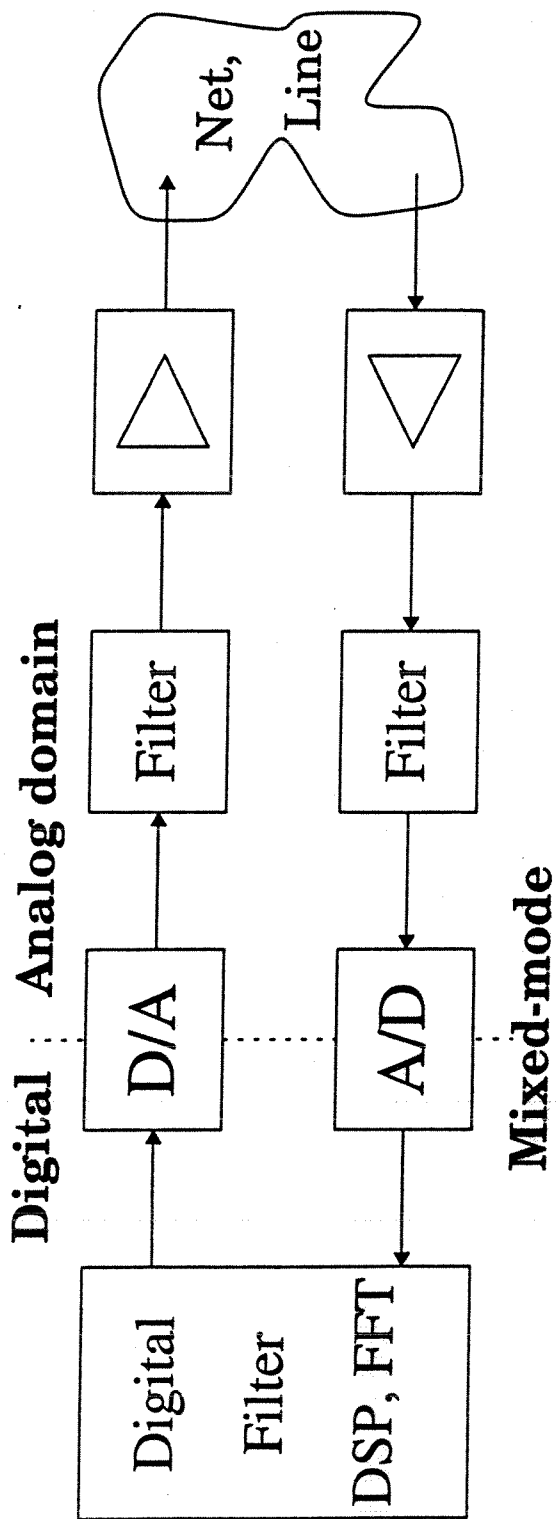
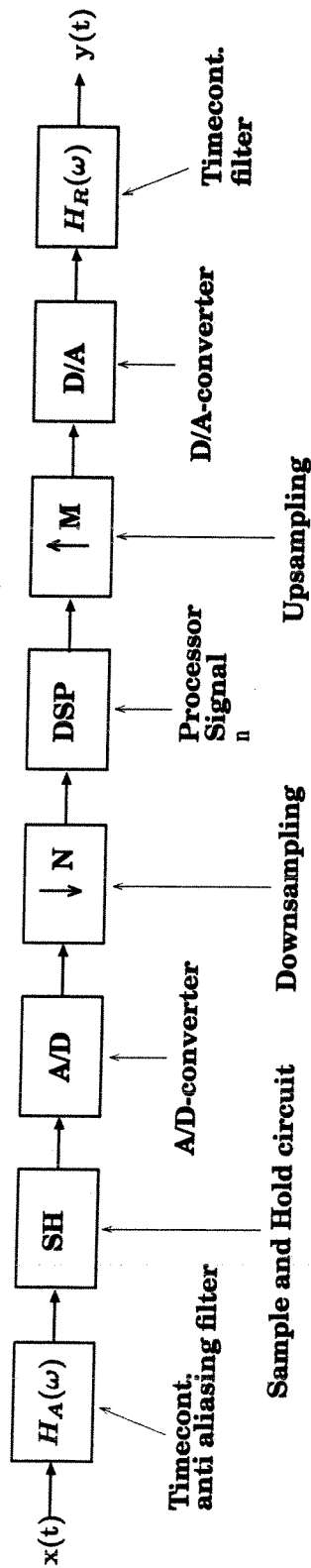


Blockdiagram. Digital Signal Processing of time continuous signals.



Traditional view of the telecommunication system.

ANALOG PERFORMANCE METRICS

TIME DOMAIN

- STEP ANSWER $g(t)$
- RISE TIME (10-90%)
- SETTLING TIME (5%)
- SLEW RATE

$$\left. \frac{dg(t)}{dt} \right|_{\max}$$

OVERSHOOT/GLITCH

POWER DISSIPATION

FREQUENCY DOMAIN

- SNR (SIGNAL NOISE RATIO)
- THD (TOTAL HARMONIC DISTORSION)
- SNDR (SIGNAL NOISE DISTORSION RATE)
- SFDR (SPURIOUS-FREE DYNAMIC RANGE)
- BANDWIDTH
- POWER DISSIPATION

2

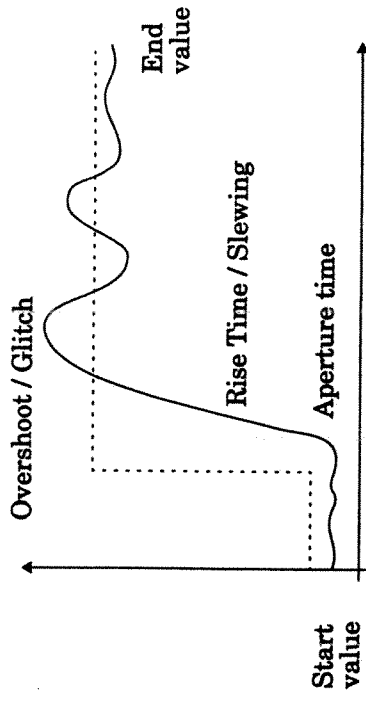


Figure 1.2: Time-domain characteristics.

