Tutorial 2: LNA Solutions

Problem 1

It is preferred in current RF designs that the input of LNA be matched to 50 Ω. The easiest way is to shunt the gate with a resistor of 50 Ω.

a) Calculate the gain A_0, input impedance and noise figure (NF) in absence of gate noise. Assume that $R_{sh} = R_s$ and the resistances R_L and R_{sh} noiseless for NF derivation.

b) What are the disadvantages of shunt resistor with reference to gain and NF?

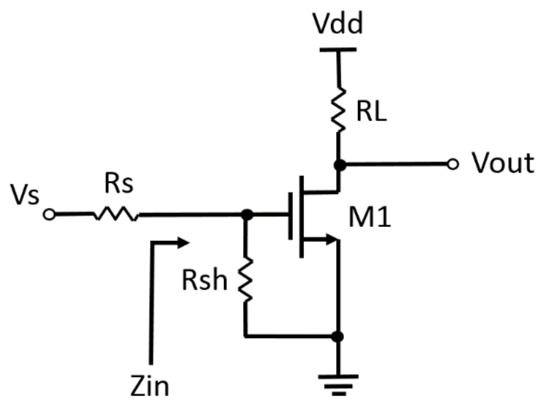

Fig. 1.1. Common-source amplifier with shunt input resistance

Solution:

a) For the gain calculation, we make use of the small-signal model shown in Fig. 1.2.

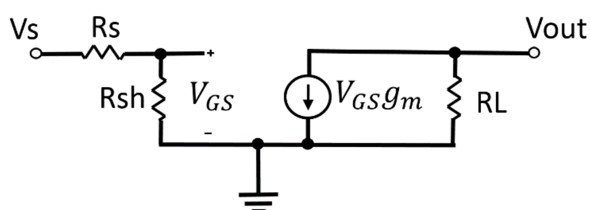

Fig. 1.2. CS small-signal model with shunt input resistance

From Fig. 1.2, we have

$$V_{GS} = V_s \frac{R_{sh}}{R_s + R_{sh}} \tag{1.1}$$

$$V_{out} = -V_{GS} g_m R_L \tag{1.2}$$

Substituting (2.1) into (2.2) and solving for ${V_{out}}/{V_s}$, then expression A_0 can be expressed as

$$\Rightarrow A_0 = \frac{V_{out}}{V_s} = -g_m R_L \left(\frac{R_{sh}}{R_s + R_{sh}}\right) \tag{1.3}$$

To find the input impedance, we make use of a test signal as shown in Fig. 1.3.

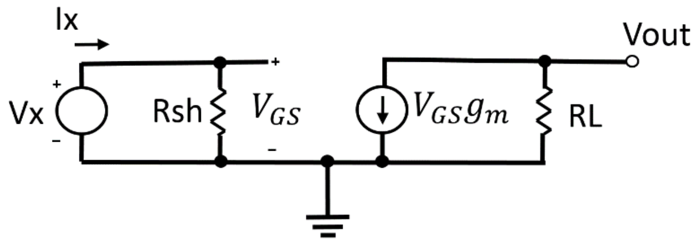

Fig. 1.3. CS small-signal model with test signal

From Fig. 1.2., the input impedance can be expressed as

$$Z_{in} = \frac{V_x}{I_X} = R_{sh} \tag{1.4}$$

Finally, the NF is equal to

$$NF = \frac{\overline{V_{n,out}^2}}{A_0^2} \cdot \frac{1}{4KTR_s} \tag{1.5}$$

The gain A_0 was already calculated, remaining only the total output noise $\overline{V_{n,out}^2}$. For this purpose, we make use of the noise circuit representation shown in Fig. 1.4.

