Exercises for Tutorial 3: Frequency Response

1) Problem 6.6. in the course book.

a)

KCL at Vx and Vy:

$$I_{in} - (V_x - V_y)sC_2 = 0 \leftrightarrow V_x sC_2 = I_{in} + V_y s\, C_2 \ (1)$$
$$(V_y - V_x)sC_2 + g_m V_x = 0 \leftrightarrow V_y sC_2 = V_x(sC_2 - g_m) \ (2)$$

(1) in (2):

$$V_x sC_2 = I_{in} + V_x(sC_2 - g_m) \leftrightarrow I_{in} = g_m V_x$$
$$I_{in} = (V_{in} - V_x)sC_2 \leftrightarrow V_x = (V_{in} sC_2 - I_{in})\frac{1}{sC_1}$$
$$I_{in} = g_m(V_{in} sC_1 - I_{in})\frac{1}{sC_1} \leftrightarrow I_{in}(sC_1 + g_m) = g_m sC_1 V_{in}$$
$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{g_m + sC_1}{g_m sC_1}$$

b)

$$I_{in} = (g_{m1} + g_{m2})V_x + \frac{V_y}{r_{o1} \parallel r_{o2}} \leftrightarrow V_y = (r_{o1} \parallel r_{o2})I_{in} - (g_{m1} + g_{m2})(r_{o1} \parallel r_{o2})V_x \ (1)$$
$$I_{in} = (V_x - V_y)sC_2 \leftrightarrow V_x = \frac{I_{in}}{sC_2} + V_y \ (2)$$
$$I_{in} = (V_{in} - V_x)sC_1 \leftrightarrow V_{in} = \frac{I_{in}}{sC_1} + V_x \ (3)$$

Set $g_m = g_{m1} + g_{m2}$ and $r_o = r_{o1} \parallel r_{o2}$

(1), (2): $V_x = \frac{I_{in}}{sC_2} + r_{o1} I_{in} - g_m r_o V_x \leftrightarrow V_x = \frac{\left(\frac{1}{sC_2} + r_o\right) I_{in}}{1 + g_m r_o}$ (4)

(3), (4): $V_{in} = \frac{I_{in}}{sC_1} + \frac{\left(\frac{1}{sC_2} + r_o\right) I_{in}}{1 + g_m r_o} \leftrightarrow sC_1(1 + g_m r_o)V_{in} = (1 + g_m r_o)I_{in} + sC_1\left(\frac{1}{sC_2} + r_o\right) I_{in} \leftrightarrow$

$$sC_1(1 + g_m r_o)V_{in} = \left(1 + g_m r_o + \frac{C_1}{C_2} + sC_1 r_o\right) I_{in} \leftrightarrow$$
$$sC_1 C_2(1 + g_m r_o)V_{in} = (C_2 + g_m r_o C_2 + C_1 + sC_1 C_2 r_o)I_{in}$$
$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{sC_1 C_2 r_o + C_1 + C_2 + g_m r_o C_2}{sC_1 C_2(1 + g_m r_o)}$$

c)

$$I_{in} = V_x sC_2 \leftrightarrow V_x = \frac{I_{in}}{sC_2}$$
$$I_{in} = -g_{m1}(V_x - V_{in}) + g_{mb1} V_{in} = (g_{m1} + g_{mb1})V_{in} - g_{m1} V_x \ (2)$$

(1), (2): $I_{in} = (g_{m1} + g_{mb1})V_{in} - \frac{g_{m1}}{sC_2} I_{in} \leftrightarrow (g_{m1} + sC_2)I_{in} = (g_{m1} + g_{mb}\ )sC_2 V_{in}$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{g_{m1} + sC_2}{sC_2(g_{m1} + g_{mb1})}$$

2) Problem 6.8(e) in the course book.

e)

$$C_1 = C_{gs1} + C_{sb1} + C_{gd2} + C_{db2}$$
$$C_2 = C_{gd1} + C_{db1} + C_{gs2} + C_{sb2}$$

KCL at Vx: $\frac{V_x - Vi}{R_s} + V_x sC_1 + g_{m1}V_x - g_{m2}V_{out} = 0 \leftrightarrow \left(\frac{1}{R_s} + sC_1 + g_{m1}\right)V_x = \frac{V_{in}}{R_s} + g_{m2}V_{out} \leftrightarrow$

$$V_x = \left(\frac{1}{R_s}V_{in} + g_{m2}V_{out}\right)\left(\frac{1}{R_s} + sC_1 + g_{m1}\right)^{-1} \quad (1)$$

KCL at Vout: $g_{m2}V_{out} - g_{m1}V_x + sC_2V_{out} = 0 \leftrightarrow V_x = \frac{V_{out}(g_{m2}+sC_2)}{g_{m1}}$) (2)