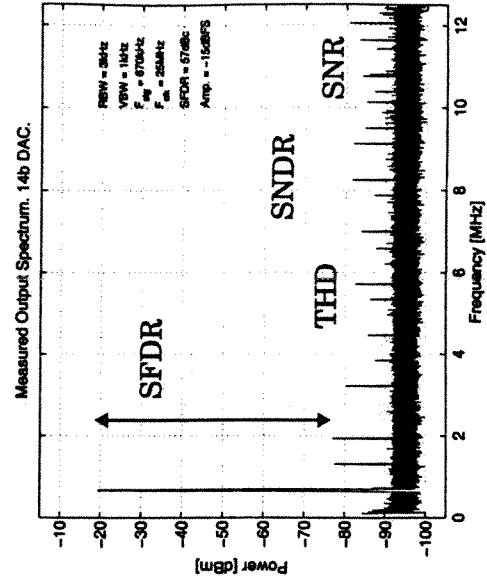
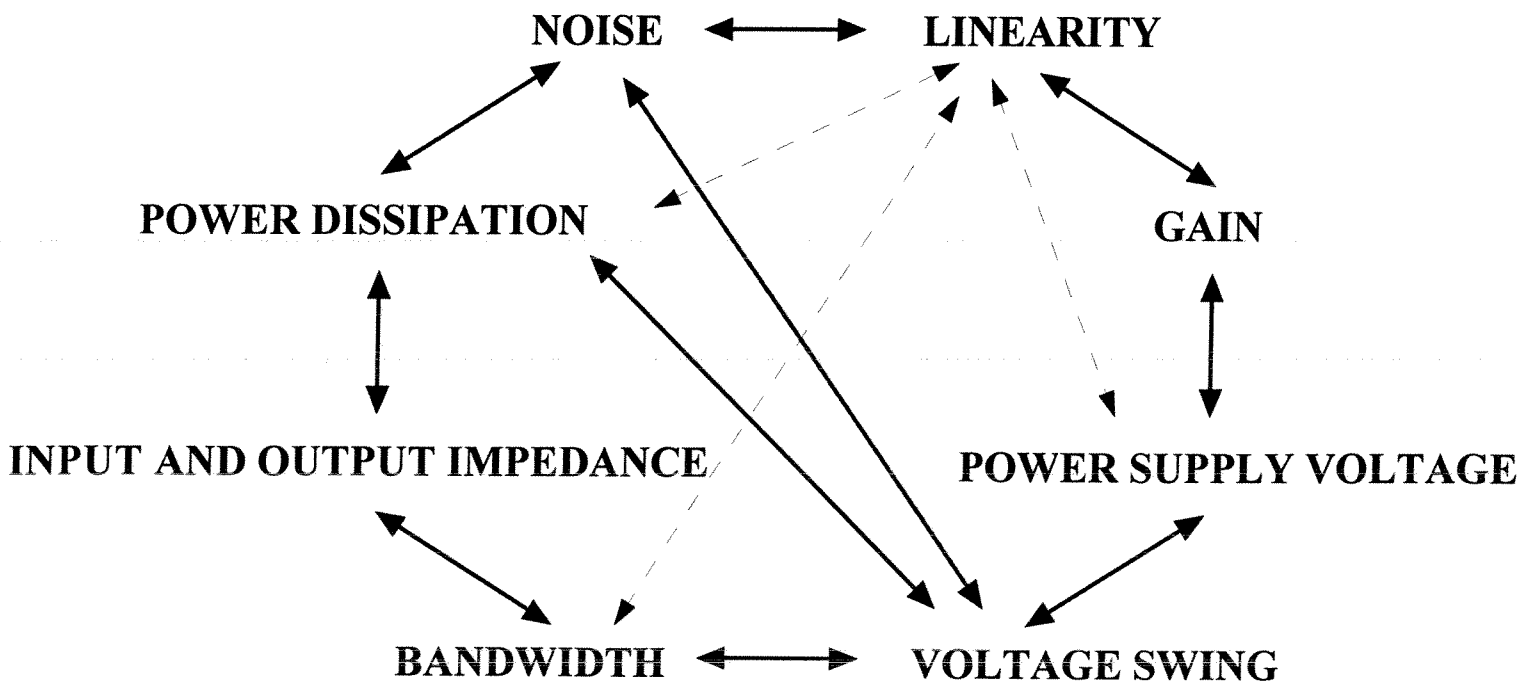


Figure 1.3: Frequency-domain characteristics.

TIME DOMAIN

- CLOCK FREQUENCY
- NUMBER OF DIGITS
- POWER DISSIPATION

DESIGN OCTAGONE



NOISE AND DISTORSION METRICS

$$SNR = 10 \log \frac{P_s}{P_n} \quad [dB]$$

$$THD = 10 \log \frac{P_s}{P_{d,harm}} \quad [dB]$$

$$SNDR = 10 \log \frac{P_s}{P_{n+d}} \quad [dB]$$

$$SFDR = 10 \log \frac{P_s}{P_{d,max}} \quad [dB]$$

P_s = signal power

P_n = noise power

$P_{d,harm}$ = power of all harmonic distortion

P_{n+d} = total power of noise and distortion

$P_{d,max}$ = power of the strongest spurious peak

Diode Equations

Reverse-Biased Diode (Abrupt Junction)

$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{\Phi_0}}}$	$Q = 2C_{j0}\Phi_0\sqrt{1 + \frac{V_R}{\Phi_0}}$
$C_{j0} = \sqrt{\frac{qK_s\epsilon_0 N_D N_A}{2\Phi_0(N_A + N_D)}}$	$C_{j0} = \sqrt{\frac{qK_s\epsilon_0 N_D}{2\Phi_0}}$ if $N_A \gg N_D$
$\Phi_0 = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$	

Forward-Biased Diode

$I_D = I_S e^{V_D/V_T}$	$I_S = A_D q n_i^2 \left(\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right)$
$V_T = \frac{kT}{q} \approx 26 \text{ mV at } 300^\circ\text{K}$	