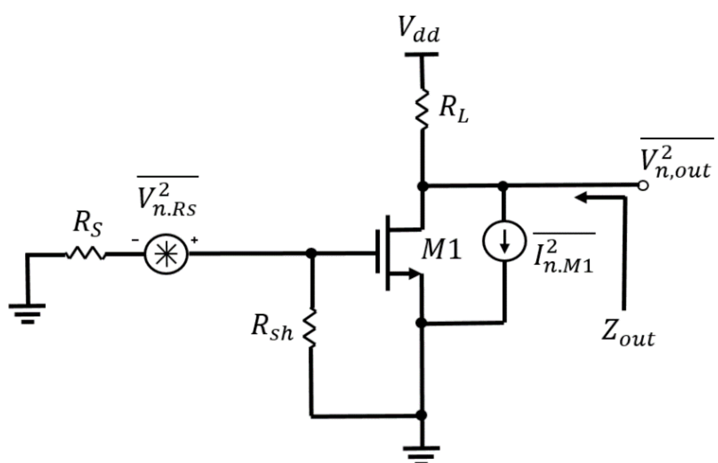

Fig. 1.4. CS noise circuit representation

Notice that R_{sh} and R_L are noiseless. The total output noise can be expressed as

$$\overline{V_{n,out}^2} = \overline{V_{n,Rs}^2} A_0^2 + \overline{I_{n,M1}^2} \cdot |Z_{out}|^2$$

$$\Rightarrow \overline{V_{n,out}^2} = 4KTR_s A_0^2 + 4kT\gamma g_m R_L^2 \tag{1.6}$$

where Z_{out} is equal to R_L. Substituting (1.6) into (1.5), we obtain for $R_{sh} = R_s$.

$$NF = \frac{4KTR_s A_0^2 + 4kT\gamma g_m R_L^2}{A_0^2 \cdot 4KTR_s}$$

$$\Rightarrow NF = 1 + \frac{\gamma (R_s + R_{sh})^2}{g_m R_s R_{sh}^2} \tag{1.7}$$

Since $R_{sh} = R_s$, NF in (1.7) can be simplified to

$$\Rightarrow NF = 2 + \frac{4\gamma}{g_m R_s} \blacksquare \tag{1.8}$$

b) The utilization of the shunt resistor reduces the voltage gain by a factor $R_{sh}/(R_s + R_{sh})$ for this LNA. Considering input impedance matching, the gain would be reduce by a factor of 2!

From (1.8), notice that there is factor of 2, showing that the NF is higher than 3 dB with the presence of the shunt resistance R_{sh} at the input.

Problem 2

The inductor source degenerate amplifier shown below presents a noiseless resistance of 50 Ω for input power match.

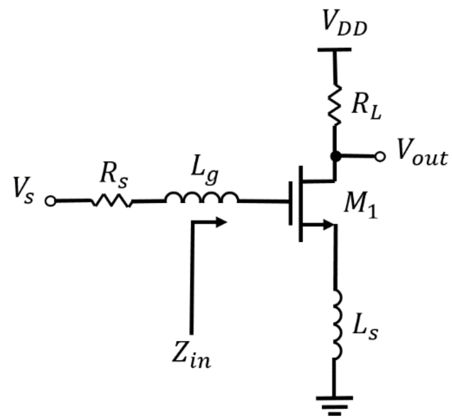

Fig. 2.1. Inductor source degenerated amplifier

a) Calculate the input impedance. How we can cancel the imaginary part of the complex input impedance so that the LNA presents 50 Ω real input resistance at input port. Neglect gate drain, gate-bulk capacitance.

b) Calculate the NF. Neglect gate-drain, gate-bulk and gate-source capacitance.

c) C_{gd} bridges the input and the output ports. The reverse isolation of this LNA is very poor. Why is reverse isolation important? Suggest a modification to improve the reverse isolation.

Solution:

a) The input impedance Z_{in} can be expressed as

$$Z_{in} = \frac{V_x}{I_x} \tag{2.1}$$

where Z_{in} can be found through the small-signal model shown in Fig. 2.2.