(2) in (1): $\frac{V_{in}}{R_s} = \left(\frac{1}{R_s} + sC_1 + g_{m1}\right)\left(\frac{sC_2+g_{m2}}{g_{m1}}\right)V_{out} - g_{m2}V_{out} \leftrightarrow$

$$g_{m1}V_{in} = \left((1 + sR_sC_1 + g_{m1}R_s)(g_{m2} + sC_2) - g_{m2}g_{m1}R_s\right)V_{out}$$
$$= (g_{m2} + sC_2 + sg_{m2}R_sC_1 + s^2R_sC_1C_2 + sg_{m1}C_sR_s)V_{out}$$
$$\frac{V_{out}}{V_{in}} = \frac{g_{m1}}{s^2R_sC_1C_2 + s(R_s(g_{m1}C_2 + g_{m2}C_1) + C_2) + g_{m2}}$$

Input impedance:

$I_x = sC_1V_x + g_{m1}V_x - g_{m2}V_{out}$ and $V_{out} = \frac{g_{m1}}{sC_2+g_{m2}}V_x$

$$I_x = \left(g_{m1} + sC_1 - \frac{g_{m1}g_{m2}}{g_{m2} + sC_2}\right)V_x = \frac{g_{m1}g_{m2} + sg_{m1}C_2 + sg_{m2}C_1 + s^2C_1C_2 - g_{m1}g_{m2}}{g_{m2} + sC_2}V_x$$
$$Z_{out} = \frac{V_x}{I_x} = \frac{g_{m2} + sC_2}{s^2C_1C_2 + s(g_{m1}C_2 + g_{m2}C_1)}$$

3) Problem 6.9(b) in the course book.

KCL at Vx:

$$g_{m1}V_{in} + g_{m2}V_x + (V_x - V_{out})sC_1 = 0 \leftrightarrow (g_{m2} + sC_1)V_x = -g_{m1}V_{in} + sC_1V_{out}$$

KCL at Vout:

$$-g_{m2}V_x + (V_{out} - V_x)sC_1 = 0 \leftrightarrow sC_1V_{out} = (g_{m2} + sC_1)V_x$$

$$sC_1V_{out} = -g_{m1}V_{in} + sC_1V_{out} \leftrightarrow (sC_1 - sC_1)V_{out} = -g_{m1}V_{in} \rightarrow$$
$$A_v = \frac{V_{out}}{V_{in}} = -\frac{g_{m1}}{sC_1 - sC_1} = -\infty \; if \; \lambda = 0$$

4) Problem 6.10(b) in the course book. Assume $r_{03} \gg R_2$.

KCL at Vout1

$$-g_{m6}V_y + g_{m3}V_x + \frac{V_{out1} - V_x}{R_2} = 0 \leftrightarrow V_{out1} = g_{m6}R_2V_y - (g_{m3}R_2 - 1)V_x \; (1)$$

KCL at Vout2

$$-g_{m6}V_Z + g_{m3}V_x + \frac{V_{out2} - V_x}{R_2} = 0 \leftrightarrow V_{out2} = g_{m6}R_2V_Z - (g_{m3}R_2 - 1)V_x \; (2)$$

(1) and (2):

$$V_{out1} - V_{out2} = g_{m6}R_2V_y - g_{m6}R_2V_z - (g_{m3}R_2 - 1)V_x + (g_{m3}R_1 - 1)V_x \leftrightarrow$$

$$V_{out1} - V_{out2} = g_{m6}R_2(V_y - V_z)\ (3)$$

KCL at Vy:

$$I_y = -g_{m6}V_y - g_{m1}(V_{in1} - V_P)\ (4)$$

$$\frac{V_y - V_z}{R_1 + \frac{1}{sC_1}} = I_y = \frac{I_y}{2} + \frac{I_y}{2} = \frac{I_y}{2} - \frac{I_z}{2}\ (5)$$

KCL at Vz:

$$I_z = -I_y = -g_{m6}V_z - g_{m1}(V_{in2} - V_P)\ (6)$$

(4), (5), (6):

$$\frac{V_y - V_z}{R_1 + \frac{1}{sC_1}} = \frac{1}{2}\left[-g_{m6}V_y - g_{m1}(V_{in1} - V_P) + g_{m6}V_z + g_{m1}(V_{in2} - V_P)\right]$$

$$= -\frac{1}{2}g_{m6}(V_y - V_z) - \frac{1}{2}g_{m1}(V\_in1 - V\_in2\,) \leftrightarrow$$

$$V\_in1 - V\_in2\,) \leftrightarrow$$

$$V_y - V_z = -\frac{g_{m1}R_1}{g_{m6}R_1 + \frac{2R_1}{R_1 + \frac{1}{sC_1}}}(V_{in1} - V_{in2})\ (7)$$