C_j = diode depletion capacitance Φ_0 = built-in voltage PN-junction

N_D = doping concentration electrons N_A = doping concentration holes

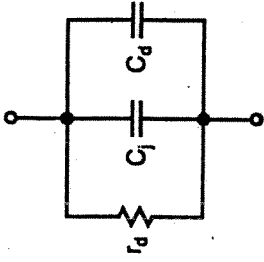
I_S = scale current V_T = thermal voltage

D_n = diffusion constant of electrons, p-side D_p = diffusion constant of holes, n-side

L_n = diffusion length of electrons, p-side L_p = diffusion length of holes, n-side

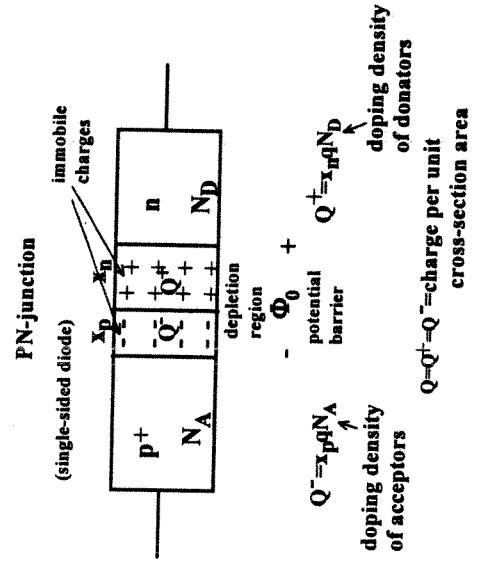
Small-Signal Model of Forward-Biased Diode

5



$r_d = \frac{V_T}{I_D}$	$C_T = C_d + C_j$
$C_d = \tau_T \frac{I_D}{V_T}$	$C_j \approx 2C_{j0}$
$\tau_T = \frac{L_n^2}{D_n}$	

τ_T = transit time for diode



Constants

$q = 1.602 \times 10^{-19} \text{ C}$	$k = 1.38 \times 10^{-23} \text{ JK}^{-1}$
$n_i = 1.1 \times 10^{16} \text{ carriers/m}^3 \text{ at } T = 300 \text{ }^\circ\text{K}$	$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$
$K_{ox} \cong 3.9$	$K_s \cong 11.8$
$\mu_n = 0.05 \text{ m}^2/\text{V} \cdot \text{s}$	$\mu_p = 0.02 \text{ m}^2/\text{V} \cdot \text{s}$

q = elektron charge

k = Boltzmann's constant

n_i = carrier concentration
intrinsic silicon

ϵ_0 = permittivity
for free space

K_{ox} = relative permittivity for SiO₂

K_s = relative permittivity for Si

μ_n = mobility of electrons

μ_p = mobility of holes

Table 1.1 Important SPICE parameters for modelling diodes

SPICE Parameter	Model Constant	Brief Description	Typical Value
IS	I_s	Transport saturation current	10^{-17} A
RS	R_d	Series resistance	30 Ω
TT	τ_T	Diode transit time	12 ps
CJ	C_{j0}	Capacitance at 0-V bias	0.01 pF
MJ	m_j	Diode grading coefficient exponent	0.5
PB	Φ_0	Built-in diode contact potential	0.9 V

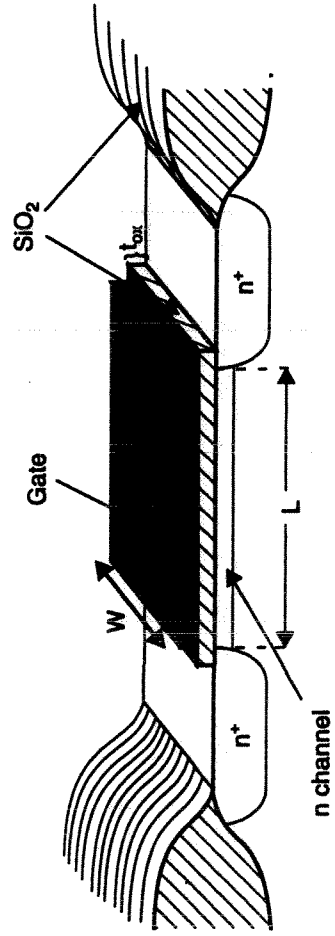
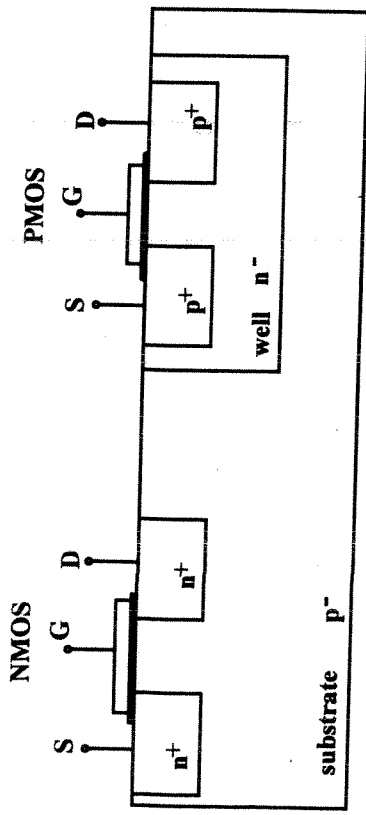


Fig. 1.10 The important dimensions of a MOS transistor.

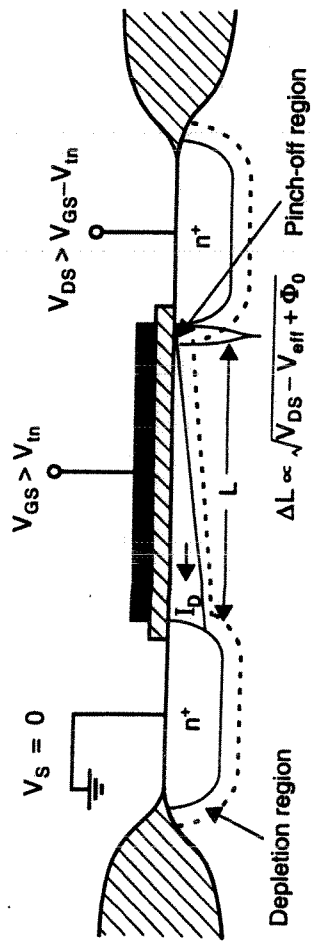


Fig. 1.15 Channel length shortening for $V_{DS} > V_{eff}$.