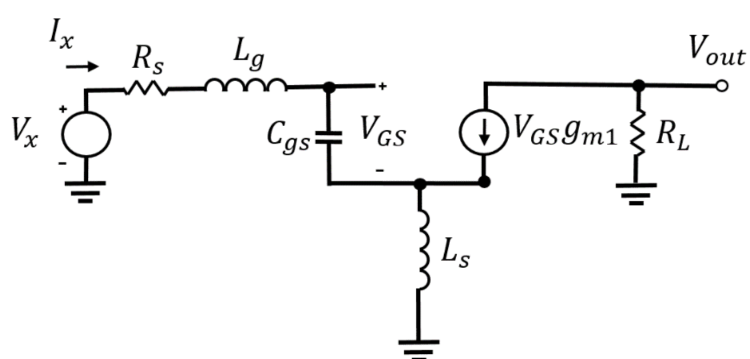

Fig. 2.2 Small-signal model with test signal at the input

From Fig. 2.2, we have,

$$V_x = I_x \left(Z_{L_g} + Z_{C_{gs}} + Z_{L_s} + Z_{C_{gs}} Z_{L_s} g_{m1} \right) \quad (2.2)$$

$$\Rightarrow Z_{in} = \frac{V_x}{I_x} = j\omega\left(L_g + L_s\right) + \frac{1}{j\omega C_{gs}} + \frac{L_s g_{m1}}{C_{gs}}$$

$$\Rightarrow Z_{in} = \frac{L_s g_{m1}}{C_{gs}} + j\frac{\omega^2 C_{gs}\left(L_g + L_s\right) - 1}{C_{gs}} \quad (2.3)$$

From (2.3), notice that the factor $\omega^2 C_{gs}\left(L_g + L_s\right)$ has to equal 1 to cancel the imaginary part of Z_{in}. The frequency that satisfies this condition can be expressed as

$$f = \frac{1}{2\pi\sqrt{C_{gs}\left(L_g + L_s\right)}} \quad (2.4)$$

On the other hand, since we want to have a 50-Ω input impedance, then

$$\Rightarrow Z_{in} = \frac{L_s g_{m1}}{C_{gs}} = 50\ \Omega$$

b) The NF is expressed as

$$NF = \frac{\overline{V_{n,out}^2}}{|A_0|^2} \cdot \frac{1}{4KTR_s} \quad (2.5)$$

The circuit with the noise sources is shown in Fig. 2.3.

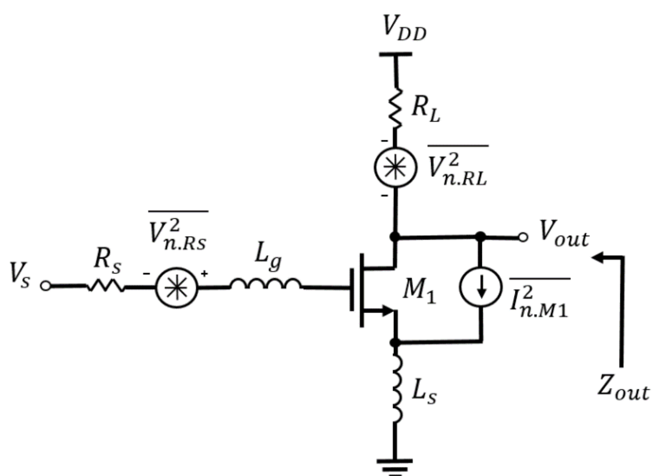

Fig. 2.3. Inductor source degenerated amplifier with noise sources

The total contribution at the output can be expressed as

$$\overline{V_{n,out}^2} = \overline{V_{n,R_s}^2} A_0^2 + \overline{V_{n,R_L}^2} + \overline{I_{n,M1}^2} |Z_{out}|^2 \quad (2.6)$$

The output impedance $Z_{out} = {}^{V_x}/_{I_x}$ can be found through the small-signal model show in Fig 2.4.

