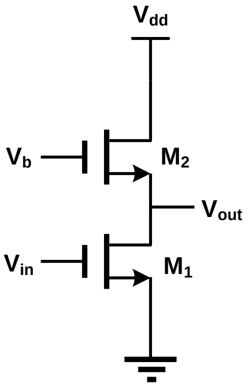
Tutorial 6: Repetition Solutions

Problem 1. Noise

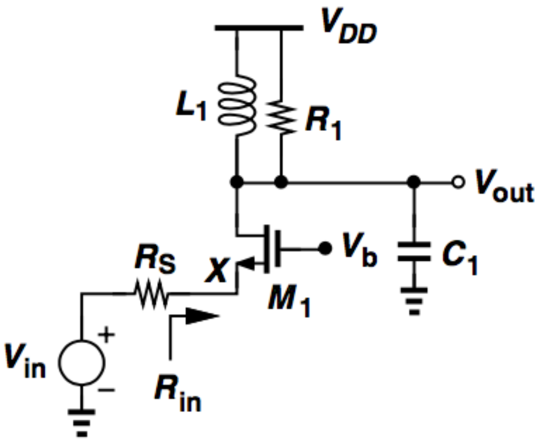
A cascode amplifier stage is shown in figure below. Assume that both transistors are long-channel devices and V_b is the bias voltage for M_2. Furthermore, assume that $gm_1 \neq gm_2$ and $\lambda \neq 0$. Determine the input-referred noise voltage. Consider only the thermal noise sources and ignore the gate noise of the transistors.



A cascode stage.

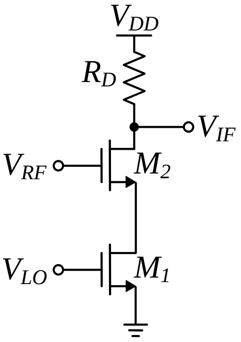
Problem 2. LNA

For the common-gate LNA shown below, matched to 50 Ohm, compute the noise figure at the output resonance frequency.



Problem 3. Mixer

The circuit shown below is a dual-gate mixer used in traditional microwave design. Assume abrupt edges and a 50 % duty cycle for the LO and neglect channel-length modulation and body effect. Also assume M1 is and ideal switch and has no noise contribution.



a) Compute the voltage conversion gain of the circuit.

b) Derive the expression for the noise figure of the mixer.

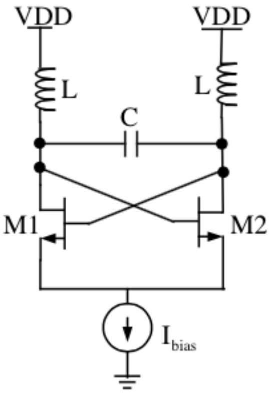
Problem 4. Oscillator

4) A negative-resistance oscillator operating at 2.4 GHz frequency is shown in Fig. 4. The resonant circuit is implemented using inductor $L = 5$ nH with $Q = 10$ and a variable capacitor C. Assume that we can neglect all parasitics associated with the transistors.

(a) What is the minimum width of two identical transistors M_1 and M_2 to ensure the oscillation? (2 p)

(b) How much should the variable capacitance C be varied to enable tuning from 2.4 GHz to 2.5 GHz? (1 p)

(c) For a capacitance corresponding to 2.4 GHz oscillation frequency, if an additional inductor of 5 nH is connected in parallel with the capacitor C, how much will the oscillation frequency change (in percentage)? (2 p)



$$R_P = (Q^2+1)R_S$$

$$I_d = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{gs}-V_t)^2$$

$$V_{DD} = 2.5V$$

$$\mu_n C_{ox} = 110\mu A/V^2$$

$$I_{bias} = 2mA$$

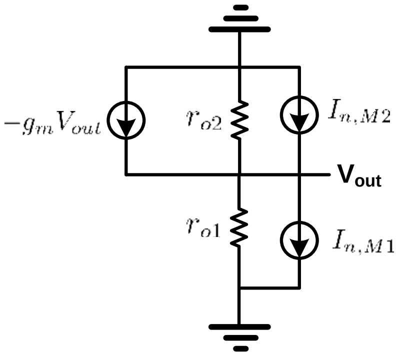
Fig. 4. A negative-resistance oscillator.

