# Diode Equations

## *Reverse-Biased Diode (Abrupt Junction)*

| | |
|---|---|
| $C_j = \dfrac{C_{j0}}{\sqrt{1+\dfrac{V_R}{\Phi_0}}}$ | $Q = 2C_{j0}\Phi_0\sqrt{1+\dfrac{V_R}{\Phi_0}}$ |
| $C_{j0} = \sqrt{\dfrac{qK_s\varepsilon_0}{2\Phi_0}\dfrac{N_DN_A}{N_A+N_D}}$ | $C_{j0} = \sqrt{\dfrac{qK_s\varepsilon_0 N_D}{2\Phi_0}}$ if $N_A >> N_D$ |
| $\Phi_0 = \dfrac{kT}{q}\ln\left(\dfrac{N_AN_D}{n_i^2}\right)$ | |

## *Forward-Biased Diode*

| | |
|---|---|
| $I_D = I_S e^{V_D/V_T}$ | $I_S = A_D q n_i^2\left(\dfrac{D_n}{L_nN_A}+\dfrac{D_p}{L_pN_D}\right)$ |
| $V_T = \dfrac{kT}{q} \cong 26$ mV at 300 °K | |

$C_j$ = diode depletion capacitance

$\Phi_0$ = built-in voltage PN-junction

$N_D$ = doping concentration electrons

$N_A$ = doping concentration holes

$I_S$ = scale current

$V_T$ = thermal voltage

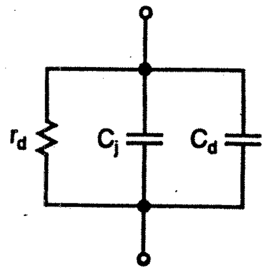
$D_n$ = diffusion constant of electrons, p-side

$D_p$ = diffusion constant of holes, n-side

$L_n$ = diffusion length of electrons, p-side

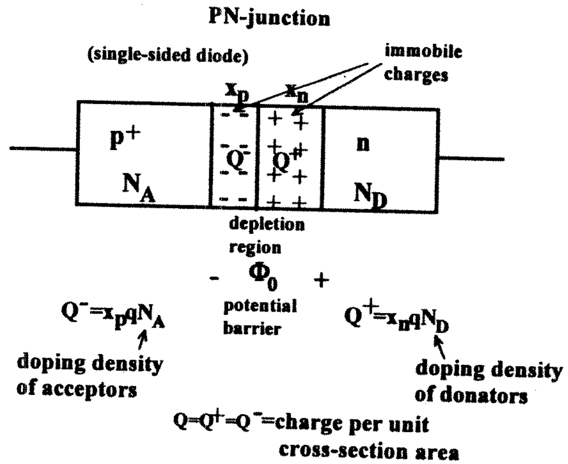
$L_p$ = diffusion length of holes, n-side

## *Small-Signal Model of Forward-Biased Diode*




| | |
|---|---|
| $r_d = \dfrac{V_T}{I_D}$ | $C_T = C_d + C_j$ |
| $C_d = \tau_T\dfrac{I_D}{V_T}$ | $C_j \cong 2C_{j0}$ |
| $\tau_T = \dfrac{L_n^2}{D_n}$ | |

$\tau_T$ = transit time for diode



**Constants**

| | |
|---|---|
| $q = 1.602 \times 10^{-19}$ C | $k = 1.38 \times 10^{-23}$ JK$^{-1}$ |
| $n_i = 1.1 \times 10^{16}$ carriers/m$^3$ at $T = 300$ °K | $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m |
| $K_{ox} \cong 3.9$ | $K_s \cong 11.8$ |
| $\mu_n = 0.05$ m$^2$/V · s | $\mu_p = 0.02$ m$^2$/V · s |