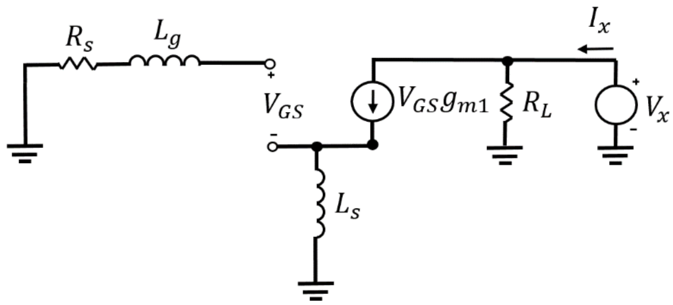

Fig. 2.4. Small-signal model with test signal at the output

From Fig. 2.4, Z_{out} is equal to

$$\Rightarrow Z_{out} = \frac{V_x}{I_X} = R_L \qquad (2.7)$$

On the other hand, finding the expression for the gain A_0. Using the small-signal model from Fig. 2.5. Notice that the parasitic capacitances are neglected.

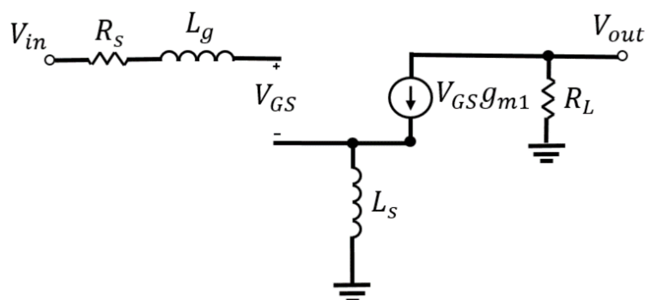

Fig. 2.5. Small-signal model

From Fig. 2.5 we have

$$V_{in} = V_{GS}(1 + Z_{L_s}g_{m1})$$

$$\Rightarrow V_{GS} = \frac{V_{in}}{(1 + Z_{L_s}g_{m1})} \qquad (2.8)$$

On the other hand, V_{GS} is equal to

$$V_{GS} = \frac{-V_{out}}{g_{m1}R_L} \qquad (2.9)$$

Substituting (2.9) into (2.8) and solving for A_0

$$A_0 = \frac{V_{out}}{V_{in}} = \frac{-g_{m1}R_L}{1 + Z_{L_s}g_{m1}} \qquad (2.10)$$

Finally, substituting (2.6) and (2.10) into (2.5) with Z_{out} equal to R_L, the NF is expressed as

$$NF = \frac{\overline{V_{n,R_S}^2}\left|\frac{g_{m1}R_L}{1+Z_{L_s}g_{m1}}\right|^2 + \overline{V_{n,R_L}^2} + \overline{I_{n,M1}^2}|Z_{out}|^2}{4KTR_S\left|\frac{g_{m1}R_L}{1+Z_{L_s}g_{m1}}\right|^2}$$

$$\Rightarrow NF = \frac{4KTR_S\left|\frac{g_{m1}R_L}{1+Z_{L_s}g_{m1}}\right|^2 + 4KTR_L + 4KT\gamma g_{m1}{R_L}^2}{4KTR_S\left|\frac{g_{m1}R_L}{1+Z_{L_s}g_{m1}}\right|^2}$$

$$\Rightarrow NF = 1 + \frac{R_L}{R_S\left|\frac{g_{m1}R_L}{1+Z_{L_s}g_{m1}}\right|^2} + \frac{\gamma g_{m1}{R_L}^2}{R_S\left|\frac{g_{m1}R_L}{1+Z_{L_s}g_{m1}}\right|^2} \quad (2.11)$$

c) The reverse isolation is important to avoid leakage from the output to the input port that can lead to instability issues in the circuit as well as out-of-band power emission, causing interference in other frequency bands. A modification in the circuit that improves the reverse isolation is shown in Fig. 2.6.