Solutions

Problem 1. Noise

Alt. 1 ($gm_1 = gm_2$)

There are two thermal noise sources as shown below.



Since for long-channel devices:

$$\overline{I^2_{n,M1}} = 4kT\gamma g_m$$
$$\overline{I^2_{n,M2}} = 4kT\gamma g_m$$

Then because of uncorrelation, we can use superposition. Using KCL at the output node for $I_{n,M1}$:

$$(g_m V_{out} + I_{n,M1}) \cdot (r_{o1} \parallel r_{o2}) = V_{out} \rightarrow \overline{V^2_{out}} = \overline{I^2_{n,M1}} \cdot (r_{o1} \parallel r_{o2} \parallel 1/g_m)^2$$

Similarly, we get the effect of $I_{n,M2}$ at the output. Then by superposition:

$$\overline{V^2_{n,out}} = 8kT\gamma g_m (r_{o1} \parallel r_{o2} \parallel 1/g_m)^2$$

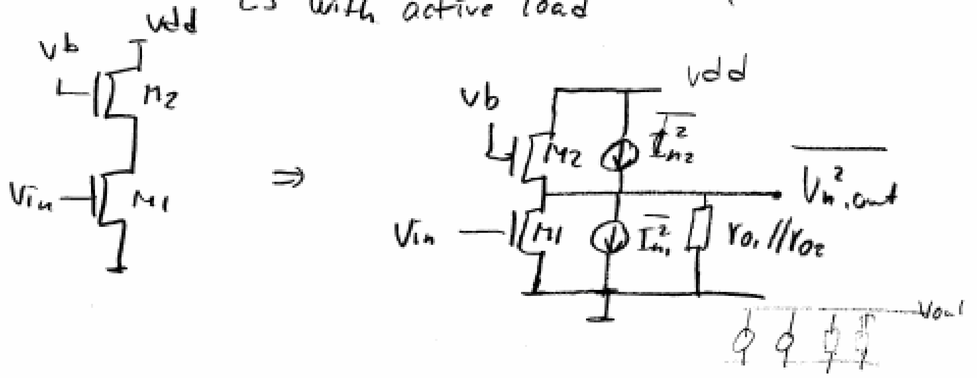
To get the input-referred noise voltage we should divide the total output noise by the gain square ($g_m^2 (r_{o1} \parallel r_{o2} \parallel 1/g_m)^2$:

$$\overline{V^2_{n,in}} = \frac{\overline{V^2_{n,out}}}{A_v^2} = \frac{8kT\gamma g_m (r_{o1} \parallel r_{o2} \parallel 1/g_m)^2}{g_m^2 (r_{o1} \parallel r_{o2} \parallel 1/g_m)^2} = \frac{8kT\gamma}{g_m}$$

A1t2.

Similar to Example 7.10 in Razavi Analog CMOS: (p. 226)

CS with active load



Sources are uncorrelated, can be added.

$$\overline{V^2_{n,out}} = 4kT\gamma\,(g_{m1}+g_{m2})\,(r_{o1}//r_{o2})^2$$

$$(V = i_{noise}\cdot r_o)$$

Since $\overline{I^2_{n1}} = 4kT\gamma g_{m1}$, $\overline{I^2_{n2}} = 4kT\gamma g_{m2}$

For a CS with current-source load: $A_v = -g_m \cdot R_D$

$$A_v = -g_{m1}(r_{o1}//r_{o2})$$ (Razavi CMOS p. 58)

$$\overline{V^2_{n,in}} = \frac{4kT\gamma\,(g_{m1}+g_{m2})\cdot(r_{o1}//r_{o2})^2}{\underbrace{A_v^2}_{=(-g_{m1}\cdot(r_{o1}//r_{o2}))^2}} = \frac{4kT\gamma\,(g_{m1}+g_{m2})}{g_{m1}^2}$$