$q$ = elektron charge

$k$ = Bolzmann's constant

$n_i$ = carrier concentration intrinsic silicon

$\varepsilon_0$ = permittivity for free space

$K_{ox}$ = relative permittivity for $SiO_2$

$K_S$ = relative permittivity for Si

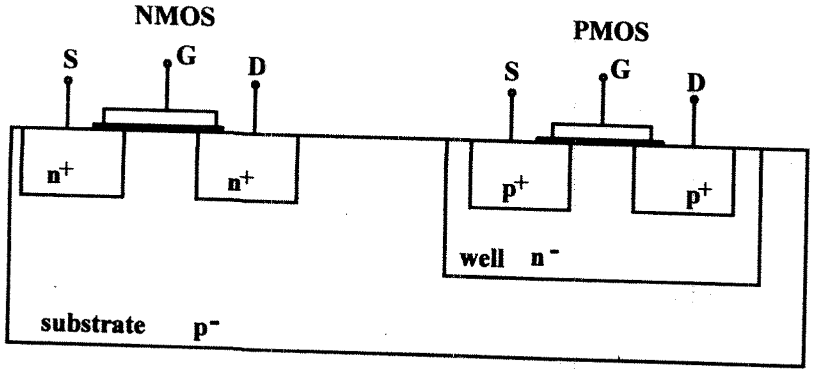
$\mu_n$ = mobility of electrons

$\mu_p$ = mobility of holes



**Table 1.1 Important SPICE parameters for modelling diodes**

| SPICE Parameter | Model Constant | Brief Description | Typical Value |
|---|---|---|---|
| IS | $I_S$ | Transport saturation current | $10^{-17}$ A |
| RS | $R_d$ | Series resistance | 30 Ω |
| TT | $\tau_T$ | Diode transit time | 12 ps |
| CJ | $C_{j0}$ | Capacitance at 0-V bias | 0.01 pF |
| MJ | $m_j$ | Diode grading coefficient exponent | 0.5 |
| PB | $\Phi_0$ | Built-in diode contact potential | 0.9 V |



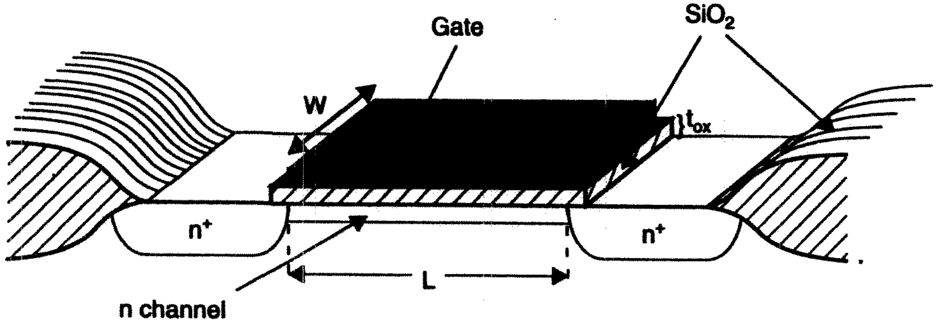




Fig. 1.10 The important dimensions of a MOS transistor.

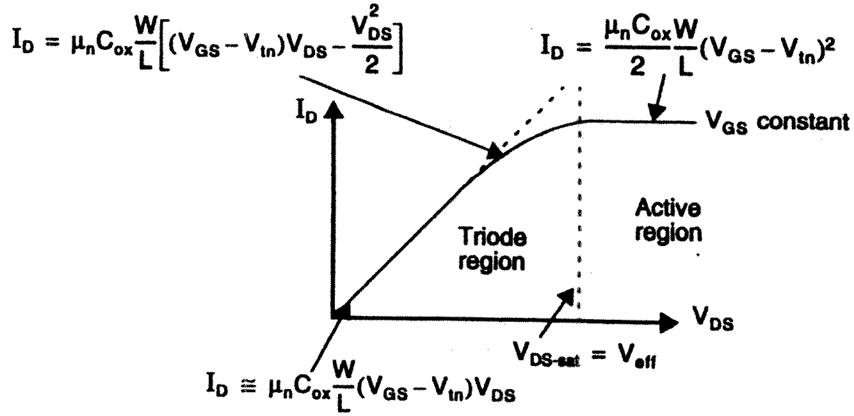



Fig. 1.14 The $I_D$ versus $V_{DS}$ curve for an ideal MOS transistor. For $V_{DS} > V_{DS\text{-sat}}$, $I_D$ is approximately constant.

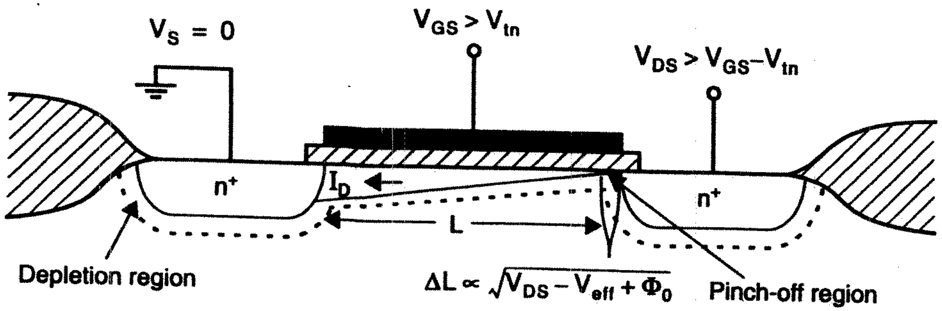



Fig. 1.15 Channel length shortening for $V_{DS} > V_{eff}$.