(7) in (3):

$$V_{out1} - V_{out2} = -\frac{g_{m1}g_{m6}R_1R_2}{g_{m6}R_1 + \frac{2R_1}{R_1 + \frac{1}{sC_1}}}(V_{in1} - V_{in}\ ) \rightarrow$$

$$A_v = \frac{V_{out1} - V_{out2}}{V_{in1} - V_{in2}} = -\frac{g_{m1}g_{m6}R_1R_2}{g_{m6}R_1 + \frac{2R_1}{R_1 + \frac{1}{sC_1}}}$$

Low frequencies:

$$A_{v,low} \rightarrow -g_{m1}R_2\ as\ s \rightarrow 0$$

High frequencies:

$$A_{v,high} \rightarrow -\frac{g_{m1}g_{m6}R_1R_2}{2 + g_{m6}R_1}\ as\ s \rightarrow \infty$$

5) Figure 6 shows an amplifier schematic. For simplicity we can ignore all parasitics of M_1 and M_2 and we assume that the dominant pole occurs at the output node. Also, we assume $g_m \gg 1/r_0$. Find the product $|A_0|\omega_{-3dB}$, where A_0 is the DC gain and ω_{-3dB} is the 3 dB cut-off frequency. Assume $\gamma = 0$.

KCL at Vx and Vout gives:

$$\frac{V_{out}}{V_x} = -\frac{g_{m2}}{sC + \frac{1}{r_{o2}} + \frac{1}{R}}\ and\ \frac{V_x}{V_{in}} = 1 \rightarrow$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m2}}{sC + \frac{1}{r_{o2}} + \frac{1}{R}} = -\frac{g_{m2}r_{o2}R}{sCRr_{o2} + r_{o2} + R} = -\frac{g_{m2}r_{o2}R}{r_{o2} + R}\frac{1}{1 + \frac{s}{\frac{r_{o2} + R}{Cr_{o2}R}}}$$

$$A_0 = \{s = 0\} = -g_{m2}(r_{o2} \parallel R)\ and\ \omega_{p1} = \frac{r_{o2} + R}{C r_{o2} R} = \frac{1}{r_{o2} \parallel R}\frac{1}{C} \rightarrow$$

$$|A_0|\omega_{p1} = g_{m2}\,(r_{o2} \parallel R)\frac{1}{C}\frac{1}{r_{o2} \parallel R} = \frac{g_{m2}}{C}$$

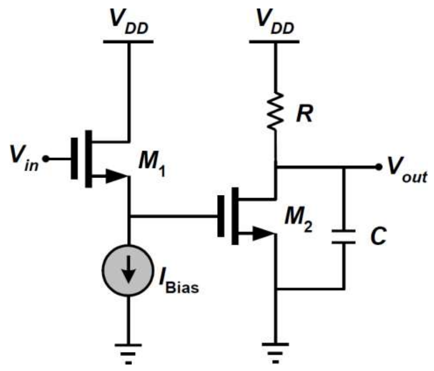

Figure 6 An amplifier schematic.

6) Figure 7 shows an amplifier schematic. For simplicity we can ignore all parasitics of $M_1 - M_4$ and we assume that the dominant pole occurs at the output node. If the input signal has an angular frequency of $\omega_i = 10^9\ rad/s$, determine the AC gain of the amplifier. Assume $g_{m1} = g_{m3} = 4\ mA/V$, $g_{m2} = g_{m4} = 1\ mA/V$, $C = 1\ pF$, $g_m \gg 1/r_0$ and $\gamma = 0$.

KCL at Vx and Vout gives:

$$\frac{V_x}{V_{in}} = \frac{g_{m2}}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} \approx 1$$

$$\frac{V_{out}}{V_x} = -\frac{g_{m3}}{g_{m4} + sC + \frac{1}{r_{o3}} + \frac{1}{r_{o4}}} \approx -\frac{g_{m3}}{g_{m4} + sC} \rightarrow$$

$$A_v(s) = \frac{V_{out}}{V_{in}} \approx -\frac{g_{m3}}{g_{m4} + sC}$$

$$|A_v(\omega_i = 10^9)| = \left|\frac{g_{m3}}{g_{m4} + sC}\right| = \left|\frac{4 \times 10^{-3}}{1 \times 10^{-3} + j10^9 \times 1 \times 10^{-12}}\right|$$

$$= \left|\frac{4}{1 + j}\right| = \left|\frac{4(1 - j)}{2}\right| = 2|1 - j| = 2\sqrt{2}$$

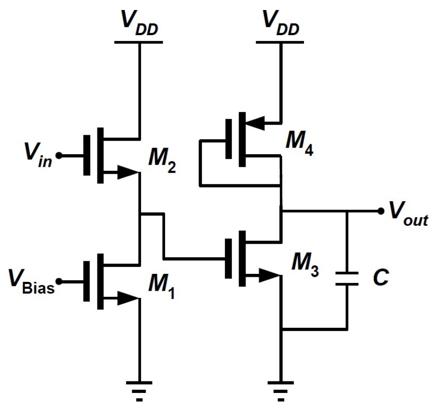

Figure 7 An amplifier schematic.