$$= 4kT\gamma\left(\frac{1}{g_{m1}}+\frac{g_{m2}}{g_{m1}^2}\right)$$

If $g_{m1} = g_{m2} \Rightarrow$

$$= 4kT\gamma\cdot\frac{2}{g_m} = \frac{8kT\gamma}{g_m}$$

Problem 2. LNA

Tu 6 LNA

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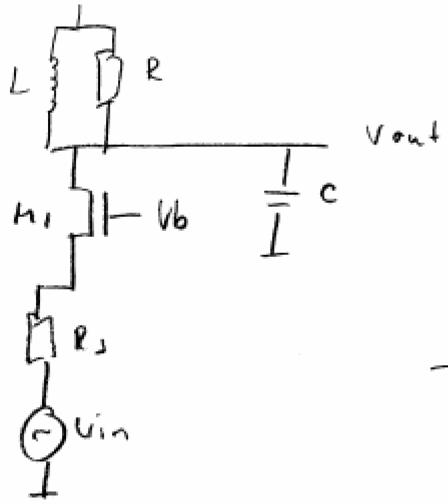
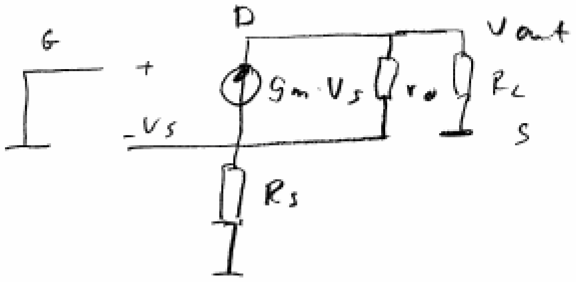


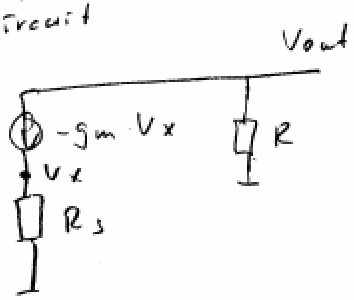
Common-gate LNA

Small-signal transist model:



⇓

Our circuit



1) Voltage gain A_v

$V_{out} = -g_m \cdot V_x \cdot R$

$(U = R \cdot I)$

$$A_x = \frac{V_{out}}{V_x} = g_m \cdot R \underset{\substack{\uparrow \\ \text{matching} \\ g_m = 1/R_s}}{=} R/R_s$$

$$V_x = V_{in} \cdot \frac{1/g_m}{R_s + 1/g_m} = V_{in} \cdot \frac{R_s}{2R_s} = \frac{V_{in}}{2}$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{2V_x} = \underline{\underline{\frac{1}{2}\frac{R}{R_s}}}$$

Noise LNA

1) R $\quad \overline{V^2_{n,out,R}} = 4kTR$

2) R_S $\quad \overline{V^2_{n,out,R_S}} = 4kTR_s \cdot A_v^2 = 4kTR_S\left(\frac{R}{2R_s}\right)^2 = \frac{kTR^2}{R_s}$

3) M_1 $\quad$ gain from V_g to $V_{out} = \frac{R_1}{R_S + \frac{1}{g_m}}$ $\quad$ * do yourself. (textbook)

noise modelled as $\overline{V^2_{n,M_1}} = \frac{4kT\gamma}{g_m}$ (voltage source)

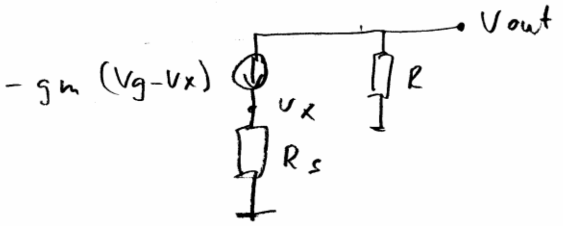
$$\Rightarrow \overline{V^2_{n,out\,M_1}} = \frac{4kT\gamma}{g_m}\left(\frac{R}{R_S + \frac{1}{g_m}}\right)^2 = \frac{4kT\gamma}{g_m}\left(\frac{g_m \cdot R}{g_m R_s + 1}\right)^2$$

$$= \left(R_s = \frac{1}{g_m}\right) \Rightarrow 4kT\gamma \cdot R_s \frac{R^2}{R_s^2 \cdot 4} = kT\gamma \cdot \frac{R^2}{R_s}$$

$$NF = \frac{4kTR + \frac{kTR^2}{R_s} + kT\gamma\frac{R^2}{R_S}}{\frac{kTR^2}{R_s}} =$$