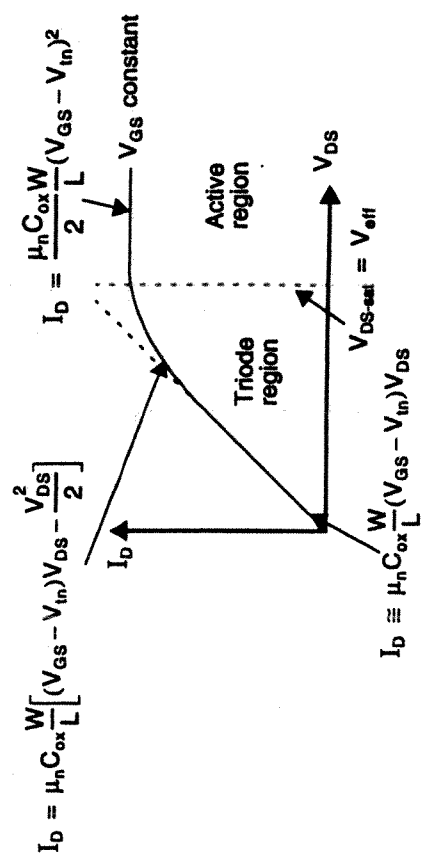


Fig. 1.14 The I_D versus V_{DS} curve for an ideal MOS transistor. For $V_{DS} > V_{DS-sat}$, I_D is approximately constant.

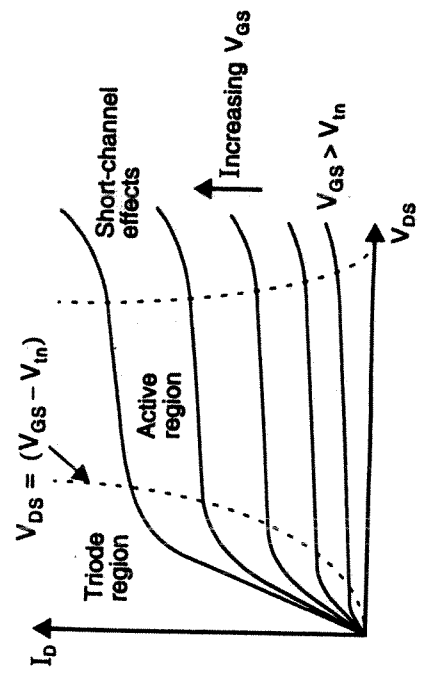


Fig. 1.16 I_D versus V_{DS} for different values of V_{GS} .

The following equations are for n-channel devices—for p-channel devices, put negative signs in front of all voltages. These equations do not account for short-channel effects (i.e., $L < 2L_{min}$).

Triode Region ($V_{GS} > V_{tn}$, $V_{DS} \leq V_{eff}$)

$I_D = \mu_n C_{ox} \left(\frac{W}{L} \right) \left[(V_{GS} - V_{tn}) V_{DS} - \frac{V_{DS}^2}{2} \right]$
$V_{eff} = V_{GS} - V_{tn} \qquad V_{tn} = V_{tn-0} + \gamma \left(\sqrt{V_{SB} + 2\Phi_F} - \sqrt{2\Phi_F} \right)$

Active (or Pinch-Off) Region ($V_{GS} > V_{tn}$, $V_{DS} \geq V_{eff}$)

$I_D = \frac{\mu_n C_{ox} W}{2 L} (V_{GS} - V_{tn})^2 [1 + \lambda (V_{DS} - V_{eff})]$
$\lambda \propto \frac{1}{L \sqrt{V_{DS} - V_{eff} + \Phi_0}} \qquad V_{tn} = V_{tn-0} + \gamma \left(\sqrt{V_{SB} + 2\Phi_F} - \sqrt{2\Phi_F} \right)$
$V_{eff} = V_{GS} - V_{tn} = \sqrt{\frac{2I_D}{\mu_n C_{ox} W/L}}$

$\Phi_F = \frac{kT}{q} \ln \left(\frac{N_A}{n_i} \right)$	$\gamma = \frac{\sqrt{2qK_{si}\epsilon_0 N_A}}{C_{ox}}$
$C_{ox} = \frac{K_{ox}\epsilon_0}{t_{ox}}$	

V_{tn} = threshold voltage n-channel

C_{ox} = gate capacitance

t_{ox} = thickness of insulating SiO_2

W = gate width

L = effective gate length

Φ_F = Fermi potential

γ = body-effect constant

λ = output impedance [V^{-1}]

V_{tp} = threshold voltage p-channel (negative)

C_{S-SW} = capacitance between source and sidewalls

C_{D-SW} = capacitance between drain and sidewalls

N_B = doping concentration bulk

Table 1.2 A reasonable set of MOS parameters for a typical 0.8- μm technology

SPICE Parameter	Model Constant	Brief Description	Typical Value
VTO	$V_{tn}:V_{tp}$	Transistor threshold voltage (in V)	0.7:-0.9
UO	$\mu_n:\mu_p$	Carrier mobility in bulk (in $\text{cm}^2/\text{V}\cdot\text{s}$)	500:175
TOX	t_{ox}	Thickness of gate oxide (in m)	1.8×10^{-8}
LD	L_D	Lateral diffusion of junction under gate (in m)	6×10^{-8}
GAMMA	γ	Body-effect parameter	0.5: 0.8
NSUB	$N_A:N_D$	The substrate doping (in cm^{-3})	$3 \times 10^{16}:7.5 \times 10^{16}$
PHI	$ 2\phi_F $	Surface inversion potential (in V)	0.7
PB	Φ_0	Built-in contact potential of junction to bulk (in V)	0.9
CJ	C_{j0}	Junction-depletion capacitance at 0-V bias (in F/m^2)	$2.5 \times 10^{-4}:4.0 \times 10^{-4}$
CJSW	C_{j-sw0}	Sidewall capacitance at 0-V bias (in F/m)	$2.0 \times 10^{-10}:2.8 \times 10^{-10}$
MJ	m_j	Bulk-to-junction exponent (grading coefficient)	0.5
MJSW	m_{j-sw}	Sidewall-to-junction exponent (grading coefficient)	0.3