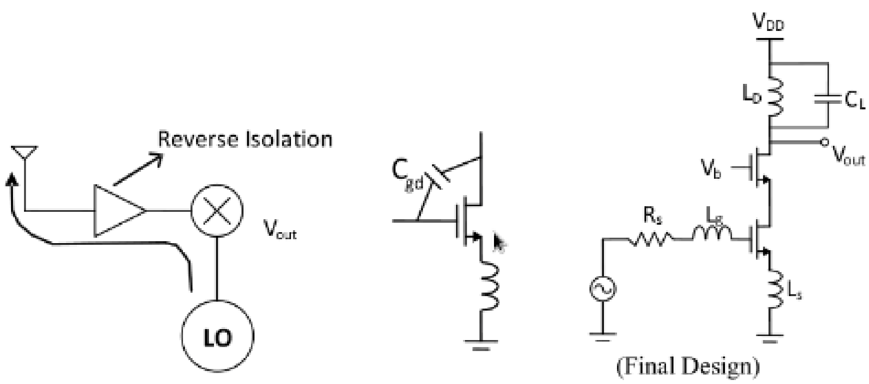

Fig. 2.6. Circuit modification to improve reverse isolation

Problem 3

A common-source low noise amplifier (LNA) with feedback is shown in the figure below. R_s is the input source resistance. Assume that the transistors are long-channel devices and $\lambda = 0$.

a) Determine the input impedance R_{in} of the LNA.
b) Calculate the voltage gain, $A_0 = {V_{out}}/{V_{in}}$ of the LNA after matching if $R_F = 10R_s$.
c) Derive an expression for the output noise of the LNA contributed by R_s after matching. Assume $R_F \gg R_s$.

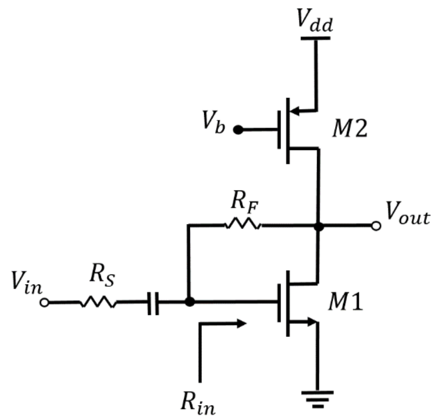

Fig. 3.1 CS stage with resistive feedback

Solution:

a) The input resistance can be found from the small-signal circuit shown in Fig 3.2.

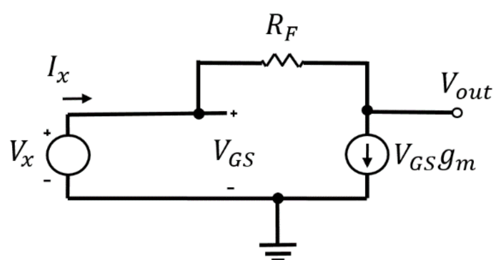

Fig. 3.2. Small-signal model test signal at the input

From Fig 3.2., we have

$$I_x = V_{GS}g_{m1} \tag{3.1}$$

$$V_x = V_{GS} \tag{3.2}$$

Substituting (3.1) into (3.2) and solving for ${V_x}/{I_x}$, we have

$$R_{in} = \frac{V_x}{I_x} = \frac{1}{g_{m1}} \tag{3.3}$$

b) To find the voltage gain $A_0 = {}^{V_{out}}/_{V_{in}}$, we use the small-signal model shown in Fig 3.3.

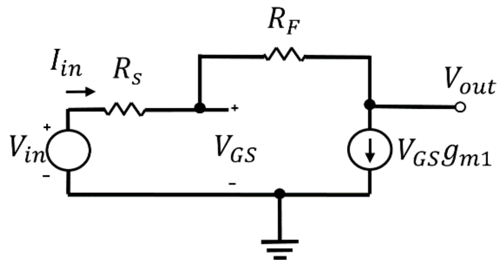

Fig. 3.3. Small-signal model

From Fig 3.3., we have

$$V_{in} - I_{in}(R_s + R_F) = V_{out} \tag{3.4}$$

$$I_{in} = g_{m1}V_{GS} \tag{3.5}$$

$$V_{GS} = V_{in} - I_{in}R_s \tag{3.6}$$

Substituting (3.6) into (3.5) and solving for I_{in}, we have

$$I_{in} = g_{m1}(V_{in} - R_sI_{in})$$

$$\Rightarrow I_{in} = \frac{g_{m1}V_{in}}{(1 + g_{m1}R_s)} \tag{3.7}$$

Now, substituting (3.7) into (3.4) and solving for ${}^{V_{out}}/_{V_{in}}$, we have