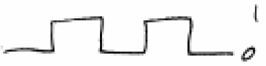
$$= \underline{1 + 4\frac{R_s}{R} + \gamma}$$

$* \; A_v = \frac{g_m \cdot R_0}{1+g_m \cdot R_s}$



Problem 3. Mixer

a)



LO

See T3; prob4

$$i_{LO}(t) = \frac{1}{2} + \frac{2}{\pi}\cos\omega_{LO}(t) - \frac{2}{3\pi}\cos 3\omega_{LO}(t) + \dots$$

$$i_{RF} = I_{bias} + I_{RF}\cdot\cos\omega_{RF}(t)$$

$$i_{IF} = i_{LO}\cdot i_{RF} = \left(\frac{1}{2} + \frac{2}{\pi}\cos\omega_{LO} - \dots\right)\left(I_{bias} + I_{RF}\cdot\cos\omega_{RF}\right)$$

= [For IF, we are only interest in $(\omega_{RF}-\omega_{LO})$ compoent] $\Rightarrow \frac{2 I_{RF}}{\pi}\cdot\cos\omega_{LO}\cdot\cos\omega_{RF}$

$$= \frac{2}{\pi} I_{RF}\left(\overset{\text{upmix}}{\frac{1}{2}\cos(\omega_{RF}+\omega_{LO})} + \overset{\text{downmix}}{\frac{1}{2}\cos(\omega_{RF}-\omega_{LO})}\right) \Rightarrow$$

$$i_{IF} = \frac{I_{RF}}{\pi}\cos(\omega_{RF}-\omega_{LO})$$

$$\text{conv. gain} = \left|\frac{v_{IF}(t)}{v_{RF}(t)}\right| = \left|\frac{R_D\cdot\frac{I_{RF}}{\pi}\cos(\omega_{RF}-\omega_{LO})}{\frac{1}{g_m} I_{RF}\cdot\cos\omega_{RF}}\right| =$$

$$= \boxed{\frac{1}{\pi} g_m\cdot R_D}$$

b) Noise source: R_D, R_s, M_2

M_2: $\overline{V^2_{n,out,M2}} = R_D^2 \cdot \overline{i^2_{n,out\,M2}} = R_D^2 \cdot 4kT\gamma g_m$

R_D: $\overline{V^2_{n,out,RD}} = 4kTR_D$

R_S: $\overline{V^2_{n,out,RS}} = 4kTR_S\,(g_m \cdot R_D)^2$

$$NF = \frac{SNR_{in}}{SNR_{out}} = \frac{\frac{S_{in}}{N_{in}}}{\frac{S_{out}}{N_{out}}} = \frac{S_{in}}{S_{out}} \cdot \frac{N_{out}}{N_{in}} = \underset{\uparrow}{\frac{1}{G_c}} \cdot \frac{N_{out}}{N_{in}}$$

gain for noise from input to output

$$G_c = \frac{1}{\pi} g_m \cdot R_D \quad (\text{from a}))$$

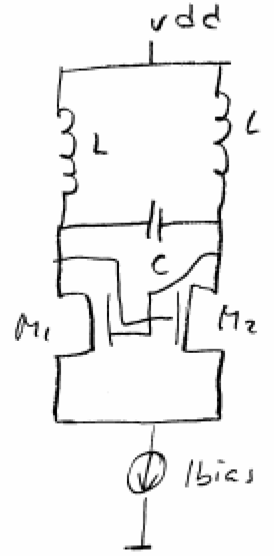
$$\Rightarrow NF = \frac{1}{\left(\frac{1}{\pi} g_m \cdot R_D\right)^2} \cdot \frac{4kT\gamma g_m R_D^2 + 4kTR_D + 4kTR_S (g_m \cdot R_D)^2}{4kTR_S}$$

$$= \pi^2 \frac{\gamma g_m \cdot R_D^2 + R_D + R_S\, g_m^2 \cdot R_D^2}{g_m^2 \cdot R_D^2 \cdot R_S} =$$

$$= \pi^2 \left(1 + \frac{1}{g_m^2 \cdot R_D \cdot R_S} + \frac{\gamma}{g_m R_S}\right)$$

