources.



The equations are written as:

$$RI_0 = \frac{R}{R_i}(E - V_1)$$

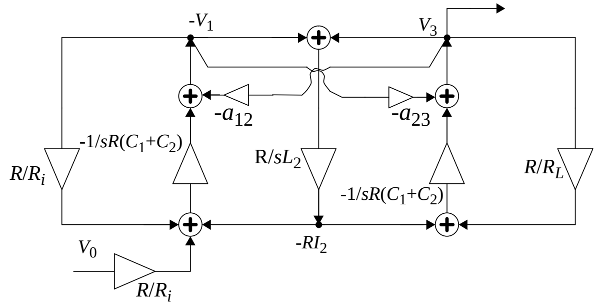
$$V_1 = \frac{1}{sR(C_1 + C_2)}(RI_0 - RI_2') + \frac{C_2}{C_1 + C_2}V_3$$

$$RI_2' = \frac{R}{sL_2}(V_1 - V_3)$$

$$V_3 = \frac{1}{sR(C_2 + C_3)}(RI_2' - RI_4) + \frac{C_2}{C_2 + C_3}V_1$$

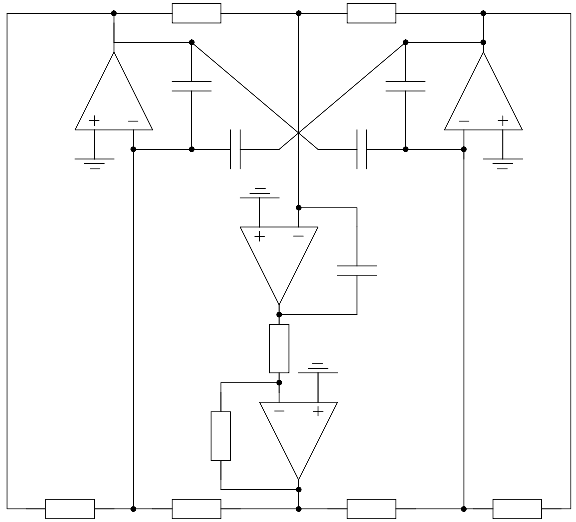
$$RI_4 = \frac{R}{R_L}V_3$$

The signal flow chart is given by



Realization

The a_{ij} terms can be realized by using capacitors instead of resistances. This realizes a negative and scaled signal flow.



Component values are found using the same manner as for the previous exercise. The resistances can be implemented by using transistors.