Typical Values for a 0.8- μm Process

$V_{tn} = 0.8 \text{ V}$	$V_{tp} = -0.9 \text{ V}$
$\mu_n C_{ox} = 90 \mu\text{A}/\text{V}^2$	$\mu_p C_{ox} = 30 \mu\text{A}/\text{V}^2$

$C_{ox} = 1.9 \times 10^{-3} \text{ pF}/(\mu\text{m})^2$	$C_j = 2.4 \times 10^{-4} \text{ pF}/(\mu\text{m})^2$
$C_{j-sw} = 2.0 \times 10^{-4} \text{ pF}/\mu\text{m}$	$C_{gs(\text{overlap})} = 2.0 \times 10^{-4} \text{ pF}/\mu\text{m}$
$\phi_F = 0.34 \text{ V}$	$\Phi_0 = 0.9 \text{ V}$
$\gamma = 0.5 \text{ V}^{1/2}$	$t_{ox} = 0.02 \mu\text{m}$
$N_B = 6 \times 10^{21} \text{ impurities}/\text{m}^3$	

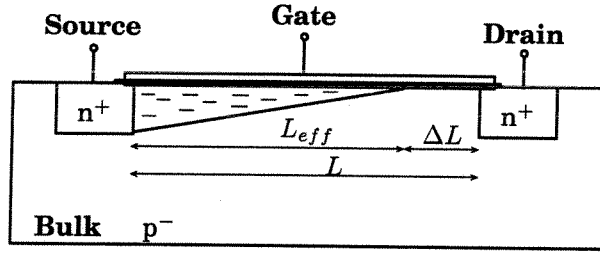
Derivation. RELATION BETWEEN I_D AND V_{DS} WHEN CHANNEL LENGTH MODULATION

At pinch-off limit:

$$I_{D,sat} = \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L} \right) (V_{GS} - V_{tn})^2 \quad (1)$$

When $V_{DS} > V_{DS,sat}$:

$$I_D = \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L_{eff}} \right) (V_{GS} - V_{tn})^2 \quad (2)$$



Assumptions: $V_{GS} > V_{tn}$, $V_{DS} > V_{GS} - V_{tn} = V_{eff}$. $N_D \gg N_A \Rightarrow \Delta L = x_p + x_n \approx x_p$.

$$L_{eff} = L - \Delta L \quad (3)$$

$$\begin{aligned} \Delta L \approx x_p &= \frac{Q_-}{q \cdot N_A} = \frac{1}{q \cdot N_A} \sqrt{2qK_s \epsilon_0 (\phi_0 + V_R) \frac{N_A N_D}{N_A + N_D}} = \sqrt{\frac{2K_s \epsilon_0 (\phi_0 + V_R)}{q} \cdot \frac{N_D}{N_A (N_A + N_D)}} \\ &\approx \sqrt{\frac{2K_s \epsilon_0 (\phi_0 + V_R)}{q} \cdot \frac{1}{N_A}} = k_{ds} \sqrt{V_R + \phi_0} = k_{ds} \sqrt{V_{DS} - V_{eff} + \phi_0} \end{aligned} \quad (4)$$

$$k_{ds} = \sqrt{\frac{2K_s \epsilon_0}{q N_A}} \quad V_R = V_{DS} - V_{DS,sat} = V_{DS} - V_{eff} \quad V_{eff} = V_{GS} - V_{tn}$$

- V_{DS} increase $\Rightarrow \Delta L$ increase $\Rightarrow L_{eff}$ decrease $\Rightarrow I_D$ increase.

- Determine the relation between I_D and V_{DS} !

$$(2) \Rightarrow I_D = \frac{A}{L_{eff}} (V_{GS} - V_{tn})^2 \quad (5)$$

$$(1) \Rightarrow I_{D,sat} = \frac{A}{L} (V_{GS} - V_{tn})^2 \quad (6)$$

Taylorseries around $I_{D,sat}$:

$$I_D = I_{D,sat} + \left. \frac{\partial I_D}{\partial L_{eff}} \right|_{L_{eff}=L} \cdot \left. \frac{\partial L_{eff}}{\partial V_{DS}} \right|_{V_{DS}=V_{DS,sat}} \cdot \Delta V_{DS} \quad (7)$$

Derivating (5) and (3) gives:

$$\begin{aligned}\frac{\partial I_D}{\partial L_{eff}} &= -\frac{A}{L_{eff}^2} (V_{GS} - V_{tn})^2 \\ \frac{\partial L_{eff}}{\partial V_{DS}} &= \frac{\partial(L - \Delta L)}{\partial V_{DS}} = -\frac{\partial \Delta L}{\partial V_{DS}} = -\frac{k_{ds}}{2} \frac{1}{\sqrt{V_{DS} - V_{eff} + \phi_0}} \\ \Delta V_{DS} &= V_{DS} - V_{DS,sat} = V_{DS} - V_{eff}\end{aligned}\quad (8)$$

(6), (7) and (8) give:

$$\begin{aligned}I_D &= \frac{A}{L} (V_{GS} - V_{tn})^2 + \left(-\frac{A}{L^2} (V_{GS} - V_{tn})^2\right) \left(-\frac{k_{ds}}{2} \frac{1}{\sqrt{V_{DS,sat} - V_{eff} + \phi_0}}\right) (V_{DS} - V_{eff}) \\ &= \frac{A}{L} (V_{GS} - V_{tn})^2 \left(1 + \frac{k_{ds}}{2L} \cdot \frac{1}{\sqrt{\phi_0}} \cdot (V_{DS} - V_{eff})\right) \\ &= \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L}\right) (V_{GS} - V_{tn})^2 (1 + \lambda(V_{DS} - V_{eff})) \\ &= \alpha \cdot V_{eff}^2 (1 + \lambda(V_{DS} - V_{eff}))\end{aligned}$$

