TSEI03 Formulas 2019

Equations for the MOS transistor

NMOS PMOS

Definition of source (*S*) and drain (*D*)

NMOS: $V_S \le V_D$ PMOS: $V_S \ge V_D$

Voltage notations

$$V_{GS} = V_G - V_S,\ V_{DS} = V_D - V_S,\ V_{SB} = V_S - V_B,\ V_{GT} = V_{GS} - V_T$$

Threshold voltage

$$V_T = V_{T0} + \gamma\left(\sqrt{|V_{SB} - 2\Phi_F|} - \sqrt{2|\Phi_F|}\right)$$

Unified model

NMOS: $V_{GT} \le 0$ (PMOS: $V_{GT} \ge 0$) $\Rightarrow$ Subthreshold region ($I_D \approx 0$)

NMOS: $V_{GT} \ge 0$ (PMOS: $V_{GT} \le 0$) $\Rightarrow I_D = k' \frac{W}{L} V_{min}\left(|V_{GT}| - \frac{V_{min}}{2}\right)(1 + \lambda V_{DS})$

$V_{min} = min(|V_{GT}|, |V_{DS}|, |V_{DSAT}|)$

$V_{min} = |V_{GT}|$ $\Rightarrow$ saturation region

$V_{min} = |V_{DS}|$ $\Rightarrow$ resistive (linear, triode) region ($\lambda = 0$)

$V_{min} = |V_{DSAT}|$ $\Rightarrow$ velocity saturation region

V_{DSAT} dependency on channel length

$$V_{DSAT} = L\xi_c$$

Subthreshold region

NMOS: $V_{GT} \le 0 \Rightarrow I_{Dn} = I_{0n} \frac{W}{L} e^{\frac{q(V_{GSn} - V_{Tn})}{nkT}} \left(1 - e^{-\frac{qV_{DSn}}{kT}}\right)(1 + \lambda_n V_{DSn})$

PMOS: $V_{GT} \ge 0 \Rightarrow I_{Dp} = I_{0p} \frac{W}{L} e^{\frac{q(V_{SGp} - |V_{Tp}|)}{nkT}} \left(1 - e^{-\frac{qV_{SDp}}{kT}}\right)(1 + \lambda_p V_{DSp})$

Model parameters for 0.25 μm CMOS devices

Parameters for drain current calculations

	V_{T0} [V]	γ [$\sqrt{\text{V}}$]	V_{DSAT} [V]	k' [μA/V^2]	λ	Φ_F [V]
NMOS	0.43	0.40	0.63	115	0.06	–0.30
PMOS	–0.40	–0.40	–1.00	–30	–0.10	0.30

Parameters for capacitance calculations

	C_{ox} [fF/μm^2]	C_O [fF/μm]	C_j [fF/μm^2]	m_j	ϕ_b [V]	C_{jsw} [fF/μm]	m_{jsw}	ϕ_{bsw} [V]
NMOS	6	0.31	2.0	0.50	0.9	0.28	0.44	0.9
PMOS	6	0.27	1.9	0.48	0.9	0.22	0.32	0.9

Gate capacitance

Overlap capacitance

$$C_{GSO} = C_{GDO} = C_{ox}x_dW = C_OW$$

Channel capacitance

$$C_{GC} = C_{GCB} + C_{GCS} + C_{GCD}$$

Condition	C_{GCB}	C_{GCS}	C_{GCD}
$V_{GTn} \le 0$, $V_{GTp} \ge 0$	$C_{ox}WL$	0	0
$V_{GTn} > 0$, $V_{GTp} < 0$, $\lvert V_{DS}\rvert \le \lvert V_{GT}\rvert$	0	$C_{ox}WL/2$	$C_{ox}WL/2$
$V_{GTn} > 0$, $V_{GTp} < 0$, $\lvert V_{GT}\rvert \le \lvert V_{DS}\rvert$	0	$2C_{ox}WL/3$	0

Junction capacitance

Junction capacitance as function of voltage

$$C_j(V) = \frac{C_{j0}}{(1 - V/\phi_0)^m}$$

Average capacitance during transition from V_1 to V_2

$$C_{eq} = K_{eq} C_{j0} = \frac{-\phi_0^m}{(V_2 - V_1)(1-m)} [(\phi_0 - V_2)^{(1-m)} - (\phi_0 - V_1)^{(1-m)}] C_{j0}$$

Dynamic power consumption

$$P = afC_{tot}V_{dd}^2$$

Switch functions

$$S_N = \overline{F(A, B, \ldots)},\ S_P = F(\bar{A}, \bar{B}, \ldots)$$

Boolean algebra

De Morgans' theorem

$$\overline{X + Y + Z + \ldots} = \bar{X}\bar{Y}\bar{Z}\ldots, \qquad \overline{XYZ\ldots} = \bar{X} + \bar{Y} + \bar{Z} + \ldots$$

Expansion in sum

$$f(X, Y, Z, \ldots) = Xf(1, Y, Z, \ldots) + \bar{X}f(0, Y, Z, \ldots)$$

Expansion in product

$$f(X, Y, Z, \ldots) = [X + f(0, Y, Z, \ldots)][\bar{X} + f(1, Y, Z, \ldots)]$$

Transmission line

Characteristic impedance

$$Z_0 = \sqrt{l/c}$$

Velocity of wave

$$\upsilon = 1/\sqrt{lc}$$

Reflection coefficient for a transmission line (Z_0) terminated by a load (Z_L)

$$\Gamma = (Z_L - Z_0)/(Z_L + Z_0)$$

Elmore delay

P_i = "the path between node 0 and i".

$P_{ij} = P_i \cap P_j$ = "the common part of the paths P_i and P_j".

R_{ij} = "the sum of all resistances in P_{ij}".

Time constant from node 0 to i: $\tau_{di} = \sum_{j=0}^{n} R_{ij} C_j$. Propagation delay: $t_{pi} \approx 0.69\tau_{di}$.

Sizing of cascaded inverters

For minimal propagation delay find the best solution to $1 = e^{(1+\gamma/k)}/k$, where
k ="tapering factor", N = "number of inverters", $F = C_L/C_{g1} = k^N$ and $\gamma = C_{int1}/C_{g1}$.