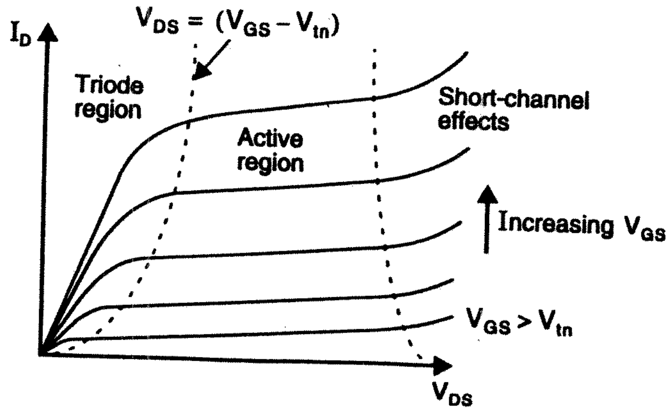



Fig. 1.16 $I_D$ versus $V_{DS}$ for different values of $V_{GS}$.

## MOS Transistor Equations

The following equations are for n-channel devices—for p-channel devices, put negative signs in front of all voltages. These equations do not account for short-channel effects (i.e., $L < 2L_{min}$).

***Triode Region*** ($V_{GS} > V_{tn}$, $V_{DS} \le V_{eff}$)

| | |
|---|---|
| $I_D = \mu_n C_{ox}\left(\frac{W}{L}\right)\left[(V_{GS}-V_{tn})V_{DS} - \frac{V_{DS}^2}{2}\right]$ | |
| $V_{eff} = V_{GS} - V_{tn}$ | $V_{tn} = V_{tn\text{-}0} + \gamma(\sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F})$ |

***Active (or Pinch-Off) Region*** ($V_{GS} > V_{tn}$, $V_{DS} \ge V_{eff}$)

| | |
|---|---|
| $I_D = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS}-V_{tn})^2[1 + \lambda(V_{DS} - V_{eff})]$ | |
| $\lambda \propto \frac{1}{L\sqrt{V_{DS} - V_{eff} + \Phi_0}}$ | $V_{tn} = V_{tn\text{-}0} + \gamma(\sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F})$ |
| $V_{eff} = V_{GS} - V_{tn} = \sqrt{\frac{2I_D}{\mu_n C_{ox} W/L}}$ | |

| | |
|---|---|
| $\phi_F = \frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right)$ | $\gamma = \frac{\sqrt{2qK_{si}\varepsilon_0 N_A}}{C_{ox}}$ |
| $C_{ox} = \frac{K_{ox}\varepsilon_0}{t_{ox}}$ | |



$V_{tn}$ = threshold voltage n-chanal

$C_{OX}$ = gate capacitance

$t_{OX}$ = thicknes of insulating $SiO_2$

$W$ = gate width

$L$ = effective gate lenght

$\Phi_F$ = Fermi potential

$\gamma$ = body-effect constant

$\lambda$ = output impedance $[V^{-1}]$

$V_{tp}$ = threshold voltage p-chanal (negative)

$C_{S\text{-}SW}$ = capacitance between source and sidewalls

$C_{D\text{-}SW}$ = capacitance between drain and sidewalls

$N_B$ = doping concentration bulk

**Table 1.2 A reasonable set of MOS parameters for a typical 0.8-μm technology**

| SPICE Parameter | Model Constant | Brief Description | Typical Value |
|---|---|---|---|
| VTO | $V_{tn}$:$V_{tp}$ | Transistor threshold voltage (in V) | 0.7:–0.9 |
| UO | $\mu_n$:$\mu_p$ | Carrier mobility in bulk (in $cm^2/V\cdot s$) | 500:175 |
| TOX | $t_{ox}$ | Thickness of gate oxide (in m) | $1.8 \times 10^{-8}$ |
| LD | $L_D$ | Lateral diffusion of junction under gate (in m) | $6 \times 10^{-8}$ |
| GAMMA | $\gamma$ | Body-effect parameter | 0.5: 0.8 |
| NSUB | $N_A$:$N_D$ | The substrate doping (in $cm^{-3}$) | $3 \times 10^{16}$:$7.5 \times 10^{16}$ |
| PHI | $\lvert 2\phi_F \rvert$ | Surface inversion potential (in V) | 0.7 |
| PB | $\Phi_0$ | Built-in contact potential of junction to bulk (in V) | 0.9 |
| CJ | $C_{j0}$ | Junction-depletion capacitance at 0-V bias (in $F/m^2$) | $2.5 \times 10^{-4}$:$4.0 \times 10^{-4}$ |
| CJSW | $C_{j\text{-}sw0}$ | Sidewall capacitance at 0-V bias (in F/m) | $2.0 \times 10^{-10}$:$2.8 \times 10^{-10}$ |
| MJ | $m_j$ | Bulk-to-junction exponent (grading coefficient) | 0.5 |
| MJSW | $m_{j\text{-}sw}$ | Sidewall-to-junction exponent (grading coefficient) | 0.3 |