$$V_{in} - \frac{V_{in}g_{m1}(R_s + R_F)}{(1 + g_{m1}R_s)} = V_{out}$$

$$\Rightarrow V_{in}\left[1 - \frac{V_{in}g_{m1}(R_s + R_F)}{(1 + g_{m1}R_s)}\right] = V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{(1 + g_{m1}R_s) - g_{m1}(R_s + R_F)}{(1 + g_{m1}R_s)} \tag{3.8}$$

Due to matching, we have that $R_s = \frac{1}{g_{m1}}$. Besides, $R_F = 10R_s$. So, substituting for R_s and R_F in (3.8) and simplifying, the gain becomes

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{(1 + 1) - g_{m1}(11R_s)}{(1 + 1)} = \frac{2 - 11}{2} = -\frac{9}{2}$$

c) The output noise contribution from the resistance R_s corresponds to the product between the noise generated by the resistance and A_0^2. This can be expressed as follows

$$\overline{V_{n,out,Rs}^2} = \overline{V_{n,R_s}^2} A_0^2 \tag{3.9}$$

From (3.8), after some simplification we have for

$$\Rightarrow A_0 = \frac{1}{2}\left[1 - \frac{R_F}{R_s}\right] \tag{3.10}$$

Then, $\overline{V_{n,out,Rs}^2}$ can be expressed as

$$\overline{V_{n,out,Rs}^2} = KTR_s\left[1 - \frac{R_F}{R_s}\right]^2 \approx KTR_s\left[-\frac{R_F}{R_s}\right]^2 = \frac{KTR_F{}^2}{R_s} \ \blacksquare \tag{3.11}$$

Problem 4

In the common-source stage shown below, determine

a) Input impedance, R_{in}.
b) Closed-loop gain.
c) Noise Figure

Assume that the channel-length modulation is NOT neglected and matching at the input.

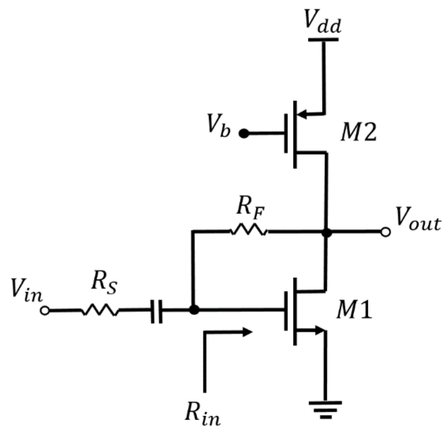

Fig. 4.1 CS stage with resistive feedback

Solution:

a) Using the small-signal model with test signal connected at the input as shown in Fig. 4.2 to find R_{in}, we have

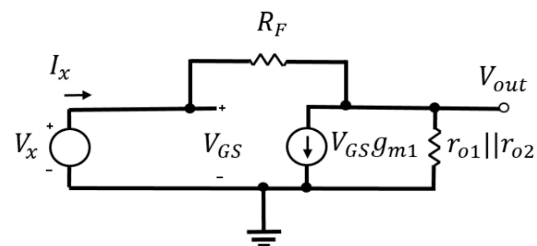

Fig. 4.2. CS small-signal model with resistive feedback and test signal at the input

The input impedance is expressed as follows

$$R_{in} = \frac{V_x}{I_X} \tag{4.1}$$

From Fig. 4.2, we can derive the following equations

$$V_x - V_{out} = I_x R_F \tag{4.2}$$

$$\frac{V_{out}}{r_{o1}||r_{o2}} + V_x g_m = I_x \tag{4.3}$$

Solving for V_{out} in (4.2) and substituting in (4.3), we have

$$V_x - (I_x - V_x g_m) r_{o1}||r_{o2} = I_x R_F$$

$$\Rightarrow V_x(1 + g_m r_{o1}||r_{o2}) = I_x(R_F + r_{o1}||r_{o2})$$

$$\Rightarrow R_{in} = \frac{V_x}{I_x} = \frac{(R_F + r_{o1}||r_{o2})}{(1 + g_m r_{o1}||r_{o2})} \blacksquare \qquad (4.4)$$

b) The closed-loop gain can be found through the small-signal model shown in Fig. 4.3.

