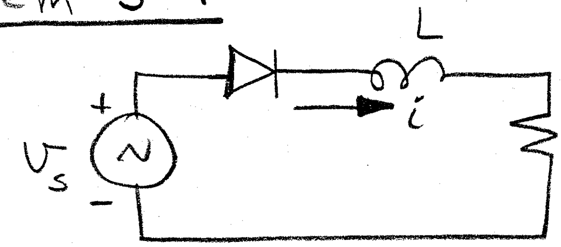
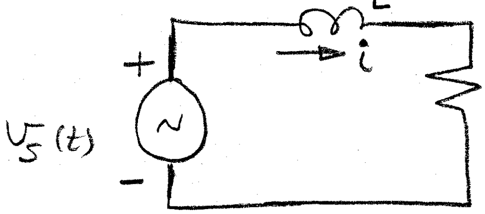
Problem 5-1



$V_s = 120V$ at 60Hz
$L = 10mH$, $R = 5\Omega$
$\omega = 2\pi f = 377$ rad/s

To obtain $i(t)$, we can consider the following equivalent circuit, which applies only when the current is flowing



$$v_s(t) = \left[\hat{V}_s \sin \omega t\right] u(t)$$

In the above circuit, for $t > 0$ while the current flows

$$Ri + L\frac{di}{dt} = \hat{V}_s \sin \omega t$$

or $$\frac{di}{dt} + \frac{R}{L} i = \frac{\hat{V}_s}{L} \sin \omega t$$

Adding the forced response and the natural response,

$$i = Ae^{-\frac{R}{L}t} + \frac{\hat{V}_s}{Z} \sin(\omega t - \phi)$$

where $Z = \sqrt{R^2 + (\omega L)^2}$, $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ and A is the coefficient to be determined.

Initially at $t=0$, $i(0^-) = i(0^+) = 0$. We will use this initial condition to calculate A in the previous equation:

$$0 = A + \frac{\hat{V}_s}{Z} \sin(-\phi)$$

$$\therefore \quad A = \frac{\hat{V}_s}{Z} \sin\phi = \hat{V}_s \left(\frac{\omega L}{Z^2} \right)$$

$$\therefore \quad i(t) = \frac{\hat{V}_s \, \omega L}{Z^2} e^{-\frac{R}{L}t} + \frac{\hat{V}_s}{Z} \sin(\omega t - \phi)$$