*Typical Values for a 0.8-μm Process*

| | |
|---|---|
| $V_{tn} = 0.8\ V$ | $V_{tp} = -0.9\ V$ |
| $\mu_n C_{ox} = 90\ \mu A/V^2$ | $\mu_p C_{ox} = 30\ \mu A/V^2$ |

| | |
|---|---|
| $C_{ox} = 1.9 \times 10^{-3}\ pF/(\mu m)^2$ | $C_j = 2.4 \times 10^{-4}\ pF/(\mu m)^2$ |
| $C_{j\text{-}sw} = 2.0 \times 10^{-4}\ pF/\mu m$ | $C_{gs(overlap)} = 2.0 \times 10^{-4}\ pF/\mu m$ |
| $\phi_F = 0.34\ V$ | $\Phi_0 = 0.9\ V$ |
| $\gamma = 0.5\ V^{1/2}$ | $t_{ox} = 0.02\ \mu m$ |
| $N_B = 6 \times 10^{21}$ impurities$/m^3$ | |

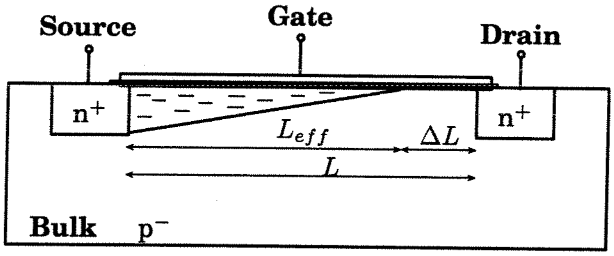
**Derivation. RELATION BETWEEN $I_D$ AND $V_{DS}$ WHEN CHANNEL LENGTH MODULATION**

At *pinch-off* limit:

$$I_{D,sat} = \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right)(V_{GS} - V_{tn})^2 \tag{1}$$

When $V_{DS} > V_{DS,sat}$:

$$I_D = \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L_{eff}}\right)(V_{GS} - V_{tn})^2 \tag{2}$$




**Assumptions:** $V_{GS} > V_{tn}$, $V_{DS} > V_{GS} - V_{tn} = V_{eff}$. $N_D >> N_A \Rightarrow \Delta L = x_p + x_n \approx x_p$.

$$L_{eff} = L - \Delta L \tag{3}$$

$$\begin{aligned}\Delta L \approx x_p &= \frac{Q_-}{q \cdot N_A} = \frac{1}{q \cdot N_A}\sqrt{2qK_s\epsilon_0(\phi_0 + V_R)\frac{N_A N_D}{N_A + N_D}} = \sqrt{\frac{2K_s\epsilon_0(\phi_0 + V_R)}{q} \cdot \frac{N_D}{N_A(N_A + N_D)}} \\ &\approx \sqrt{\frac{2K_s\epsilon_0(\phi_0 + V_R)}{q} \cdot \frac{1}{N_A}} = k_{ds}\sqrt{V_R + \phi_0} = k_{ds}\sqrt{V_{DS} - V_{eff} + \phi_0}\end{aligned} \tag{4}$$

$k_{ds} = \sqrt{\frac{2K_s\epsilon_0}{qN_A}}$ $\quad V_R = V_{DS} - V_{DS,sat} = V_{DS} - V_{eff}$ $\quad V_{eff} = V_{GS} - V_{tn}$

– $V_{DS}$ increase $\Rightarrow \Delta L$ increase $\Rightarrow L_{eff}$ decrease $\Rightarrow I_D$ increase.

– Determine the relation between $I_D$ and $V_{DS}$!

$$(2) \Rightarrow I_D = \frac{A}{L_{eff}}(V_{GS} - V_{tn})^2 \tag{5}$$

$$(1) \Rightarrow I_{D,sat} = \frac{A}{L}(V_{GS} - V_{tn})^2 \tag{6}$$

Taylorseries around $I_{D,sat}$:

$$I_D = I_{D,sat} + \left.\frac{\partial I_D}{\partial L_{eff}}\right|_{L_{eff}=L} \cdot \left.\frac{\partial L_{eff}}{\partial V_{DS}}\right|_{V_{DS}=V_{DS,sat}} \cdot \Delta V_{DS} \tag{7}$$

Derivating (5) and (3) gives:

$$\begin{aligned}
\frac{\partial I_D}{\partial L_{eff}} &= -\frac{A}{L_{eff}^2}(V_{GS}-V_{tn})^2 \\
\frac{\partial L_{eff}}{\partial V_{DS}} &= \frac{\partial(L-\Delta L)}{\partial V_{DS}} = -\frac{\partial \Delta L}{\partial V_{DS}} = -\frac{k_{ds}}{2}\frac{1}{\sqrt{V_{DS}-V_{eff}+\phi_0}} \\
\Delta V_{DS} &= V_{DS}-V_{DS,sat} = V_{DS}-V_{eff}
\end{aligned} \tag{8}$$