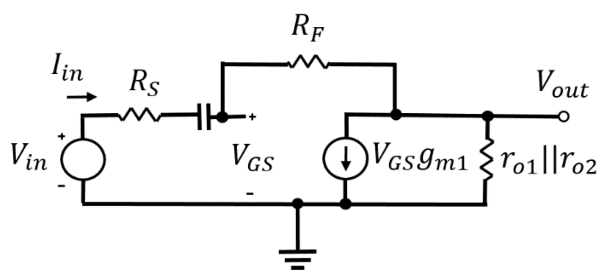

Fig. 4.3. Small-signal model

From Fig. 4.3, we have

$$I_{in} - g_{m1}V_{GS} - \frac{V_{out}}{r_{o1}||r_{o2}} = 0 \qquad (4.5)$$

$$V_{GS} = V_{in} - I_{in}R_s \qquad (4.6)$$

$$I_{in} = \frac{V_{in} - V_{out}}{R_s + R_F} \qquad (4.7)$$

Substituting (4.7) into (4.6), we have

$$V_{GS} = V_{in} - R_s\frac{(V_{in} - V_{out})}{R_s + R_F} \qquad (4.8)$$

Now, substituting (4.7) and (4.8) into (4.5) and solving for V_{out}/V_{in},

$$\frac{V_{in} - V_{out}}{R_s + R_F} - g_{m1}\left[V_{in} - R_s\frac{(V_{in} - V_{out})}{R_s + R_F}\right] - \frac{V_{out}}{r_{o1}||r_{o2}} = 0$$

$$\Rightarrow \frac{V_{in}}{(R_s + R_F)} - g_{m1}V_{in} + \frac{g_{m1}R_sV_{in}}{(R_s + R_F)} = \frac{V_{out}}{(R_s + R_F)} + \frac{g_{m1}R_sV_{out}}{(R_s + R_F)} + \frac{V_{out}}{r_{o1}||r_{o2}}$$

$$\Rightarrow V_{in}[1 + g_{m1}R_s - g_{m1}(R_s + R_F)] = V_{out}\left[\frac{(r_{o1}||r_{o2}) + g_{m1}R_s(r_{o1}||r_{o2}) + (R_s + R_F)}{(r_{o1}||r_{o2})}\right]$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{[1 + g_{m1}R_s - g_{m1}(R_s + R_F)](r_{o1}||r_{o2})}{(r_{o1}||r_{o2}) + g_{m1}R_s(r_{o1}||r_{o2}) + (R_s + R_F)} \qquad (4.9)$$

Considering matching impedance at the input, then (4.9) can be simplified to

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{\left[1 + {R_F}/{R_S}\right](r_{o1}||r_{o2})}{[2(r_{o1}||r_{o2}) + (R_s + R_F)]} \tag{4.10}$$

c) The NF is expressed as

$$NF = \frac{\overline{V_{n,out}^2}}{A_0^2} \cdot \frac{1}{4KTR_s} \tag{4.11}$$

The circuit with the noise sources is shown in Fig. 4.4.