I.e.:

$$I_D = \alpha \cdot V_{eff}^2 (1 + \lambda(V_{DS} - V_{eff})) \quad (9)$$

where $\alpha = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L}$ and channellength-modulation constant

$$\lambda = \frac{k_{ds}}{2L \cdot \sqrt{\phi_0}}; \quad k_{ds} = \sqrt{\frac{2K_s \epsilon_0}{qN_A}}$$

Matching

Differentiate the saturation current

$$I_D = K' \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

$$\frac{\Delta I_D}{I_D} = \frac{\Delta K'}{K'} + \frac{\Delta W}{W} - \frac{\Delta L}{L} + 2 \frac{\Delta V_{GS} - \Delta V_T}{V_{GS} - V_T} + \frac{\Delta V_{DS} \lambda + V_{DS} \lambda}{1 + \lambda V_{DS}}$$

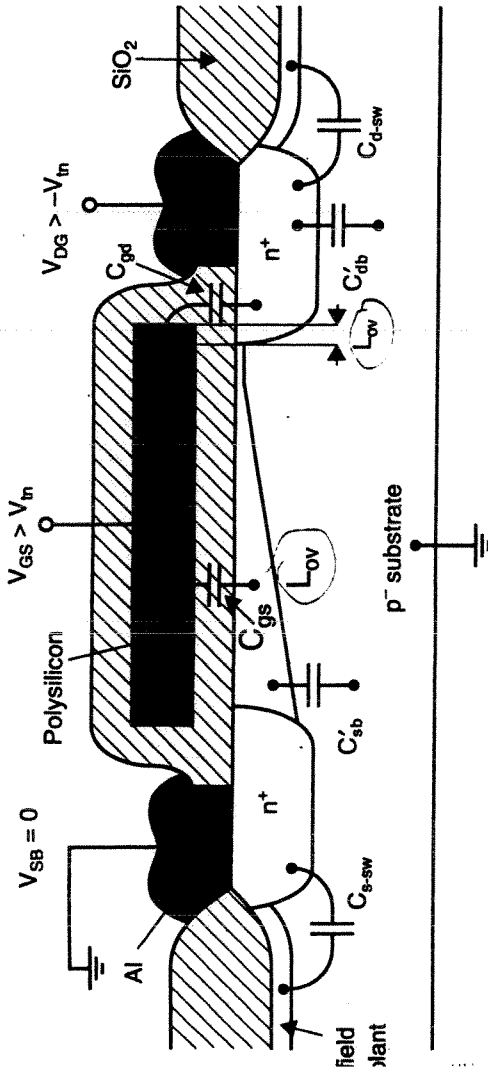
Rule of thumb 1

$V_{GS} - V_T = V_{eff} > 0.15 - 0.25$ to obtain good V_T -matching.

Rule of thumb 2

Choose $L \geq 1.5 L_{min}$ for good β -matching.

Choose $W \gg W_{min}$ for high gain and good β -matching.



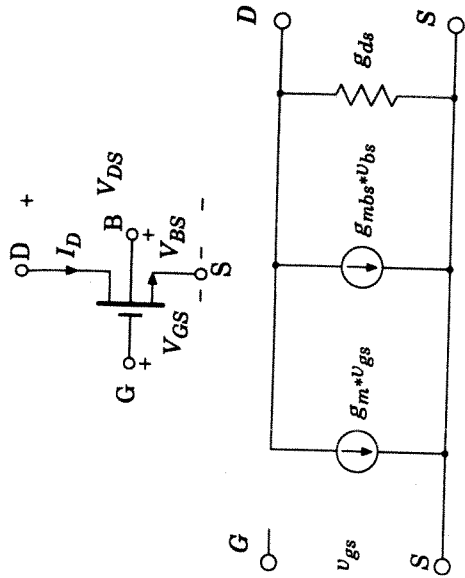
1.20 A cross section of an n-channel MOS transistor showing the small-signal capacitances.

Table 1.2 A reasonable set of MOS parameters for a typical 0.8- μ m technology

SPICE Parameter	Model Constant	Brief Description	Typical Value
VTO	V_{tn}, V_{tp}	Transistor threshold voltage (in V)	0.7; -0.9
UO	μ_n, μ_p	Carrier mobility in bulk (in $\text{cm}^2/\text{V}\cdot\text{s}$)	500; 175
TOX	t_{ox}	Thickness of gate oxide (in m)	1.8×10^{-8}
LD	L_D	Lateral diffusion of junction under gate (in m)	6×10^{-8}
GAMMA	γ	Body-effect parameter	0.5; 0.8
NSUB	N_A, N_D	The substrate doping (in cm^{-3})	3×10^{16} ; 7.5×10^{16}
PHI	$ \phi_F $	Surface inversion potential (in V)	0.7
PB	Φ_0	Built-in contact potential of junction to bulk (in V)	0.9
CJ	C_{j0}	Built-in contact capacitance at 0-V bias (in F/m^2)	2.5×10^{-4} ; 4.0×10^{-4}
CJSW	C_{jsw0}	Sidewall capacitance at 0-V bias (in F/m)	2.0×10^{-10} ; 2.8×10^{-10}
MJ	m_j	Built-to-junction exponent (grading coefficient)	0.5
MJSW	m_{jsw}	Sidewall-to-junction exponent (grading coefficient)	0.3

Small-signal model (Linear region)

$$I_D = \beta \left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2} \right]$$

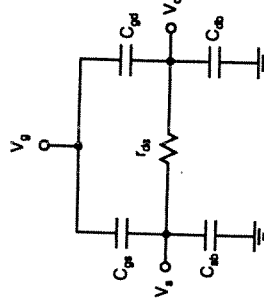


$$g_{ds} = \frac{\partial I_D}{\partial V_{DS}} = \beta [V_{GS} - V_T - V_{DS}]$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \beta V_{DS} \approx 0$$