(6), (7) and (8) give:

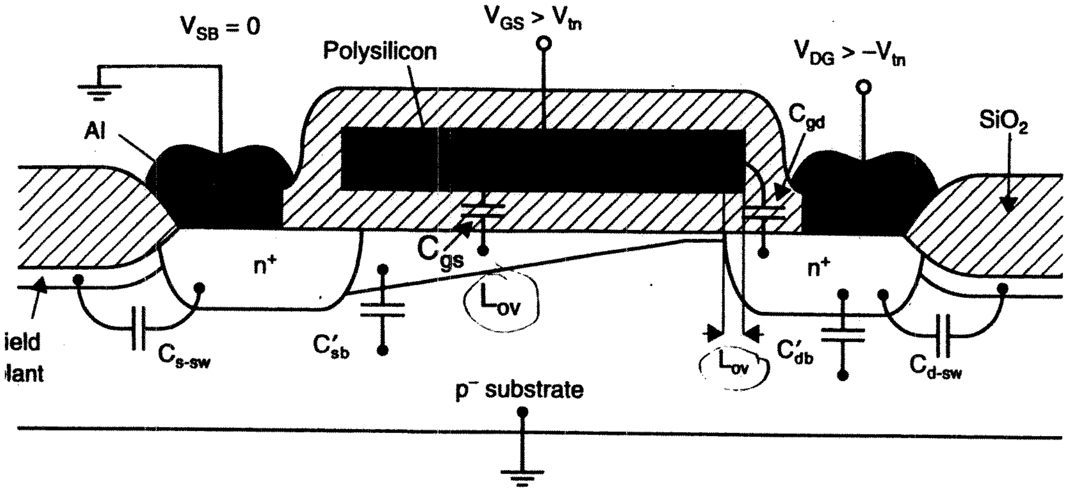
$$\begin{aligned}
I_D &= \frac{A}{L}(V_{GS}-V_{tn})^2 + \left(-\frac{A}{L^2}(V_{GS}-V_{tn})^2\right)\left(-\frac{k_{ds}}{2}\frac{1}{\sqrt{V_{DS,sat}-V_{eff}+\phi_0}}\right)(V_{DS}-V_{eff}) \\
&= \frac{A}{L}(V_{GS}-V_{tn})^2\left(1+\frac{k_{ds}}{2L}\cdot\frac{1}{\sqrt{\phi_0}}\cdot(V_{DS}-V_{eff})\right) \\
&= \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right)(V_{GS}-V_{tn})^2(1+\lambda(V_{DS}-V_{eff})) \\
&= \alpha\cdot V_{eff}^2(1+\lambda(V_{DS}-V_{eff}))
\end{aligned}$$

I.e.:

$$I_D = \alpha\cdot V_{eff}^2(1+\lambda(V_{DS}-V_{eff})) \tag{9}$$

where $\alpha = \frac{\mu_n C_{ox}}{2}\cdot\frac{W}{L}$ and channellength-modulation constant

$\lambda = \frac{k_{ds}}{2L\cdot\sqrt{\phi_0}}$ ; $k_{ds} = \sqrt{\frac{2K_s\epsilon_0}{qN_A}}$




1.20 A cross section of an n-channel MOS transistor showing the small-signal capacitances.

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| TOX | $t_{ox}$ | Thickness of gate oxide (in m) | $1.8 \times 10^{-8}$ |
| LD | $L_D$ | Lateral diffusion of junction under gate (in m) | $6 \times 10^{-8}$ |
| GAMMA | $\gamma$ | Body-effect parameter | 0.5: 0.8 |
| NSUB | $N_A$:$N_D$ | The substrate doping (in $cm^{-3}$) | $3 \times 10^{16}$:$7.5 \times 10^{16}$ |
| PHI | $\|2\phi_F\|$ | Surface inversion potential (in V) | 0.7 |
| PB | $\Phi_0$ | Built-in contact potential of junction to bulk (in V) | 0.9 |
| CJ | $C_{j0}$ | Junction-depletion capacitance at 0-V bias (in $F/m^2$) | $2.5 \times 10^{-4}$:$4.0 \times 10^{-4}$ |
| CJSW | $C_{j-sw0}$ | Sidewall capacitance at 0-V bias (in F/m) | $2.0 \times 10^{-10}$:$2.8 \times 10^{-10}$ |
| MJ | $m_j$ | Bulk-to-junction exponent (grading coefficient) | 0.5 |
| MJSW | $m_{j-sw}$ | Sidewall-to-junction exponent (grading coefficient) | 0.3 |

