Exercise 1-5 RMS current calculation

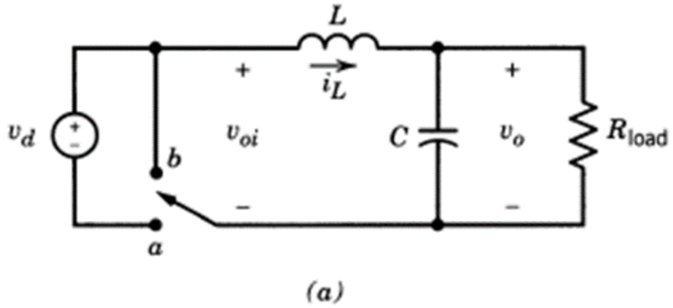

Figure 1

Vd=20V
Duty cycle D=0.75
fs = 300kHz
L=1.3 µH
C=50 µF
Pload = 240W
V0=15V

The inductor current is defined with an average value given by Pload and Vo: I_{Lav} = 240/15=16A.

The switched voltage voi is defined by the duty cycle D as shown below.

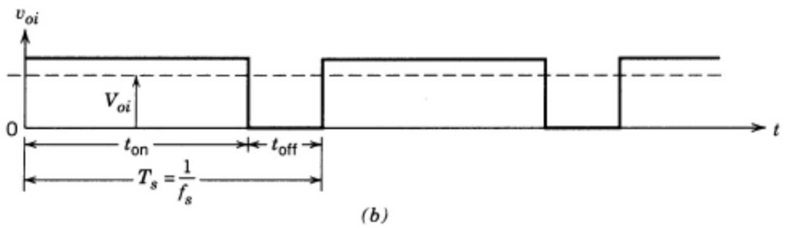

Figure 2

Ts=1/300kHz = 3.33µs

Ton = D*Ts=0.75*3.33µs = 2.5µs

The ripple current is defined by the voltage across the inductor, $v_L = v_{oi} - v_o$

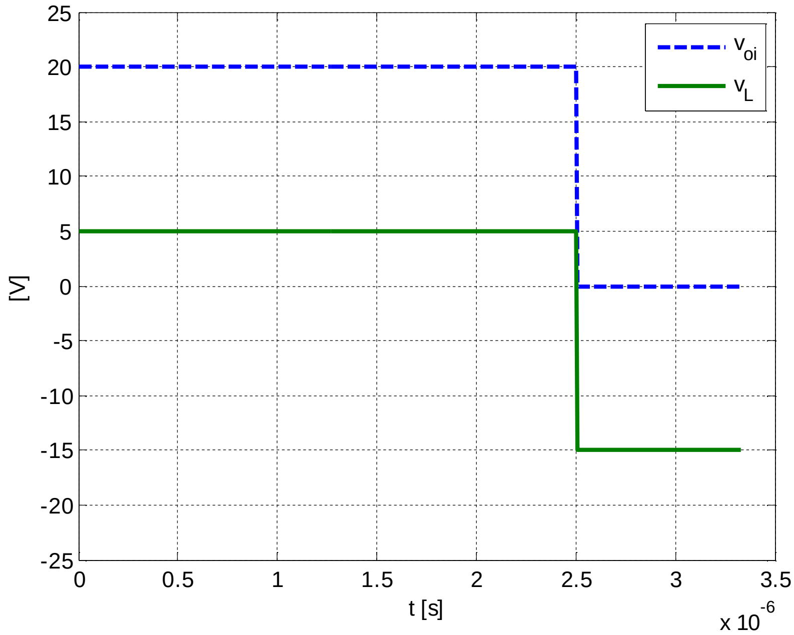

Figure 3

The current gets two slopes:

- Rising slope with a $dI1/dt = v_{L1}/L = 5/1.3$ [A/µs]
- Dropping slope $dI2/dt = v_{L2}/L = -15/1.3$ [A/µs]

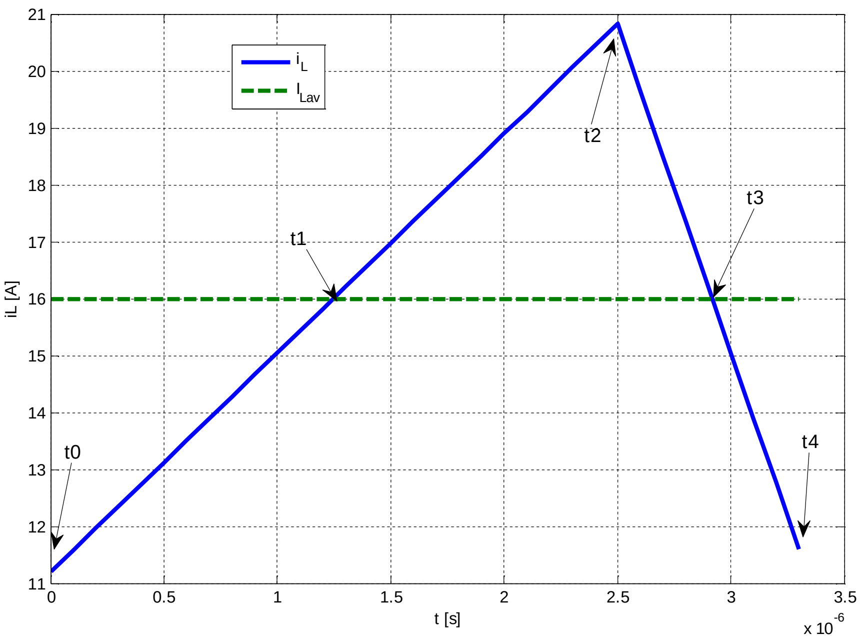

Figure 4

t1=1.25µs, t2=2.5µs, t3=2.9µs, t4=3.3µs

The RMS current of the inductor current can be divided in a dc-part (I_{Lav}) and a ripple-part (i_{Lr}). The total rms is defined as:

$$I_{LRMS} = \sqrt{I_{Lav}^2 + i_{LrRMS}^2}$$

The general formula for the RMS current is.

$$I = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

The ripple current RMS can be calculated by considering the four separate triangles, each with a magnitude of ΔI, for the time intervals (t0..t1), (t1..t2), (t2..t3), (t3..t4).

$$\Delta I = \frac{\mathrm{dI1}}{dt} t1 = \frac{5}{1.3} 1.25 = 4.8A$$

Considering the triangle (t1..t2) where we shift the time to get a zero point at t1, gives:

$$I_{LrRMS2} = \sqrt{\frac{1}{t2 - t1}\int_{t1}^{t2} \left(\Delta I \cdot \frac{t - t1}{t2 - t1}\right)^2 dt} = \{\tilde{t} = t - t1\} = \sqrt{\frac{1}{t2 - t1}\int_{0}^{t2-t1} \left(\Delta I \cdot \frac{\tilde{t}}{t2 - t1}\right)^2 dt}$$
$$= \sqrt{\frac{\Delta I^2}{(t2 - t1)^3}\left[\frac{\tilde{t}^3}{3}\right]_0^{t2 - t1}} = \frac{\Delta I}{\sqrt{3}}$$

The total RMS for the ripple current becomes the same since it is the sum of the four triangles divided by 4 to get the total RMS for a full cycle.

$$I_{LrRMS} = \frac{\Delta I}{\sqrt{3}} = \frac{4.8}{\sqrt{3}} = 2.8A$$

The total RMS current;

$$I_{LRMS} = \sqrt{I_{Lav}^2 + i_{LrRMS}^2} = \sqrt{16^2 + 2.8^2} = 16.2A$$