TSEI03 Homework 1: Solution

For this circuit, we have $V_{GS} = V_{in}$ and $V_{DS} = V_{out} = V_{DD} - RI_D$. Further, $V_{SB} = 0 \Rightarrow V_T = V_{T0}$, and $\lambda = 0 \Rightarrow$ current can be written

$$V_{in} \le V_{T0} \Rightarrow I_D = 0 \;\rightarrow\; \langle \text{cutoff} \rangle$$

$$V_{in} > V_{T0} \Rightarrow I_D = k'\frac{W}{L} V_{min}\left(V_{in} - V_{T0} - \frac{V_{min}}{2}\right),$$

where $V_{min} = \min\left(V_{in} - V_{T0},\ V_{DD} - RI_D,\ V_{DSAT}\right) \;\rightarrow\; \langle \text{saturated, resistive, velocity sat.} \rangle$

1. $V_{in} = 0$ V

$V_{in} \le V_{T0} \Rightarrow I_D = 0 \Rightarrow V_{out} = V_{DD}$

MOSFET is cutoff, $I_D = 0$, and $V_{out} = 2.5$ V

2. $V_{in} = 0.5$ V

$V_{in} > V_{T0}$, guess MOSFET is saturated $\Rightarrow V_{min} = V_{in} - V_{T0} = 0.07$ V

$$I_D = k'\frac{W}{L}\frac{\left(V_{in} - V_{T0}\right)^2}{2} = 1.7\ \mu\text{A}$$

$$V_{out} = V_{DD} - RI_D = 2.49\ \text{V}$$

Check mode: $V_{min} = \min\left(0.07,\ 2.49,\ 0.63\right)$ V $\rightarrow$ saturated is correct

MOSFET is saturated, $I_D = 1.7$ µA, and $V_{out} = 2.49$ V

3. $V_{in} = 1.0$ V

$V_{in} > V_{T0}$, guess MOSFET is saturated $\Rightarrow V_{min} = V_{in} - V_{T0} = 0.57$ V

$$I_D = k'\frac{W}{L}\frac{\left(V_{in} - V_{T0}\right)^2}{2} = 112\ \mu\text{A}$$

$$V_{out} = V_{DD} - RI_D = 1.93\ \text{V}$$

Check mode: $V_{min} = \min\left(0.57,\ 1.93,\ 0.63\right)$ V $\rightarrow$ saturated is correct

MOSFET is saturated, $I_D = 112$ µA, and $V_{out} = 1.93$ V

4. $V_{in} = 1.5$ V

$V_{in} > V_{T0}$, guess MOSFET is saturated $\Rightarrow V_{min} = V_{in} - V_{T0} = 1.07$ V

$$I_D = k'\frac{W}{L}\frac{\left(V_{in} - V_{T0}\right)^2}{2} = 395\ \mu\text{A}$$

$$V_{out} = V_{DD} - RI_D = 0.52\ \text{V}$$

Check mode: $V_{min} = \min\left(1.07,\ 0.52,\ 0.63\right)$ V $\rightarrow$ saturated is wrong

Reguess MOSFET is velocity saturated $\Rightarrow V_{min} = V_{DSAT} = 0.63$ V

$$I_D = k'\frac{W}{L} V_{DSAT}\left(V_{in} - V_{T0} - \frac{V_{DSAT}}{2}\right) = 328\ \mu\text{A}$$

$$V_{out} = V_{DD} - RI_D = 0.86\ \text{V}$$

Check mode: $V_{\text{min}} = \min\left(1.07,\ 0.86,\ 0.63\right)$ V $\rightarrow$ velocity saturated is correct

MOSFET is velocity saturated, I_{D} = 328 µA, and V_{out} = 0.86 V

5. V_{in} = 2.0 V

$V_{\text{in}} > V_{\text{T0}}$, guess MOSFET is velocity saturated $\Rightarrow V_{\text{min}} = V_{\text{DSAT}} = 0.63 \text{ V} \Rightarrow$

$$I_{\text{D}} = k'\frac{W}{L}V_{\text{DSAT}}\left(V_{\text{in}} - V_{\text{T0}} - \frac{V_{\text{DSAT}}}{2}\right) = 546 \text{ µA}$$

$V_{\text{out}} = V_{\text{DD}} - RI_{\text{D}} = -0.23$ V $\rightarrow$ velocity saturated is incorrect

Reguess MOSFET is resistive $\Rightarrow V_{\text{min}} = V_{\text{out}} \Rightarrow$

$$Rk'\frac{W}{L}\frac{1}{2}V_{\text{out}}^2 - \left[1 + Rk'\frac{W}{L}\left(V_{\text{in}} - V_{\text{T0}}\right)\right]V_{\text{out}} + V_{\text{DD}} = 0 \Rightarrow$$

$$V_{\text{out}} = \begin{cases} 0.44 \text{ V} \\ \left(3.28 \text{ V} > V_{DD}\right) \end{cases} \Rightarrow I_{\text{D}} = 412 \text{ µA}$$

Check mode: $V_{\text{min}} = \min\left(1.57,\ 0.44,\ 0.63\right)$ V $\rightarrow$ resistive is correct

MOSFET is resistive, I_{D} = 412 µA, and V_{out} = 0.44 V

6. V_{in} = 2.5 V

$V_{\text{in}} > V_{\text{T0}}$, guess MOSFET is resistive $\Rightarrow V_{\text{min}} = V_{\text{out}} \Rightarrow$

$$Rk'\frac{W}{L}\frac{1}{2}V_{\text{out}}^2 - \left[1 + Rk'\frac{W}{L}\left(V_{\text{in}} - V_{\text{T0}}\right)\right]V_{\text{out}} + V_{\text{DD}} = 0 \Rightarrow$$

$$V_{\text{out}} = \begin{cases} 0.33 \text{ V} \\ \left(4.39 \text{ V} > V_{DD}\right) \end{cases} \Rightarrow I_{\text{D}} = 434 \text{ µA}$$

Check mode: $V_{\text{min}} = \min\left(2.07,\ 0.33,\ 0.63\right)$ V $\rightarrow$ resistive is correct

MOSFET is resistive, I_{D} = 434 µA, and V_{out} = 0.33 V

Plot $V_{\text{out}}(V_{\text{in}})$