# Matching

Differentiate the saturation current

$$I_D = K'\frac{W}{L}(V_{GS} - V_T)^2(1 + \lambda V_{DS}) \text{ this gives}$$

$$\frac{\Delta I_D}{I_D} = \frac{\Delta K'}{K'} + \frac{\Delta W}{W} - \frac{\Delta L}{L} + 2\frac{\Delta V_{GS} - \Delta V_T}{V_{GS} - V_T} + \frac{\Delta V_{DS}\lambda + V_{DS}\lambda}{1 + \lambda V_{DS}}$$

# Rule of thumb 1

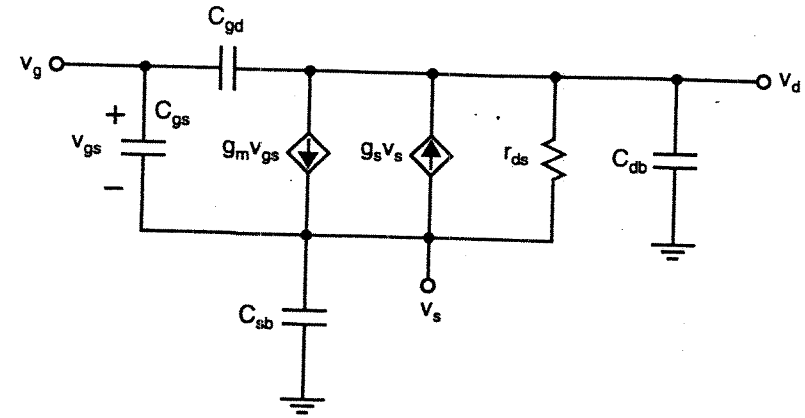
$V_{GS} - V_T = V_{eff} > 0.15–0.25$ to obtain good $V_T$-matching.

# Rule of thumb 2

Choose $L \geq 1.5 L_{min}$ for good β-matching.

Choose $W \gg W_{min}$ for high gain and good β-matching.

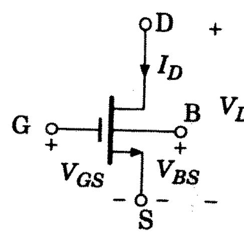
*Small-Signal Model (Active Region)*

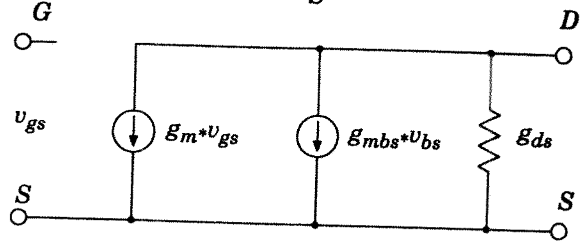



| | |
|---|---|
| $g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)V_{eff}$ | $g_m = \sqrt{2\mu_n C_{ox}(W/L)I_D}$ |
| $g_m = \frac{2I_D}{V_{eff}}$ | $g_s = \frac{\gamma g_m}{2\sqrt{V_{SB} + \lvert 2\phi_F \rvert}}$ |
| $r_{ds} = \frac{1}{\lambda I_D}$ | $g_s \cong 0.2g_m$ |
| $\lambda = \frac{k_{rds}}{2L\sqrt{V_{DS} - V_{eff} + \Phi_0}}$ | $k_{rds} = \sqrt{\frac{2K_s\varepsilon_0}{qN_A}}$ |
| $C_{gs} = \frac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$ | $C_{gd} = WL_{ov}C_{ox}$ |
| $C_{sb} = (A_s + WL)C_{js} + P_sC_{j\text{-}sw}$ | $C_{js} = \frac{C_{j0}}{\sqrt{1 + V_{SB}/\Phi_0}}$ |
| $C_{db} = A_dC_{jd} + P_dC_{j\text{-}sw}$ | $C_{jd} = \frac{C_{j0}}{\sqrt{1 + V_{DB}/\Phi_0}}$ |

## Small-signal model (Linear region)

$$I_D = \beta\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$



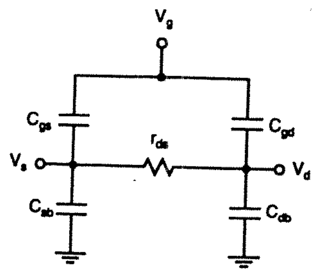




$$g_{ds} = \frac{\partial I_D}{\partial V_{DS}} = \beta[V_{GS} - V_T - V_{DS}]$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \beta V_{DS} \approx 0$$

$$g_{mbs} = \frac{\partial I_D}{\partial V_{BS}} = \eta g_m \approx 0$$

***Small-Signal Model in Triode Region (for $V_{DS} << V_{eff}$)***




| | |
|---|---|
| $r_{ds} = \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)V_{eff}}$ | |
| $C_{gd} = C_{gs} \cong \frac{1}{2}WLC_{ox} + WL_{ov}C_{ox}$ | $C_{sb} = C_{db} = \frac{C_{j0}(A_s + WL/2)}{\sqrt{1 + \frac{V_{sb}}{\Phi_0}}}$ |