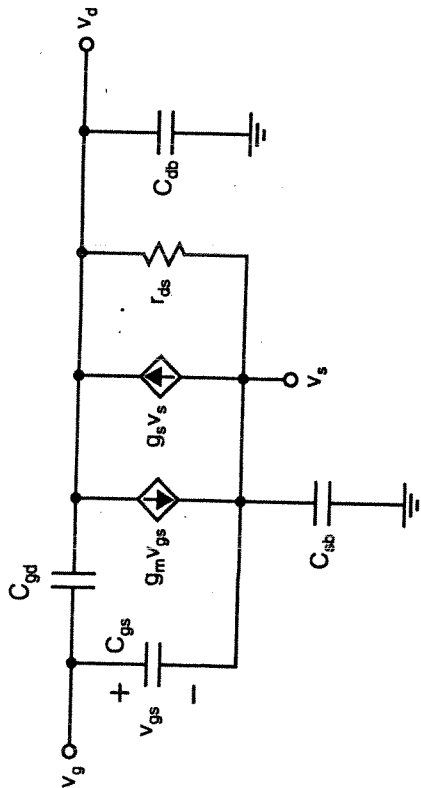
$$g_{mbs} = \frac{\partial I_D}{\partial V_{BS}} = \eta g_m \approx 0$$

Small-Signal Model in Triode Region (for $V_{DS} \ll V_{eff}$)



$r_{ds} = \frac{1}{\mu_n C_{ox} \left(\frac{W}{L}\right) V_{eff}}$	
$C_{gd} = C_{gs} \approx \frac{1}{2} W L C_{ox} + W L_{ov} C_{ox}$	$C_{db} = C_{db} = \frac{C_{j0}(A_s + WL/2)}{\sqrt{1 + \frac{V_{db}}{\Phi_0}}}$

Small-Signal Model (Active Region)



$g_m = \mu_n C_{ox} \left(\frac{W}{L}\right) V_{eff}$	$g_m = \sqrt{2\mu_n C_{ox} (W/L) I_D}$
$g_m = \frac{2I_D}{V_{eff}}$	$g_s = \frac{\gamma g_m}{2\sqrt{V_{SB} + 2\Phi_F }}$
$r_{ds} = \frac{1}{\lambda I_D}$	$g_s \approx 0.2g_m$
$\lambda = \frac{k_{rds}}{2L\sqrt{V_{DS} - V_{eff} + \Phi_0}}$	$k_{rds} = \sqrt{\frac{2K_s \epsilon_0}{qN_A}}$
$C_{gs} = \frac{2}{3} W L C_{ox} + W L_{ov} C_{ox}$	$C_{gd} = W L_{ov} C_{ox}$
$C_{sb} = (A_s + WL) C_{js} + P_s C_{j-sw}$	$C_{js} = \frac{C_{j0}}{\sqrt{1 + V_{SB}/\Phi_0}}$
$C_{db} = A_d C_{jd} + P_d C_{j-sw}$	$C_{jd} = \frac{C_{j0}}{\sqrt{1 + V_{DB}/\Phi_0}}$

DETERMINE g_m AND g_{ds}

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_t} \quad g_{ds} = \frac{1}{r_{ds}} = \lambda I_{DQ} \quad \frac{g_m}{g_{ds}} = \frac{2}{\lambda(V_{GSQ} - V_t)}$$

Regard a 0.35 μm CMOS-process ($L_{\min} = 0.35 \mu\text{m}$):

	NMOS L=1 μm	PMOS L=1 μm	
λ	0.03	0.05	[1/V]
V_t	0.47	0.62	[V]
μ	400	130	[cm ² /Vs]
C_{ox}	$4.5 \cdot 10^{-7}$	$4.5 \cdot 10^{-7}$	[F/cm ²]

For good matching choose $V_{GSQ} - V_t$ in the interval [0.15,0.25] V.

Choose e.g.. $V_{GSQ} - V_t = 0.25$ V

$$\text{NMOS: } \frac{g_m}{g_{ds}} = \frac{2}{0.03 \cdot 0.25} \approx 267$$

$$\text{PMOS: } \frac{g_m}{g_{ds}} = \frac{2}{0.05 \cdot 0.25} = 160$$

Determine g_m : $g_m = \mu C_{ox} \frac{W}{L} (V_{GSQ} - V_t)$

Suppose: $W = 10 \mu\text{m}$, $L = 1 \mu\text{m}$ och $V_{GSQ} - V_t = 0.25$ V

$$\text{NMOS: } g_m = 400 \cdot 4.5 \cdot 10^{-7} \cdot 10 \cdot 0.25 = 0.45 \cdot 10^{-3} \text{ [S]}$$

$$\text{PMOS: } g_m = 130 \cdot 4.5 \cdot 10^{-7} \cdot 10 \cdot 0.25 \approx 0.15 \cdot 10^{-3} \text{ [S]}$$

Determine g_{ds} :

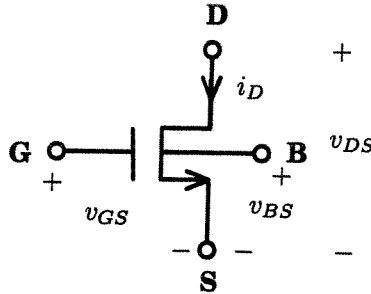
$$\text{NMOS: } g_{ds} = \frac{g_m}{267} \approx 1.69 \cdot 10^{-6} \text{ [S]} \Rightarrow r_{ds} \approx 0.59 \text{ M}\Omega$$

$$\text{PMOS: } g_{ds} = \frac{g_m}{160} \approx 0.9 \cdot 10^{-6} \text{ [S]} \Rightarrow r_{ds} \approx 1.09 \text{ M}\Omega$$