Please note that in the noise calc, gain of 1 is used for the noise → output → input, while for the signal, the gain = G_c

Problem 4. Oscillator



$R_p = (Q^2+1)R_s$

$f = 2.4\ \text{GHz}$

$L = 5\ \text{nH}$

$Q = 10$

$V_{DD} = 2.5\ \text{V}$

$\mu_n C_{ox} = 110\ \mu A/V^2$

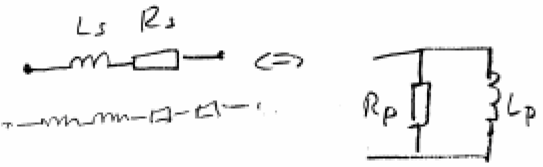
$I_{bias} = 2\ \text{mA}$

$$I_d = \frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{gs}-V_T)^2$$

$$\left(\Rightarrow g_m \approx \sqrt{2\mu_n \cdot C_{ox} \cdot I_D \cdot \frac{W}{L}}\right)$$

a) $\frac{W}{L}_{min}$ for M_1, M_2 to oscillate?

Given L is L_s (or $\frac{1}{2}\cdot L_s$)



$$\omega_p = \frac{1}{\sqrt{L_p \cdot C}}$$

$$Q = \frac{\omega L_s}{R_s} \Rightarrow R_s = \frac{\omega \cdot L_s}{Q} \rightarrow \frac{2\pi \cdot f \cdot L}{10} = \frac{2\pi \cdot 2{,}4 \cdot 10^9 \cdot 5 \cdot 10^{-9}}{10}$$

$$= 7{,}54\ \Omega$$

$$R_p = (Q^2+1)\cdot 2R_s = 1523\ \Omega$$

Oscillation : $R_{neg} = R_p = \frac{2}{g_m} \Leftrightarrow g_m = 1{,}31\ \frac{mA}{V}$

$I_d = \frac{I_{bias}}{2}$, $g_m^2 = 2\mu_n C_{ox} \cdot I_D \cdot \frac{W}{L}$

$$\frac{W}{L} = \frac{g_m^2}{\mu_n C_{ox} \cdot I_{bias}} = 7.84$$

b) For high Q $L_p \approx L_s$ (here $L_p \sim 2L_s$)

Parallel resonance $\omega = \frac{1}{\sqrt{L_p \cdot C}} \approx \frac{1}{\sqrt{2L_s \cdot C}}$

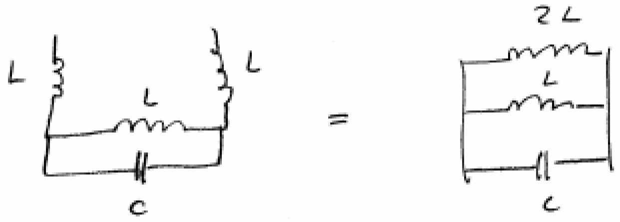
$C = \frac{1}{\omega^2 2L_s}$ $f = 2.4$ GHz ⇒ $C = 0{,}440$ pF

2,5 GHz $C = 0{,}405$ pF

$$\text{Tuning}^{*} = \frac{0{,}440 - 0.405}{\frac{0{,}440 + 0.405}{2}} = 8{,}3\ \%$$

* defined here in
the middl of the
range

c) ⇒



$$L_{eq} = \frac{2L \cdot L}{2L + L} = \frac{2}{3} L = 3.33\ \text{nH}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \left[\begin{array}{l} C = 0{,}440\ \text{pF} \\ L = 3{,}33\,\text{nH} \end{array}\right) \Rightarrow f = 4.16\ \text{GHz}$$

$$\text{Change} = \frac{4.16 - 2.4}{2.4} = 73\ \%$$