## DETERMINE $g_m$ AND $g_{ds}$

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_t} \qquad g_{ds} = \frac{1}{r_{ds}} = \lambda I_{DQ} \qquad \frac{g_m}{g_{ds}} = \frac{2}{\lambda(V_{GSQ} - V_t)}$$

Regard a 0.35 µm CMOS-process ($L_{min} = 0.35$ µm):

| | NMOS L=1 µm | PMOS L=1 µm | |
|---|---|---|---|
| $\lambda$ | 0.03 | 0.05 | [1/V] |
| $V_t$ | 0.47 | 0.62 | [V] |
| $\mu$ | 400 | 130 | [cm$^2$/Vs] |
| $C_{ox}$ | $4.5 \cdot 10^{-7}$ | $4.5 \cdot 10^{-7}$ | [F/cm$^2$] |

For good matching choose $V_{GSQ} - V_t$ in the interval [0.15,0.25] V.
Choose e.g.. $V_{GSQ} - V_t = 0.25$ V

NMOS: $\frac{g_m}{g_{ds}} = \frac{2}{0.03 \cdot 0.25} \approx 267$

PMOS: $\frac{g_m}{g_{ds}} = \frac{2}{0.05 \cdot 0.25} = 160$

**Determine** $g_m$ : $g_m = \mu C_{ox} \frac{W}{L}(V_{GSQ} - V_t)$

Suppose: $W = 10$ µm, $L = 1$ µm och $V_{GSQ} - V_t = 0.25$ V

NMOS: $g_m = 400 \cdot 4.5 \cdot 10^{-7} \cdot 10 \cdot 0.25 = 0.45 \cdot 10^{-3}$ [S]

PMOS: $g_m = 130 \cdot 4.5 \cdot 10^{-7} \cdot 10 \cdot 0.25 \approx 0.15 \cdot 10^{-3}$ [S]

**Determine** $g_{ds}$ :

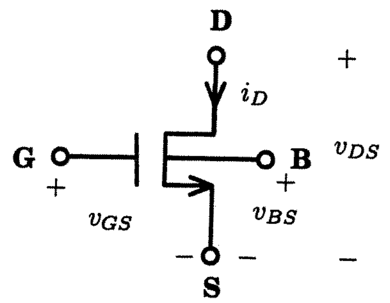
NMOS: $g_{ds} = \frac{g_m}{267} \approx 1.69 \cdot 10^{-6}$ [S] $\Rightarrow r_{ds} \approx 0.59$ MΩ

PMOS: $g_{ds} = \frac{g_m}{160} \approx 0.9 \cdot 10^{-6}$ [S] $\Rightarrow r_{ds} \approx 1.09$ MΩ

## DERIVATION OF A SMALL SIGNAL EQUIVALENT FOR NMOS-TRANSISTOR

Regard $i_D$ as a function of $v_{GS}$, $v_{BS}$ and $v_{DS}$:

$$i_D = f(v_{GS}, v_{BS}, v_{DS}) \qquad (1)$$




$i_D$, as well as $v_{GS}$, $v_{BS}$ and $v_{DS}$ consists of an DC-part, $I_{DQ}$, $V_{GSQ}$, $V_{BSQ}$ and $V_{DSQ}$ (the quiscent point) and an AC-part $i_d$, $v_{gs}$, $v_{bs}$ and $v_{ds}$ (the small signal).

$$i_D = I_{DQ} + i_d, v_{GS} = V_{GSQ} + v_{gs}, v_{BS} = V_{BSQ} + v_{bs}, v_{DS} = V_{DSQ} + v_{ds} \qquad (2)$$

That is:

$$I_{DQ} + i_d = f(V_{GSQ} + v_{gs}, V_{BSQ} + v_{bs}, V_{DSQ} + v_{ds}) \qquad (3)$$

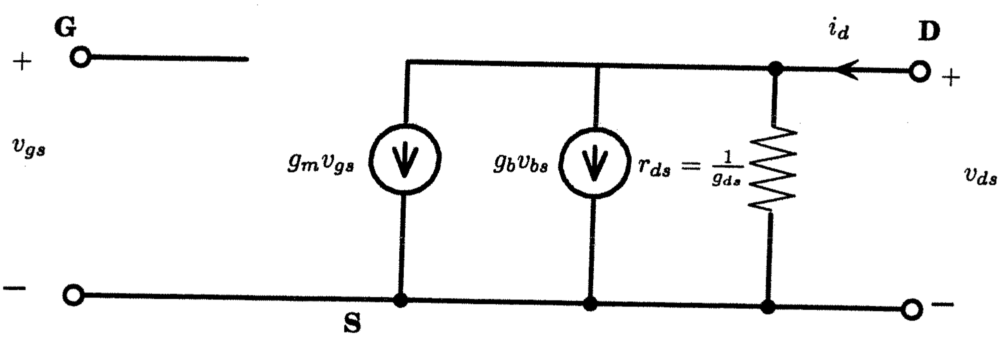
Taylor's formula for 3 variables gives:

$$I_{DQ} + i_d = \underbrace{f(V_{GSQ}, V_{BSQ}, V_{DSQ})}_{I_{DQ}} + \underbrace{\left.\frac{\partial f}{\partial v_{GS}}\right|_Q}_{g_m} \cdot v_{gs} + \underbrace{\left.\frac{\partial f}{\partial v_{BS}}\right|_Q}_{g_b} \cdot v_{bs} + \underbrace{\left.\frac{\partial f}{\partial v_{DS}}\right|_Q}_{g_{ds}} \cdot v_{ds} \qquad (4)$$

That is:

$$i_d = g_m \cdot v_{gs} + g_b \cdot v_{bs} + g_{ds} \cdot v_{ds} \qquad (5)$$

Equation (5) gives following equivalent small signal scheme:




To calculate the values of $g_m$, $g_b$ and $g_{ds}$ we first have to determine in which region the transistor is operating.

(CONT. ↦)

We presume that the transistor operates in the SATURATION region, where following relations applies: (Notice that $v_{BS} = -v_{SB}$.)

$$i_D = \alpha(v_{GS} - V_t)^2(1 + \lambda(v_{DS} - V_{eff})) \tag{6}$$

$$V_t = V_{t0} + \gamma\left(\sqrt{2|\phi_F| - v_{BS}} - \sqrt{2|\phi_F|}\right) \tag{7}$$

To determine $g_m$ we derivate (6) with respect to $v_{GS}$:

$$\frac{\partial i_D}{\partial v_{GS}} = 2\alpha(v_{GS} - V_t)(1 + \lambda(v_{DS} - V_{eff}))$$

The value of this derivative in the Q-point gives $g_m$:

$$g_m = 2\alpha(V_{GSQ} - V_t)(1 + \lambda(V_{DSQ} - V_{eff})) = \frac{2I_{DQ}}{V_{GSQ} - V_t} \tag{8}$$

Neglecting the channel-length modulation gives the approximation:

$$g_m \approx 2\sqrt{\alpha I_{DQ}}$$

To determine $g_{ds}$ we derivate (6) with respect to $v_{DS}$ and calculate the value in the Q-point.

$$\frac{\partial i_D}{\partial v_{DS}} = \lambda \cdot \alpha(v_{GS} - V_t)^2$$

That is:

$$g_{ds} = \lambda \cdot \alpha(V_{GSQ} - V_t)^2 \approx \lambda I_{DQ} \tag{9}$$

To determine $g_b$ we have to use the chain-rule for derivatives, as we don't explicit have $i_D$ as a function of $v_{BS}$.

$$\frac{\partial i_D}{\partial v_{BS}} = \frac{\partial i_D}{\partial V_t} \cdot \frac{\partial V_t}{\partial v_{BS}} = -2\alpha(v_{gs} - V_t)(1 + \lambda(v_{DS} - V_{eff})) \cdot \left(-\frac{\gamma}{2}\left(2|\phi_F| - v_{BS}\right)^{-1/2}\right)$$

That is:

$$g_b = -\underbrace{2\alpha(V_{GSQ} - V_t)(1 + \lambda(V_{DSQ} - V_{eff}))}_{g_m} \cdot \left(-\frac{\gamma}{2}\left(2|\phi_F| - V_{BSQ}\right)^{-1/2}\right) = g_m \cdot \frac{\gamma}{2\sqrt{2|\phi_F| - V_{BSQ}}} = \eta \cdot g_m \tag{10}$$

Conclusion:

$$\begin{cases} g_m = \dfrac{2I_{DQ}}{V_{GSQ} - V_t} \approx 2\sqrt{\alpha I_{DQ}} & \alpha \sim \dfrac{W}{L}; \quad g_m \sim \sqrt{\dfrac{W}{L}} \\ g_{ds} = \lambda \cdot \alpha(V_{GSQ} - V_t)^2 \approx \lambda I_{DQ} & \lambda \sim \dfrac{1}{L}; \quad g_{ds} \sim \dfrac{1}{L} \\ g_b = g_m \cdot \dfrac{\gamma}{2\sqrt{2|\phi_F| - V_{BSQ}}} = \eta \cdot g_m & \eta \approx 0.2; \quad g_b \sim \sqrt{\dfrac{W}{L}} \end{cases}$$