DERIVATION OF A SMALL SIGNAL EQUIVALENT FOR NMOS-TRANSISTOR

Regard i_D as a function of v_{GS} , v_{BS} and v_{DS} :

$$i_D = f(v_{GS}, v_{BS}, v_{DS}) \tag{1}$$



i_D , as well as v_{GS} , v_{BS} and v_{DS} consists of an DC-part, I_{DQ} , V_{GSQ} , V_{BSQ} and V_{DSQ} (the quiescent point) and an AC-part i_d , v_{gs} , v_{bs} and v_{ds} (the small signal).

$$i_D = I_{DQ} + i_d, v_{GS} = V_{GSQ} + v_{gs}, v_{BS} = V_{BSQ} + v_{bs}, v_{DS} = V_{DSQ} + v_{ds} \tag{2}$$

That is:

$$I_{DQ} + i_d = f(V_{GSQ} + v_{gs}, V_{BSQ} + v_{bs}, V_{DSQ} + v_{ds}) \tag{3}$$

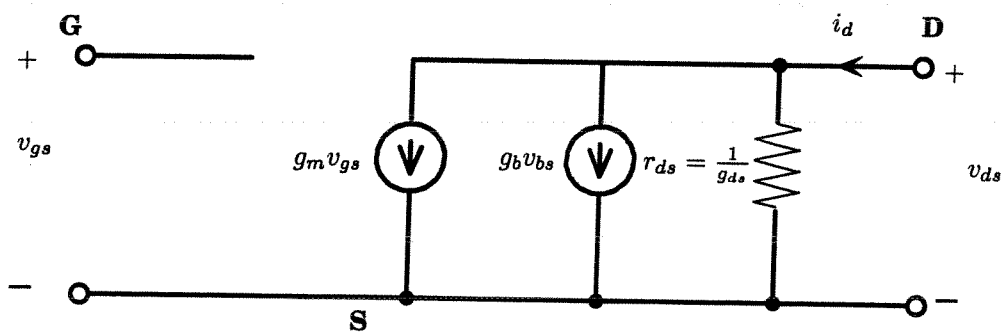
Taylor's formula for 3 variables gives:

$$I_{DQ} + i_d = \underbrace{f(V_{GSQ}, V_{BSQ}, V_{DSQ})}_{I_{DQ}} + \underbrace{\left. \frac{\partial f}{\partial v_{GS}} \right|_Q}_{g_m} \cdot v_{gs} + \underbrace{\left. \frac{\partial f}{\partial v_{BS}} \right|_Q}_{g_b} \cdot v_{bs} + \underbrace{\left. \frac{\partial f}{\partial v_{DS}} \right|_Q}_{g_{ds}} \cdot v_{ds} \tag{4}$$

That is:

$$i_d = g_m \cdot v_{gs} + g_b \cdot v_{bs} + g_{ds} \cdot v_{ds} \tag{5}$$

Equation (5) gives following equivalent small signal scheme:



To calculate the values of g_m , g_b and g_{ds} we first have to determine in which region the transistor is operating.

(CONT. →)

We presume that the transistor operates in the SATURATION region, where following relations applies: (Notice that $v_{BS} = -v_{SB}$.)

$$i_D = \alpha(v_{GS} - V_t)^2(1 + \lambda(v_{DS} - V_{eff})) \quad (6)$$

$$V_t = V_{t0} + \gamma \left(\sqrt{2|\phi_F| - v_{BS}} - \sqrt{2|\phi_F|} \right) \quad (7)$$

To determine g_m we derivate (6) with respect to v_{GS} :

$$\frac{\partial i_D}{\partial v_{GS}} = 2\alpha(v_{GS} - V_t)(1 + \lambda(v_{DS} - V_{eff}))$$

The value of this derivative in the Q-point gives g_m :

$$g_m = 2\alpha(V_{GSQ} - V_t)(1 + \lambda(V_{DSQ} - V_{eff})) = \frac{2I_{DQ}}{V_{GSQ} - V_t} \quad (8)$$

Neglecting the channel-length modulation gives the approximation:

$$g_m \approx 2\sqrt{\alpha I_{DQ}}$$

To determine g_{ds} we derivate (6) with respect to v_{DS} and calculate the value in the Q-point.

$$\frac{\partial i_D}{\partial v_{DS}} = \lambda \cdot \alpha(v_{GS} - V_t)^2$$

That is:

$$g_{ds} = \lambda \cdot \alpha(V_{GSQ} - V_t)^2 \approx \lambda I_{DQ} \quad (9)$$

To determine g_b we have to use the chain-rule for derivatives, as we don't explicit have i_D as a function of v_{BS} .

$$\frac{\partial i_D}{\partial v_{BS}} = \frac{\partial i_D}{\partial V_t} \cdot \frac{\partial V_t}{\partial v_{BS}} = -2\alpha(v_{gs} - V_t)(1 + \lambda(v_{DS} - V_{eff})) \cdot \left(-\frac{\gamma}{2} (2|\phi_F| - v_{BS})^{-1/2} \right)$$

That is:

$$g_b = \underbrace{-2\alpha(V_{GSQ} - V_t)(1 + \lambda(V_{DSQ} - V_{eff}))}_{g_m} \cdot \left(-\frac{\gamma}{2} (2|\phi_F| - V_{BSQ})^{-1/2} \right) = g_m \cdot \frac{\gamma}{2\sqrt{2|\phi_F| - V_{BSQ}}} = \eta \cdot g_m \quad (10)$$

Conclusion:

$$\left\{ \begin{array}{lll} g_m = \frac{2I_{DQ}}{V_{GSQ} - V_t} & \approx 2\sqrt{\alpha I_{DQ}} & \alpha \sim \frac{W}{L}; \quad g_m \sim \sqrt{\frac{W}{L}} \\ g_{ds} = \lambda \cdot \alpha(V_{GSQ} - V_t)^2 & \approx \lambda I_{DQ} & \lambda \sim \frac{1}{L}; \quad g_{ds} \sim \frac{1}{L} \\ g_b = g_m \cdot \frac{\gamma}{2\sqrt{2|\phi_F| - V_{BSQ}}} & = \eta \cdot g_m & \eta \approx 0.2; \quad g_b \sim \sqrt{\frac{W}{L}} \end{array} \right.$$