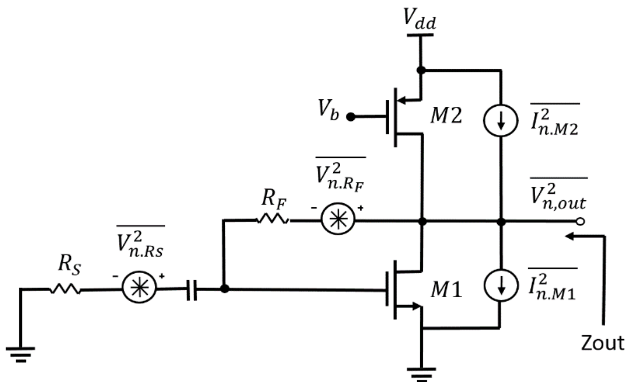

Fig. 4.4. CS circuit with noise sources

The total contribution at the output can be expressed as

$$\overline{V_{n,out}^2} = \overline{V_{n,R_s}^2}A_0^2 + \overline{V_{n,R_F}^2} + \left(\overline{I_{n,M1}^2} + \overline{I_{n,M2}^2}\right)|Z_{out}|^2 \tag{4.12}$$

The output impedance, Z_{out}, has to be found. For this purpose, using the small-signal model with the test signal connected at the output as shown in Fig. 4.5.

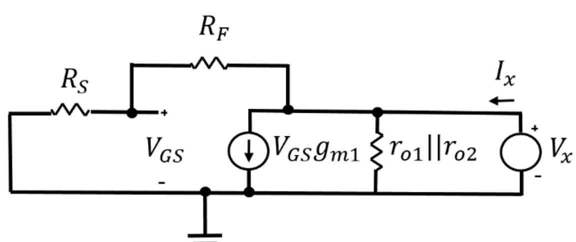

Fig. 4.5. Small-signal model with output test signal

From Fig. 4.5, we have

$$I_x = \frac{V_x}{r_{o1}||r_{o2}} + V_{GS}g_{m1} + \frac{V_x}{(R_s + R_F)} \tag{4.13}$$

$$V_{GS} = V_x \frac{R_s}{(R_s + R_F)} \tag{4.14}$$

Substituting (4.14) into (4.13) and solving for ${}^{V_x}/_{I_x}$,

$$Z_{out} = \frac{V_x}{I_x} = \left[\frac{1}{(r_{o1}||r_{o2})} + \frac{(g_{m1}R_s + 1)}{(R_s + R_F)}\right]^{-1}$$

Simplifying through the input matching

$$\Rightarrow Z_{out} = \left[\frac{(r_{o1}||r_{o2})(R_s + R_F)}{(R_s + R_F) + 2(r_{o1}||r_{o2})}\right] \tag{4.15}$$

Elaborating more for (4.12), we have

$$\overline{V_{n,out}^2} = 4KTR_S A_0^2 + 4KTR_F + 4kT\gamma(g_{m1} + g_{m2})\left[\frac{(r_{o1}||r_{o2})(R_s + R_F)}{(R_s + R_F) + (r_{o1}||r_{o2})(g_{m1}R_s + 1)}\right]^2 \tag{4.16}$$

Substituting (4.10) and (4.16) into (4.11) and after some simplification, we have

$$NF = 1 + \frac{R_F}{\left[\frac{\left(1 + {}^{R_F}/_{R_s}\right)(r_{o1}||r_{o2})}{[2(r_{o1}||r_{o2}) + (R_s + R_F)]}\right]^2 R_s} + \frac{\gamma(g_{m1} + g_{m2})}{R_s}\left[\frac{(R_s + R_F)}{\left(1 + {}^{R_F}/_{R_s}\right)}\right]^2$$

$$\Rightarrow NF = 1 + \frac{R_F}{\left(\frac{R_F}{R_s}\right)^2\left[\frac{\left({}^{R_s}/_{R_F} + 1\right)(r_{o1}||r_{o2})}{[2(r_{o1}||r_{o2}) + (R_s + R_F)]}\right]^2 R_s} + \frac{\gamma(g_{m1} + g_{m2})}{R_s}{R_s}^2$$

$$\Rightarrow NF = 1 + \frac{{}^{R_s}/_{R_F}}{\left[\frac{\left({}^{R_s}/_{R_F} + 1\right)(r_{o1}||r_{o2})}{[2(r_{o1}||r_{o2}) + (R_s + R_F)]}\right]^2} + \gamma(g_{m1} + g_{m2})R_s \;\blacksquare \tag{4.17}$$